The p-spin glass model: a holographer's perspective

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Based on **2106.03838** with **Tarek Anous (Amsterdam)**

Outline

- Review: SYK, chaos, glassiness
- The p-spin glass model
    - Replica symmetry breaking
    - Conformal limits
    - Quantum chaos (OTOCs)
- Holographic speculations
- Conclusion

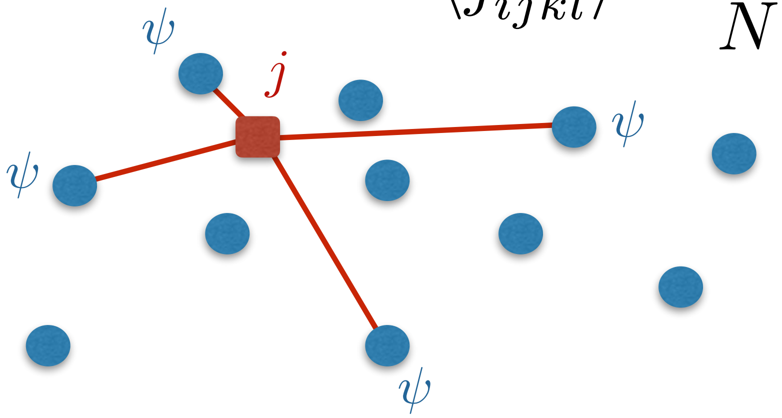
Review:
SYK, chaos, glassiness

Reminder: SYK model

- N Majorana fermions with random, Gaussian couplings

$$H = -\sum_{ijkl}^{N} j_{ijkl}\,\psi_i\psi_j\psi_k\psi_l$$

$$\langle j_{ijkl}^2\rangle \sim \frac{\mathcal{J}^2}{N^3}$$



- Solvable for $N \gg \beta J \gg 1$

- "Mean field" description at large N in terms of bilocal 2-point function

$$G(\tau,\tau') = \frac{1}{N}\sum_{i=1}^{N}\langle\psi_i(\tau)\psi_i(\tau')\rangle$$

- $\beta J \gg 1$: $S_{\text{eff}}[G]$ is approximately $\text{diff}(S^1)$ invariant: $\tau \to f(\tau)$

  - The saddle point solution breaks $\text{diff}(S^1) \to SL(2,\mathbb{R})$:

$$G_c(\tau-\tau') \propto \frac{1}{(\tau-\tau')^{2/q}}$$

[Sachdev/Ye '93] [Kitaev '15]
[Maldacena/Stanford '16] ...

- The pseudo-Goldstone associated with reparametrizations $\tau \to f(\tau)$ has a ‘Schwarzian’ effective action:

$$I_{\text{Schw.}} \propto -\frac{N}{\mathcal{J}} \int d\tau \, \{f(\tau), \tau\}$$

- This action also describes the boundary degree of freedom associated with dilaton gravity in AdS_2

[Almheiri/Polchinski ’14]
[Maldacena/Stanford/Yang ’16]

- The symmetry breaking pattern also implies a near-extremal entropy of the form

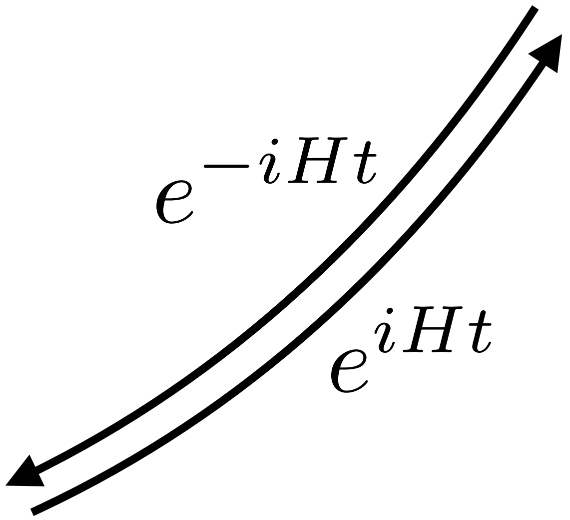
$$S = S_0 + \# \, \frac{N}{\beta \mathcal{J}}$$

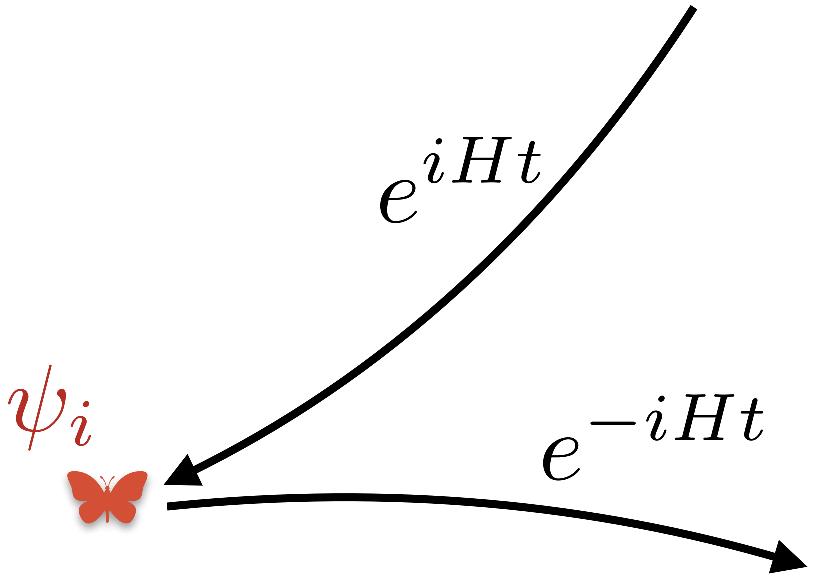
from Schwarzian

- Finally: the Schwarzian theory describes a universal contribution to out-of-time-order correlation functions (OTOCs)

Quantum butterfly effect

$\psi_i(t) = e^{iHt}\,\psi_i\,e^{-iHt}$ is ‘complicated’ even if ψ_i was ‘simple’

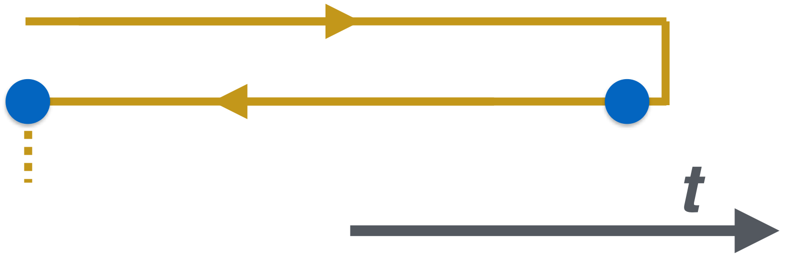




$$e^{iHt}\,\psi_i\,e^{-iHt} = \psi_i + it[H,\psi_i] - \frac{t^2}{2}[H,[H,\psi_i]] + \ldots$$

