New Phases of $\text{QCD}_3$ and $\text{QCD}_4$

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IAS

Z. Komargodski, and NS, arXiv:1706.08755;
4d pure gauge $SU(N)$

- Large $N$: 1st order transition at $\theta = \pi$.
  - $CP$ (or $T$) is spontaneously broken there [Witten]

- Finite $N$: a one-form global symmetry associated with the center of the gauge group [Gaiotto, Kapustin, NS, and Willett].
  - At $\theta = \pi$: mixed ‘t Hooft anomaly between it and $CP$.
  - ‘t Hooft anomaly matching: cannot move continuously from confinement at $\theta = 0$ to confinement at $\theta = 2\pi$.
  - Specifically, at $\theta = \pi$ deconfinement, or broken $CP$, or TQFT
  - Simplest scenario (as at large $N$): a single 1st order phase transition at $\theta = \pi$.
  - Assume it (more exotic scenarios are in the paper).
4d pure gauge $SU(N)$ at finite $T$

- Because of the anomaly, cannot move continuously from confinement at $\theta = 0$ to confinement at $\theta = 2\pi$.

- Hence, $T_{\text{deconfinement}} \leq T_{CP}$.
QCD$_4$ with one quark ($N_f = 1$)

- No chiral symmetry for massless quarks, but at infinite $N$ a massless $\eta' 
  m_{\eta'}^2 = \frac{1}{N} \Lambda^2 + O\left(\frac{1}{N^2}\right)$ [Witten]

- With massive quarks
  $$m_{\eta'}^2 = \frac{1}{N} \Lambda^2 + Re(m) \Lambda + O(1/N^2, m^2, m/N)$$

- Therefore, even at finite $N$ can find an exactly massless $\eta'$.
- For large $|m|$ the same behavior as in the pure gauge system.
- First order transition in the complex $me^{i\theta}$ plane

massless $\eta'$

“4d Ising point”
For $N_f \geq N_{CFT}(N)$ a CFT or lack of asymptotic freedom

- For $1 < N_f < N_{CFT}$ a massless theory – $SU(N_f)$ sigma model
- Turn on equal masses. As for $N_f = 1$, a first order transition line at $\theta = \pi$, which ends at the massless point [Dashen]
Now we can add a Chern-Simons term with coefficient $k$.

For large $N_f$ a non-trivial fixed point [Appelquist and Nash]

Assume that this remains the case for $N_f \geq N^*(N, k)$

Topological phases at large $|m|$

What happens for $N_f < N^*(N, k)$?

Use recently suggested dualities...
CONJECTURES AHEAD
Dual descriptions

Many references. These are some of the recent ones.

...; Giombi, Yin; Aharony, Gur-Ari, Yacoby; Giombi, Minwalla, Prakash, Trivedi, Wadia, Yin; Maldacena, Zhiboedov; Aharony, Giombi, Gur-Ari, Maldacena, Yacoby; Jain, Minwalla, Sharma, Takimi, Wadia, Yokoyama; Minwalla, Yokoyama; Yokoyama; Jain, Mandlik, Minwalla, Takimi, Wadia, Yokoyama; Inbasekar, Jain, Mazumdar, Minwalla, Umesh, Yokoyama; Jain, Minwalla, Yokoyama; Gur-Ari, Yacoby; Son; Wang, Senthil; Metlitski, Vishwanath; Barkeshli, McGreevy; Radicevic; Aharony; Karch, Tong; NS, Senthil, Wang, Witten; Hsin, NS; Kachru, Mulligan, Torroba, Wang; Metlitski, Vishwanath, Xu; Aharony, Benini, Hsin, NS; Benini, Hsin, NS ...
Dual descriptions

\(N_f\) fermions coupled to \(N_f\) scalars at \(|\Phi|^4\) point coupled to

\[SU(N)_k \leftrightarrow U\left(\frac{N_f}{2} + k\right)_{-N}\]

We take \(N\) positive and \(k\) non-negative.

The scalars are in a generalized Wilson-Fisher fixed point or a gauged version of it.

For \(N_f > 2k\) apply time reversal and then \(k \rightarrow -k\) to find another possible duality

\[SU(N)_k \leftrightarrow U\left(\frac{N_f}{2} - k\right)_N\]
$N_f$ fermions coupled to $N_f$ scalars at $|\Phi|^4$ point coupled to

- $SU(N)_k \leftrightarrow U\left(\frac{N_f}{2} + k\right)_{-N}$
- $SU(N)_k \leftrightarrow U\left(\frac{N_f}{2} - k\right)_N$

Checks:
- For large $N$ and $k$ with fixed ratio explicit calculations and relation to AdS duals (only the first one)
- For $N_f = 0$ level/rank duality (only the first one)
- Relation to SUSY dualities of $[...; \text{Giveon, Kutasov}; ...]$
- Flow to fewer flavors
- Global symmetry and ‘t Hooft anomaly matching (for simplicity, limit to $N > 2$)
Problem for $N_f > 2k$

$N_f$ fermions coupled to $N_f$ scalars at $|\Phi|^4$ point coupled to

- $SU(N)_k \leftrightarrow U\left(\frac{N_f}{2} + k\right)_{-N}$
- $SU(N)_k \leftrightarrow U\left(\frac{N_f}{2} - k\right)_N$

A mass deformation in the fermionic theory leads to a gapped system $SU(N)_{k \pm N_f/2}$ (depending on the sign of the mass).