To quantify this, compare the following two states:

$$|\psi_i(t)\psi_j(0)\rangle \qquad\qquad |\psi_j(0)\psi_i(t)\rangle$$



- The OTOC quantifies how much overlap these states have:

$$\text{OTOC } = \langle \psi_i(t)\psi_j(0)|\psi_i(t)\psi_j(0)\rangle$$

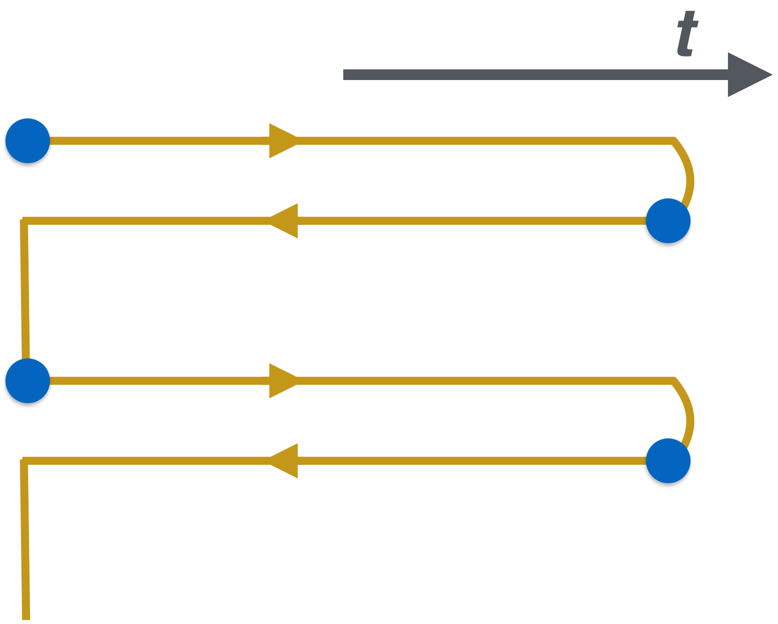
- Chaos causes this overlap to decay to 0 quickly

[Larkin/Ovchinnikov '68] [Kitaev '14] [Shenker/Stanford '14]

- The soft mode contribution in SYK is maximally chaotic:

$$\text{OTOC}_{\text{SYK}} \sim a_0 - \frac{a_1}{N}\, e^{\lambda_L(\beta J)\, t} + \dots$$

$$\lambda_L(\beta J \gg 1) = \lambda_L^{\max} \equiv \frac{2\pi}{\beta}$$



[Maldacena/Shenker/Stanford '15] [Kitaev '15]

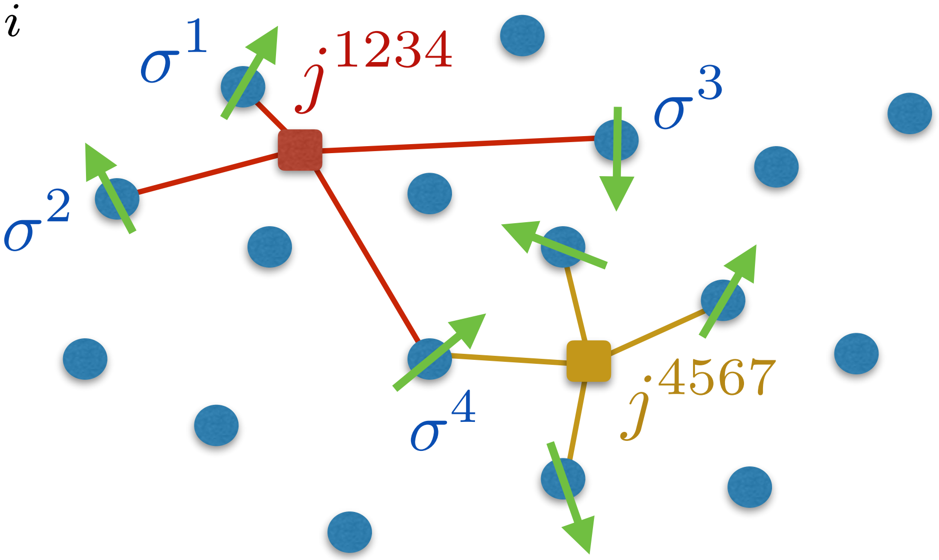
- Another signature of black hole physics

[Hayden/Preskill '07] [Sekino/Susskind '08] [Shenker/Stanford '13]

Spin glasses

- I will discuss a related model with some similar and some new features

- Roughly: replace Majorana fermions by bosonic spins σ^i constrained to live on an *N*-dimensional sphere
  [Crisanti/Sommers '92] [Cugliandolo/Grempel/da Silva Santos '01]

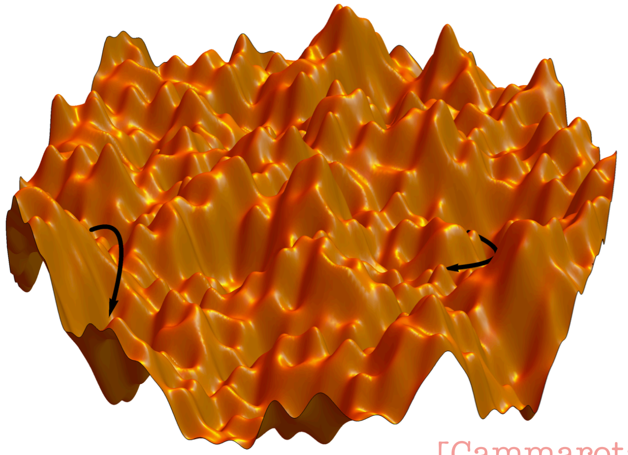


The model has <u>two</u> dimensionless couplings:

$$\beta J, \; MJ$$

~thermal fluctuations ~quantum fluctuations

- Spin glass phase: if both thermal & quantum fluctuations are weak, the system gets "stuck" in metastable states / valleys



[Cammarota]

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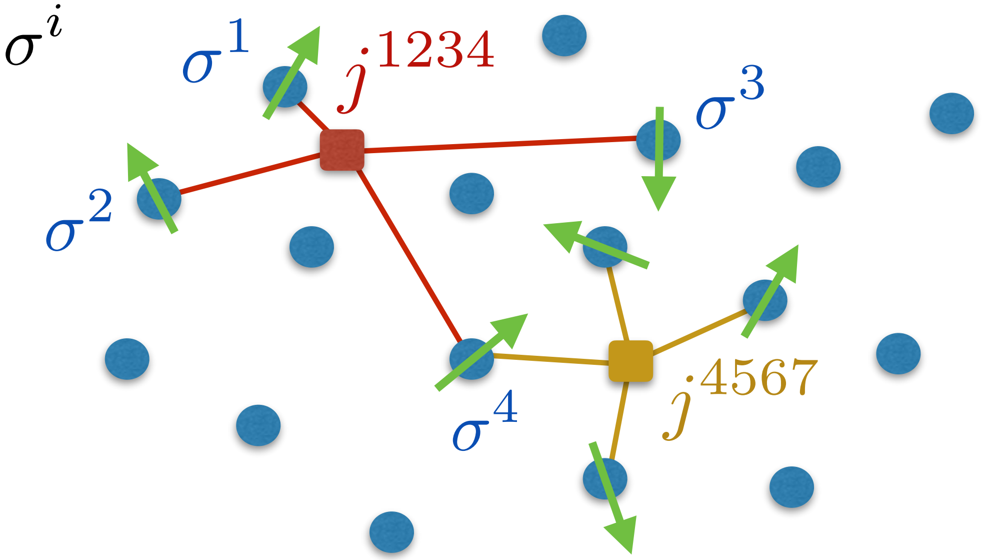

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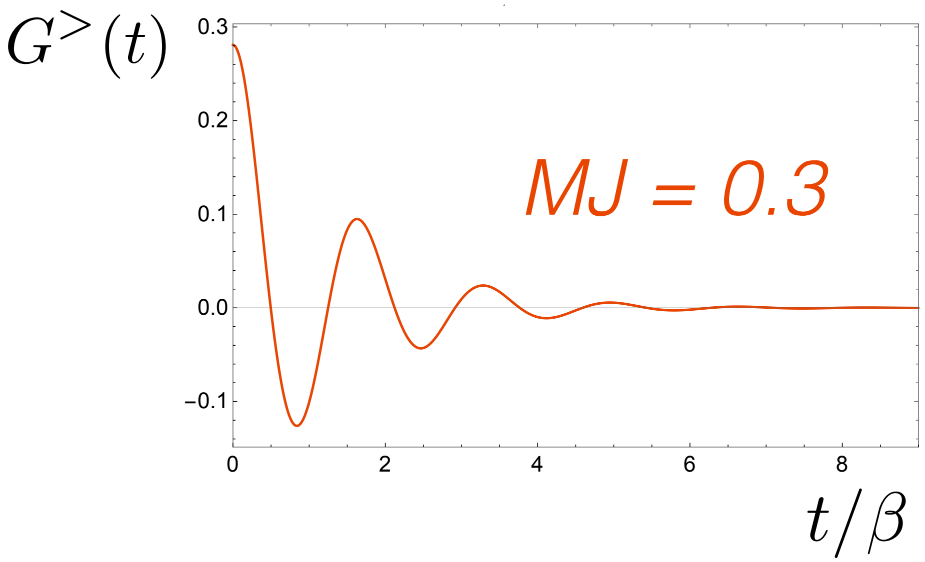
- Useful order parameter: $u \equiv \frac{1}{N}\sum_{i=1}^{N} \overline{\langle \sigma^i \rangle^2}$ [Edwards/Anderson '75]

"paramagnetic" phase: $u = 0$

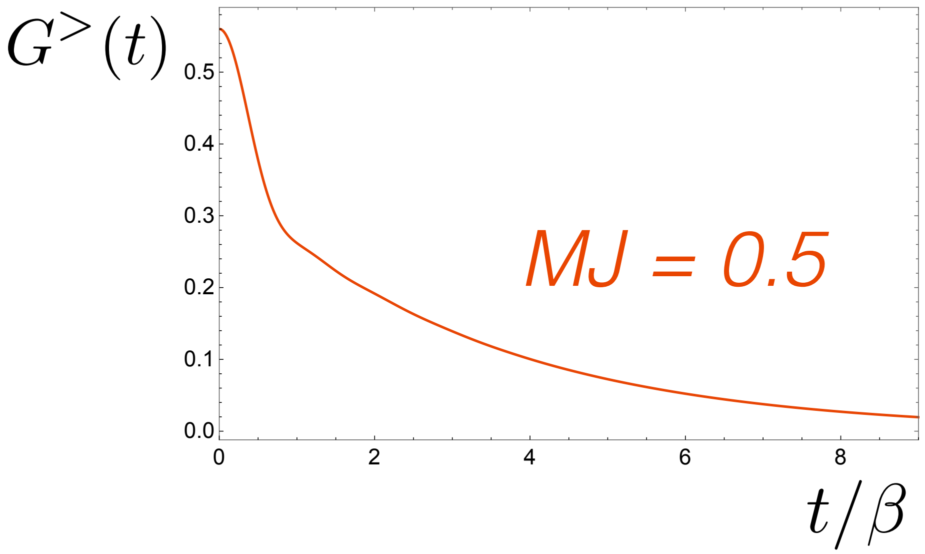
"spin glass" phase: $u > 0$

- E.g.: two-point correlation function $G^{>}(t) = \frac{1}{N}\sum \overline{\langle \sigma^i(t)\sigma^i(0)\rangle}$ (fixed βJ):



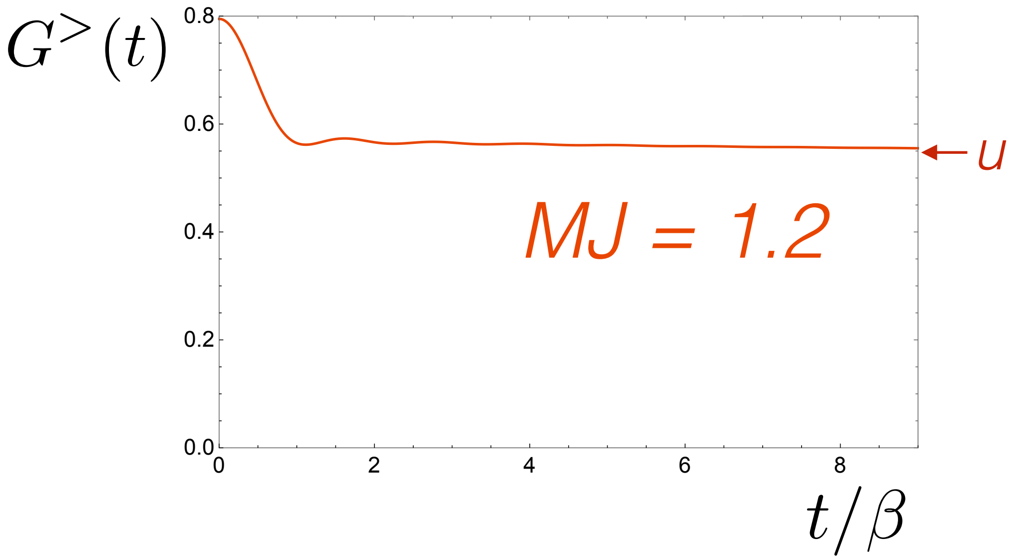
❖ Far above SG transition:

- Strong quantum fluctuations
- Relatively fast decay



❖ Near SG transition:

- Competition between thermal & quantum effects
- Slow decay ("two-step" relaxation)



❖ Below SG transition:

- Equilibration time diverges
- Asymptotic value: order parameter *u>0*

Goals

- Characteristic features of SG phase: slow dynamics, many metastable states, inability to reach equilibrium, loss of ergodicity, …

  - Universal features of the low temperature thermodynamics?