In the scalar theories for one sign it leads to a gapped theory $U\left(\frac{N_f}{2} + k\right)_{-N} \leftrightarrow SU(N)_{k + N_f/2}$, which is good.

But with the other sign the gauge symmetry is completely Higgsed and we end up with a massless theory.
$SU(N)_k$ with $N_f$ quarks \hspace{1cm} N_f \leq 2k

\[ U(k + N_f/2)_N \quad \text{with} \quad N_f \phi \]

\[ SU(N)_{k-N_f/2} \leftrightarrow U(k - N_f/2)_N \]

\[ SU(N)_{k+N_f/2} \leftrightarrow U(k + N_f/2)_N \]

\[ m < 0 \quad \text{Phase transition} \quad m > 0 \]
$SU(N)_k$ with $N_f$ quarks

$2k < N_f < N^* (N, k)$

$U(N_f/2 - k)_N$ with $N_f \phi$

$SU(N)_{k-N_f/2} \leftrightarrow U(N_f/2 - k)_N$

$SU(N)_{k+N_f/2} \leftrightarrow U(N_f/2 + k)_{-N}$

$U(N_f)$

$U(N_f)$

$m < 0$

$m > 0$

$\frac{U(N_f)}{U\left(\frac{N_f}{2} + k\right) \times U\left(\frac{N_f}{2} - k\right)}$ with $N\Gamma_{WZ}$
$SU(N)_k$ with $N_f$ quarks

$2k < N_f < N^* (N, k)$

- Three phases
  - For large $|m|$ semiclassical physics – gapped, topological.
  - For small $|m|$ a new quantum phase with global symmetry breaking
    \[ U(N_f)/U\left(\frac{N_f}{2} + k\right) \times U\left(\frac{N_f}{2} - k\right) \]
  - Each phase transition has a weakly coupled bosonic dual description

- The intermediate phase
  - Wess-Zumino term from the Chern-Simons term
  - For $k = 0$: \[ U(N_f) \rightarrow U(N_f/2) \times U(N_f/2) \] with a WZ term
  - Assuming this we can derive for other values of $k$
  - Skyrmions in the nonlinear model are monopoles in the bosonic theory and are the baryons in the fermionic theory
Return to 4d. Will soon relate to the 3d story.

Study the domain wall at the first order transition point at $\theta = \pi$.

- The theory on the domain wall needs to account for the different ‘t Hooft anomalies between the two sides of the wall.

- $SU(N)_{-1}$ (Same as on the domain wall between neighboring vacua of $\mathcal{N} = 1$ SUSY $SU(N)$ pure gauge theory.)
QCD₄ with $N_f = 1$

Study the domain wall at the transition.

• No anomaly argument
• For large negative $m e^{i \theta}$, expect $SU(N)_{-1}$
• For small mass, should be trivial – use the $\eta'$ theory
• There must be a phase transition on the domain wall.
• Same phases as in 3d

\[ SU(N)_{-1/2} \text{ with } N_f = 1 \quad \psi \leftrightarrow U(1)_N \text{ with } N_f = 1 \quad \phi \]
QCD$_4$ with $1 < N_f < N_{CFT}$

Study the domain wall at the transition.

- For large negative $m^{N_f} e^{i\theta}$ expect $SU(N)_{-1}$
- For small mass, $CP^{N_f-1}$ with $N\Gamma_{WZ}$ – use the chiral Lagrangian
- There must be a phase transition on the domain wall
- Same phases as in 3d

$$SU(N)_{-1+N_f/2} \text{ with } N_f \text{ quarks} \leftrightarrow U(1)_N \text{ with } N_f \text{ scalars}$$

- Consistent with the intermediate phase in the 3d discussion
Summary

QCD$_4$

- $N_f = 0$
  - New parity anomaly
  - Phase transition at $\theta = \pi$ for all $N$
  - $T_{\text{deconfinement}} \leq T_{CP}$

- $N_f = 1$
  - Massless $\eta'$ for all $N$

- All $N_f$
  - First order transition with domain walls
Summary

QCD\(_3\) with a Chern-Simons term

- Large \(N_f\): a second order transition separating two gapped topological phases
- Small \(N_f\): same as large \(N_f\), but with a bosonic dual
- Intermediate \(N_f\): three phases. Two of them are gapped and topological. Intermediate phase with global symmetry breaking.

Consistent with the analysis of domain walls in QCD\(_4\)

Interesting generalization to \(SO(N)/Spin(N)\) and \(Sp(N)\) gauge theories – new insights about confinement.