  - Interplay with other chaos characteristics such as OTOCs?

  - Emergent reparametrization symmetry?

  - Can we incorporate this in the $nAdS_2/nCFT_1$ paradigm?

The p-spin glass model

The p-spin model

$$P(J_{i_1\ldots i_p}) \propto \exp\left[-\frac{N^{p-1}}{p!}\frac{J^2_{i_1\ldots i_p}}{J^2}\right]$$

$$Z[J_{i_1\ldots i_p}] = \int D\sigma_i Dz \ \exp\left\{ -\int_0^\beta d\tau \left[\frac{M}{2}\dot{\sigma}_i(\tau)\dot{\sigma}_i(\tau) + \sum_{i_1<\ldots<i_p} J_{i_1\ldots i_p}\sigma_{i_1}(\tau)\ldots\sigma_{i_p}(\tau)\right] + i\int_0^\beta d\tau\, z(\tau)\left(\sum_{i=1}^{N}\sigma_i(\tau)\sigma_i(\tau) - N\right)\right\}$$

“spherical constraint”

- Dimensionless parameters: $\beta J, \ MJ$
- Nonlinear sigma-model with fixed size spherical target space
- Spherical constraint will be crucial for stability of such a bosonic model

[Crisanti/Sommers ’92] [Cugliandolo/Grempel/da Silva Santos ’01]

- First goal: compute disorder averaged ("quenched") free energy

$$\beta\overline{F} = -\int dJ_{i_1 \ldots i_p} P(J_{i_1 \ldots i_p}) \log Z[J_{i_1 \ldots i_p}]$$

- Strategy: use *replica trick*

$$\log Z = \lim_{n \to 0} \partial_n Z^n$$

$$\beta\overline{F} = -\lim_{n \to 0} \partial_n \overline{Z^n}$$

$$\overline{Z^n} = \int dJ_{i_1 \ldots i_p} P(J_{i_1 \ldots i_p}) \int D\sigma_i^a Dz^a \, \exp\left\{ -\int_0^\beta d\tau \left[\frac{M}{2} \dot{\sigma}_i^a(\tau) \dot{\sigma}_i^a(\tau) + \sum_{i_1 < \ldots < i_p} J_{i_1 \ldots i_p} \sigma_{i_1}^a(\tau) \ldots \sigma_{i_p}^a(\tau) \right] + i \int_0^\beta d\tau \, z^a(\tau) \left(\sum_{i=1}^N \sigma_i^a(\tau) \sigma_i^a(\tau) - N \right) \right\}$$

- Index *a = 1,…,n* labels the replica copy

- Introduce collective bilocal field with replica indices:

$$Q_{ab}(\tau,\tau') = \frac{1}{N}\sum_{i=1}^{N} \sigma_i^a(\tau)\sigma_i^b(\tau')$$

- After integrating out the disorder and the spins, we get an effective action for Q_{ab}. Schwinger-Dyson equation:

$$-\delta_{ab}\left[\frac{M}{2}\partial_\tau^2 + iz^a(\tau)\right] Q_{ab}(\tau,\tau') - \frac{pJ^2}{4}\int_0^\beta d\tau''\, Q_{ac}^{p-1}(\tau,\tau'')Q_{cb}(\tau'',\tau') = \frac{1}{2}\delta_{ab}\,\delta(\tau-\tau')$$

- Some notable features:
  - $Q_{a\neq b}(\tau,\tau') \to \frac{1}{N}\sum_i \overline{\langle\sigma_i^a(\tau)\rangle\langle\sigma_i^b(\tau')\rangle}$ can be non-zero
  - Two-derivative kinetic term with tunable coefficient M
  - Lagrange multiplier field $z^a(\tau)$

$$-\delta_{ab}\left[\frac{M}{2}\partial_\tau^2 + iz^a(\tau)\right]Q_{ab}(\tau,\tau') - \frac{pJ^2}{4}\int_0^\beta d\tau''\, Q_{ac}^{p-1}(\tau,\tau'')Q_{cb}(\tau'',\tau') = \frac{1}{2}\delta_{ab}\,\delta(\tau-\tau')$$

- "1-step replica symmetry breaking" ansatz (Parisi):

$$Q_{ab}(\tau,\tau') = q_r(\tau,\tau')\,\delta_{ab} + \underbrace{\left(\begin{array}{ccccccc} \multicolumn{3}{c}{\overset{m\times m}{\longleftrightarrow}} & & & & \\ u & u & u & & & & \\ u & u & u & & s & & \cdots \\ u & u & u & & & & \\ & & & u & u & u & \\ & s & & u & u & u & \\ & & & u & u & u & \\ & \vdots & & & & & \ddots \end{array}\right)}_{n\times n}$$

- Diagonal: $q(\tau,\tau') \equiv q_r(\tau,\tau') + u$ subject to $q(\tau,\tau) = 1$

- *u:* replica overlap (same cluster)

- *s:* replica overlap (different clusters) —> can set *s=0* consistently

- *m:* block size parameter

- SD equation for Q_{ab} gives equation of motion for $q(\tau, \tau')$ & u

  - After some rewritings, the equation of motion for $\hat{q}_r(k \neq 0)$ is:

$$\frac{1}{\hat{q}_r(k)} - \frac{1}{\hat{q}_r(0)} = M\left(\frac{2\pi k}{\beta}\right)^2 - J^2\left(\hat{\Lambda}_r(k) - \hat{\Lambda}_r(0)\right)$$

$$\Lambda_r(\tau) = \frac{p}{2}\left[(q_r(\tau) + u)^{p-1} - u^{p-1}\right]$$

  - In addition: 2 algebraic equations for $\hat{q}_r(0)$ and u

- What about *m* ?

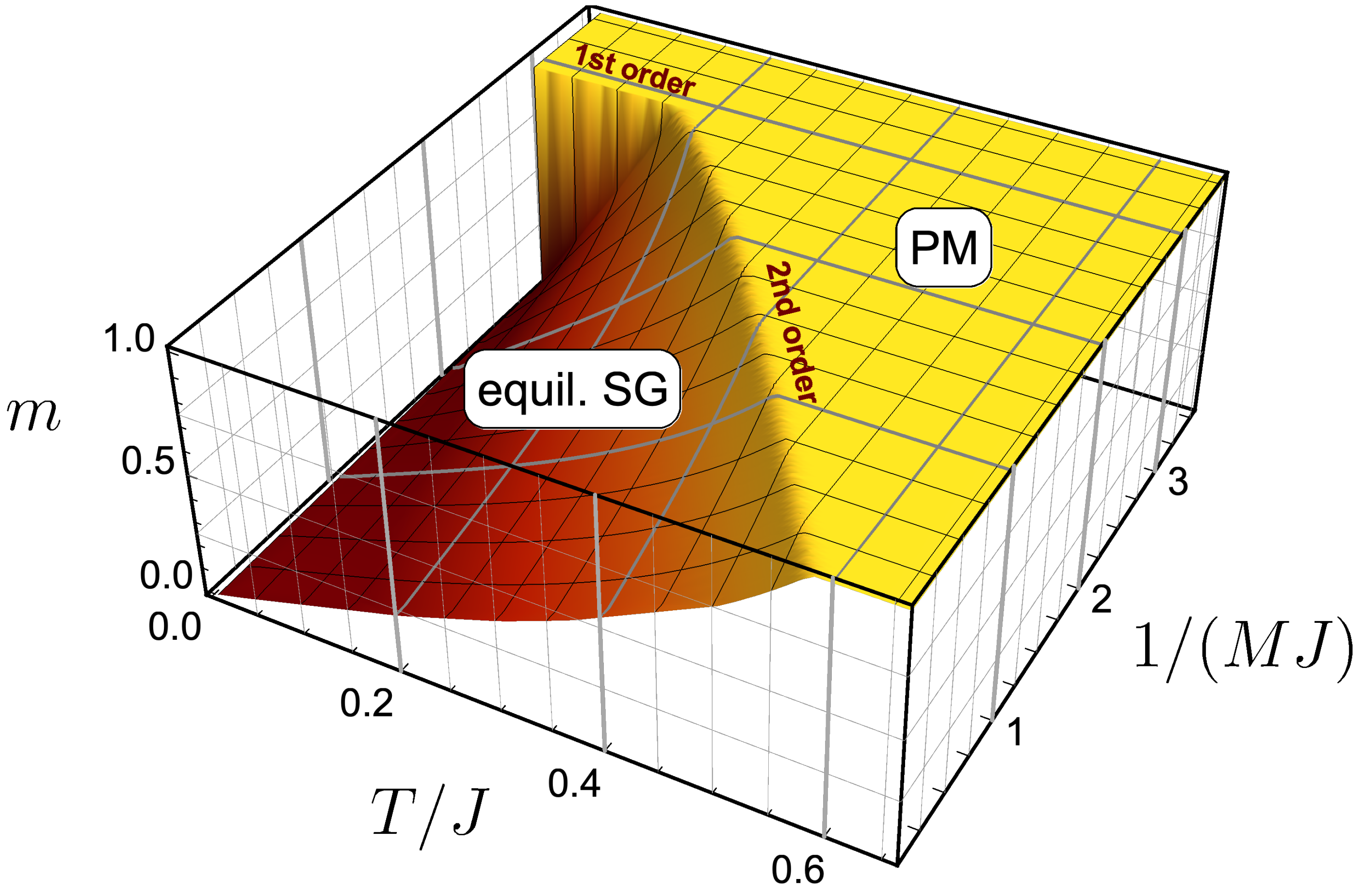
  - For replica symmetric solutions, *m* plays no role.
  - In spin glass, we could impose $\frac{\delta S_{eff}}{\delta m} = 0$ (*"equilibrium spin glass"*)

n.b.: we will later see a different, more physical condition

(recall: glassy physics means inability to reach equilibrium)

Phase diagram

- Solving these equations numerically gives:
  - Small βJ or MJ : paramagnetic phase ($u = 0,\ \ m = 1$)
  - Large βJ and MJ : spin glass (RSB: $0 < u, m < 1$)



▸ Let’s analyze low temperatures analytically…

Conformal paramagnetic solution

- For unbroken replica symmetry *(u=0, m=1):* seemingly similar to SYK

  - At strong coupling $\beta J \gg 1$, small frequencies:

$$\delta(\tau,\tau') \approx -J^2 \int_0^\beta d\tau'' \Lambda_r(\tau,\tau'')\, q_r(\tau'',\tau')\,, \qquad \Lambda_r(\tau,\tau') = \frac{p}{2}\, q_r(\tau,\tau')^{p-1}$$

  - Reparametrization invariance!
    —> spontaneously broken by the conformal solution:

$$q^c(\tau,\tau') \sim \left[\frac{\pi}{\beta \sin\left(\frac{\pi(\tau-\tau')}{\beta}\right)}\right]^{\frac{2}{p}}$$

  - Leads to Schwarzian action etc.

Conformal paramagnetic solution

- However, this solution is actually unstable

- Conformal symmetry determines the spectrum of operators appearing in the $\sigma^i\sigma^i$ OPE

[Kitaev '15] [Polchinski/Rosenhaus '16]
[Tikhanovskaya/Guo/Sachdev/Tarnopolsky '20]

Find a tower of allowed operators $\mathcal{O}_h$. E.g. for *p=3:*

$$h_0 = 2, \qquad h_{n=1,2,3,\dots} = 4.303,\ 6.404,\ 8.456,\ 10.489,\ 12.511,\ \dots$$

… and: an operator with complex dimension $h = \frac{1}{2} \pm 1.560\, i$

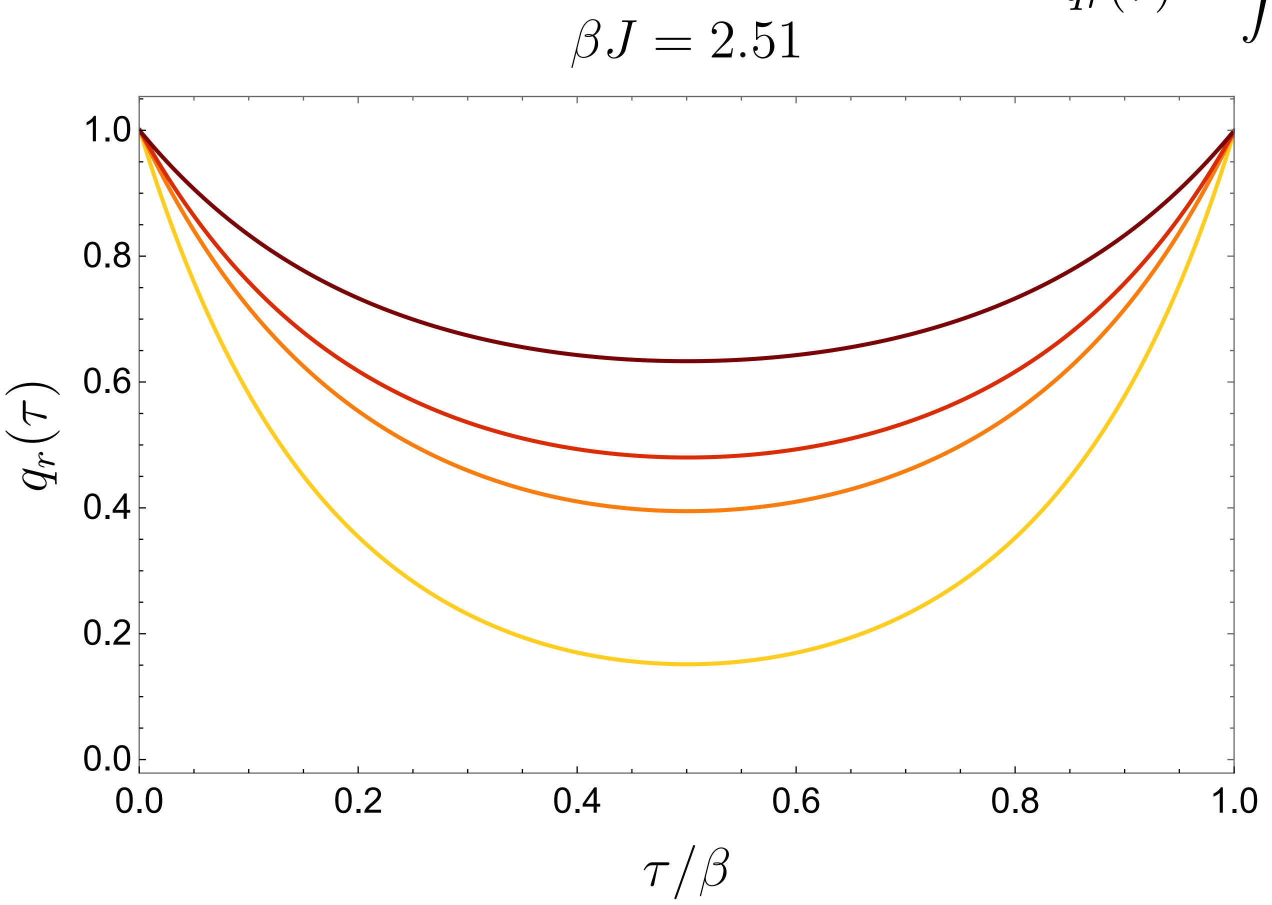
c.f. [Giombi/Klebanov/Tarnopolsky '17]

- Indeed, there exists another paramagnetic solution, which has no conformal limit

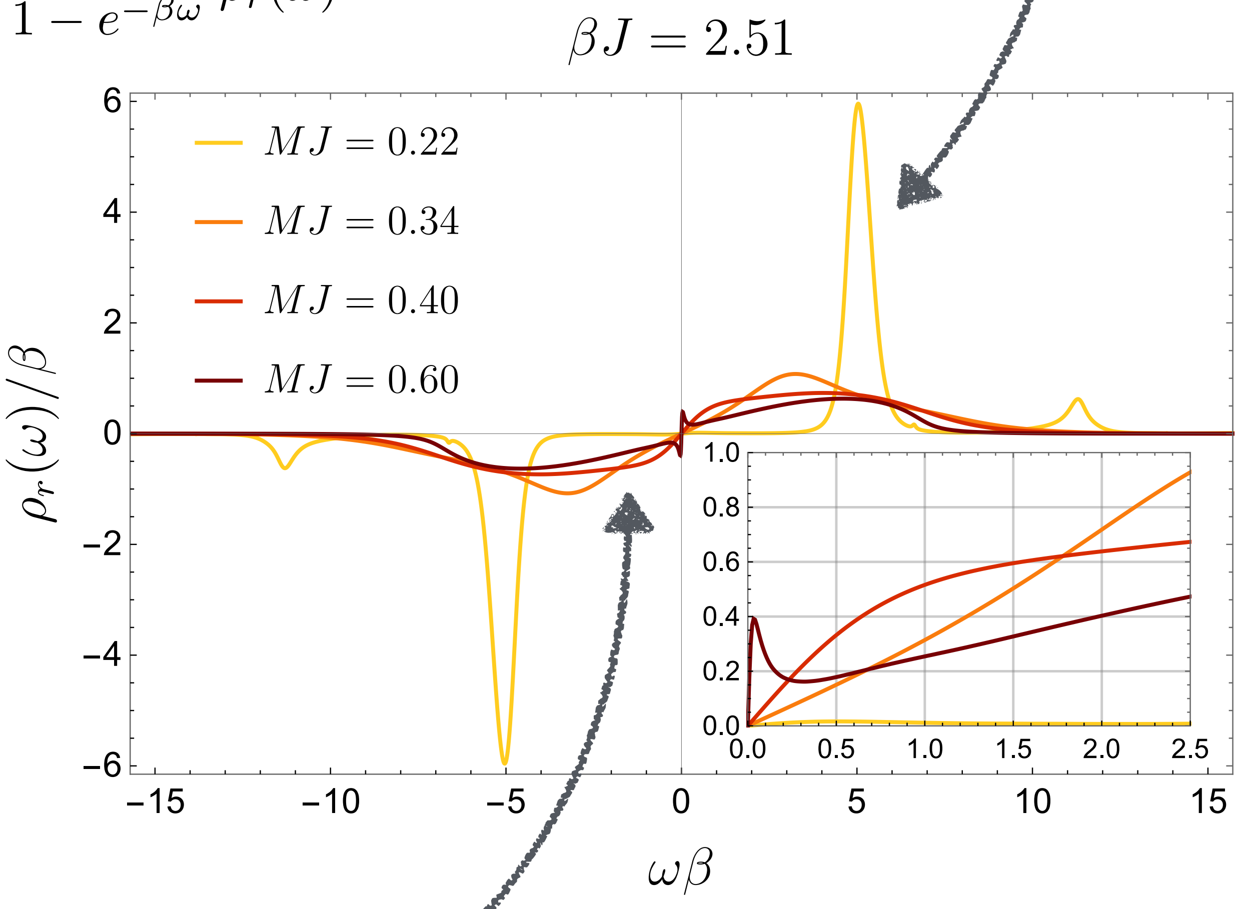
—> construct numerically

Physical paramagnetic solution

$$q_r(\tau) \equiv \int \frac{d\omega}{2\pi} \, \frac{e^{-\omega\tau}}{1-e^{-\beta\omega}} \, \rho_r(\omega)$$



$\beta J = 2.51$



$\beta J = 2.51$

far from SG transition

close to SG transition

- Gap closes near spin glass transition
- Is there a (physical) conformal solution in the SG phase?

Conformal spin glass

Approximate analytical solution

$$\frac{1}{\hat{q}_r(k)} - \frac{1}{\hat{q}_r(0)} = M\left(\frac{2\pi k}{\beta}\right)^2 - J^2\left(\hat{\Lambda}_r(k) - \hat{\Lambda}_r(0)\right)$$

- Can solve analytically at strong coupling (“deep spin glass”)

- Recall $q(\tau) \equiv q_r(\tau) + u$ and expand self-energy for $q_r(\tau) \ll u$:

$$\Lambda_r(\tau) = \frac{p}{2}\left[(q_r(\tau) + u)^{p-1} - u^{p-1}\right] = \frac{p(p-1)}{2}\, q_r(\tau)\, u^{p-2} + \ldots$$

- At first non-trivial order we can solve e.o.m. analytically:

$$\boxed{\frac{\hat{q}_r^{\approx}(\omega)}{\hat{q}_r(0)} = 1 + 2\gamma^2\omega^2 - 2\sqrt{\gamma^2\omega^2 + \gamma^4\omega^4}} \qquad \gamma \equiv \sqrt{\frac{M\hat{q}_r(0)}{4}} \qquad \omega = \frac{2\pi k}{\beta}$$

c.f. [Read/Sachdev/Ye ’95] [Cugliandolo/Grempel/da Silva Santos ’01] ...

$$\boxed{\frac{\hat{q}_r^{\approx}(\omega)}{\hat{q}_r(0)} = 1 + 2\gamma^2\omega^2 - 2\sqrt{\gamma^2\omega^2 + \gamma^4\omega^4}}$$

$$\gamma \equiv \sqrt{\frac{M\hat{q}_r(0)}{4}}$$

- Low frequency limit:

$$\frac{\hat{q}_r^{\approx}(\omega)}{\hat{q}_r(0)} = 1 - 2\gamma|\omega| + \dots$$

zero mode

conformal term: $q_r^c(\tau) = \frac{8\gamma^3}{M\pi}\frac{1}{\tau^2}$

—> conformal (dimension $\Delta = 1$)

- High frequency limit:

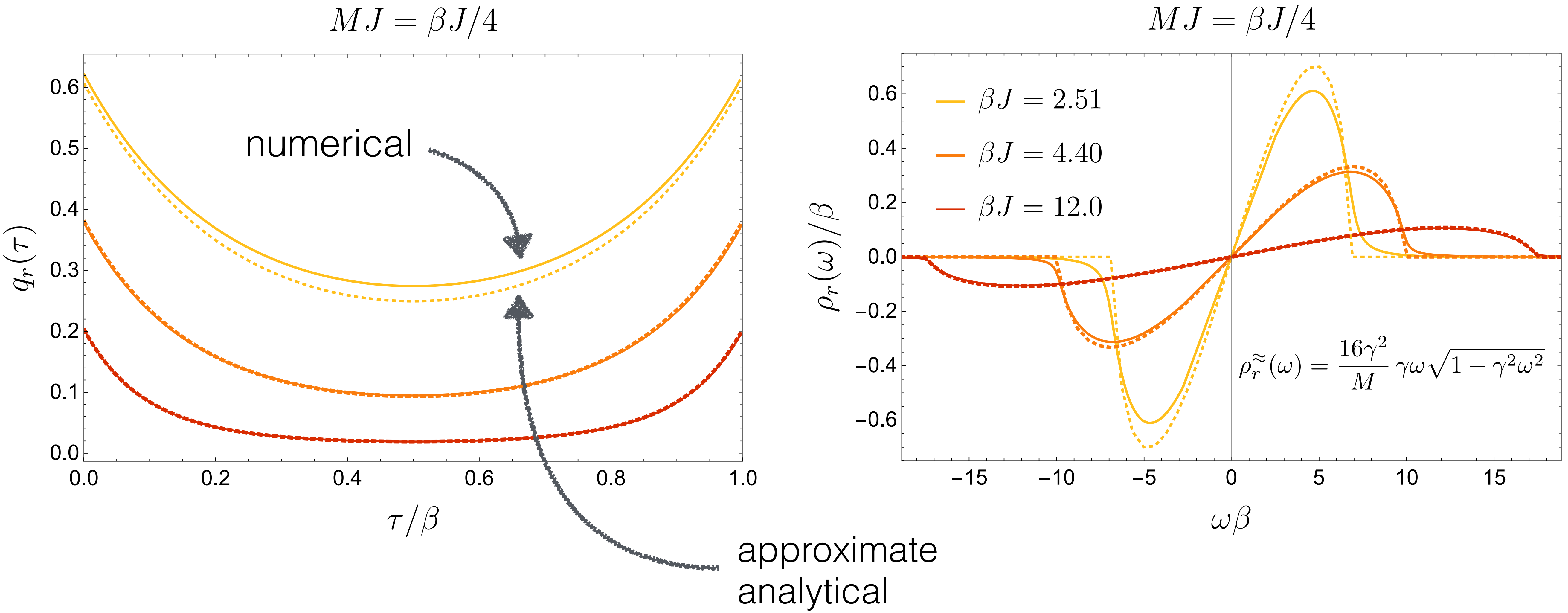
$$\frac{\hat{q}_r^{\approx}(\omega)}{\hat{q}_r(0)} = \frac{1}{4\omega^2} + \dots$$

—> finite; can consistently impose $q_r^{\approx}(\tau = 0) = 1 - u$

- Approximate solution contains UV & IR information consistently
- All parameters determined perturbatively

$$\boxed{\frac{\hat{q}_r^{\approx}(\omega)}{\hat{q}_r(0)} = 1 + 2\gamma^2\omega^2 - 2\sqrt{\gamma^2\omega^2 + \gamma^4\omega^4}}$$

$$\gamma \equiv \sqrt{\frac{M\hat{q}_r(0)}{4}}$$



- Spin glass physics for small temperatures is governed by conformal properties: power law scaling, gapless spectrum, …

Subtletly: the value of *m*

- "UV completing" the conformal solution fixes *m*. But: $\frac{\delta S_{\text{eff}}}{\delta m} \neq 0$

  —> not quite in equilibrium

  —> useful to think of *m* as an external thermodynamic parameter (like β)

  —> tune *m* to the value required for this solution to exist

- Consider a thermodynamic ensemble where we consider *m* physical copies of the system (replica symmetry is explicitly broken):

$$\frac{S_{\text{eff}}(Q^*)}{Nn} = \beta m \Phi$$

[Monasson '95]

[Mezard '99]

- Consider a thermodynamic ensemble where we consider *m* physical copies of the system (replica symmetry is explicitly broken):

$$\frac{S_{\text{eff}}(Q^*)}{Nn} = \beta m \phi$$

[Monasson '95]

[Mezard '99]

$$f = \partial_m (m\phi) \qquad \Sigma = -\partial_{1/m}(\beta\phi) = \beta m (f - \phi)$$

'complexity': counts metastable states at free energy *f*

$$\Sigma(Q^*) = \frac{1}{2}\log(p-1) - \frac{p-2}{p}$$

- As an analogy, recall usual thermodynamic identities:

$$\frac{S_{\text{eff}}(Q^*)}{Nn} = \beta f \qquad e = \partial_\beta(\beta f) \qquad s = -\partial_T f = \beta(e - f)$$

Marginal stability criterion

▶ Another way to determine *m:*

- Consider fluctuations:

$$Q_{ab} = Q^*_{ab} + \delta Q_{ab} \quad \longrightarrow \quad \delta^{(2)} S_{eff}[Q] = N \int d\tau d\tau' \, \delta Q_{ab}(\tau) \, G_{ab,cd}(\tau, \tau') \, \delta Q_{cd}(\tau')$$

- For physically sensible solutions: $G_{ab,cd}$ should have eigenvalues ≥*0*

- Determine *m* by demanding existence of a vanishing eigenvalue

—> equivalently: $\mathcal{J}^2 u^{p-2} = (\hat{q}_r(0))^{-2}$ "condition of marginal stability"

—> coincides with the value in the conformal solution!

[Cugliandolo/Kurchan '93]

[Georges/Parcollet/Sachdev '00]

Recap: signs of gravity?

- Spin glass phase has emergent conformal symmetry at strong coupling. Marginal mode on top of self-overlap:

$$q(\tau,\tau') = u + q_r^c(\tau,\tau') + \dots$$

  - Reparametrization symmetry, gapless spectrum, …

- An extensive number of nearby states is counted by the entropy-like density

$$m\bar{s} + \Sigma = \left[\frac{1}{2}\log(p-1) - \frac{p-2}{p}\right] + \dots$$

Let us now consider the quantum Lyapunov exponent as another diagnostic of gravitational physics…

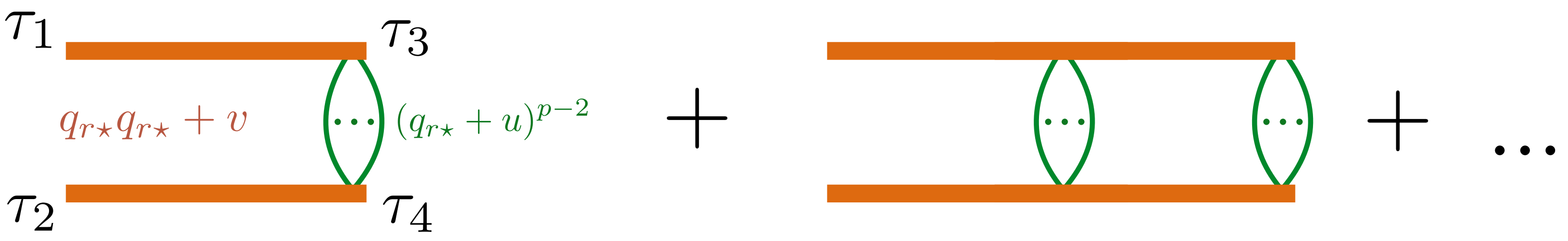
Quantum chaos

Euclidean 4-point function

- Consider $\mathcal{F}(\tau_1,\tau_2,\tau_3,\tau_4) \equiv \frac{1}{N^2}\sum_{i,j}\left\langle \sigma_i(\tau_1)\sigma_i(\tau_2)\sigma_j(\tau_3)\sigma_j(\tau_4)\right\rangle \equiv 1 + \frac{1}{N}\mathcal{F}_{\text{conn.}}$

- Connected piece is built recursively from ‘ladder diagrams’:

$$\mathcal{F}_{\text{conn.}} = \frac{1}{(\beta\mathcal{J})^2}\sum_{n\geq 1}\tilde{K}^n =$$



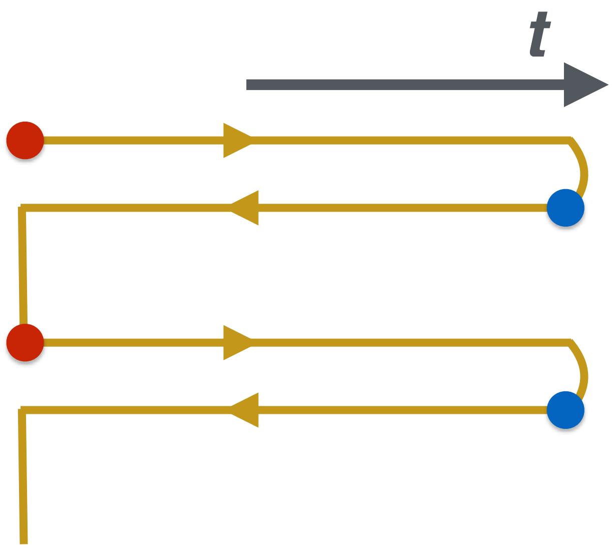
$$\tilde{K}(\tau_1,\tau_2;\tau_3,\tau_4) = (\beta\mathcal{J})^2\,[q_{r\star}(\tau_{13})\,q_{r\star}(\tau_{24})+v]\,[q_{r\star}(\tau_{34})+u]^{p-2}$$

‘rails’ offset by 4-point version of Edwards-Anderson parameter:

$$v = \frac{p}{p-2}\,\frac{m^2u^2}{m(p-1)^2-p(p-2)}$$

c.f. [Kitaev ’15][Maldacena/Stanford ’16] ... [Cheng/Swingle ’19][Mao/Chowdhury/Senthil ’19]

Out-of-time-order correlator



- We wish to compute the OTOC $(t_1 \approx t_2 \gg t_3 \approx t_4)$:

$$\mathcal{F}(t_1, t_2, t_3, t_4) \equiv \frac{1}{N^2} \sum_{i,j} \left\langle \sigma_i\left(t_1\right) \sigma_j\left(t_3\right) \rho_\beta^{1/2} \sigma_i\left(t_2\right) \sigma_j(t_4) \rho_\beta^{1/2} \right\rangle$$

  - Analytically continue the Euclidean result. Retarded ladder kernel:

$$\tilde{K}_{\mathrm{ret}}(t_1, t_2; t_3, t_4) = (\beta \mathcal{J})^2 \, q^R_{r\star}(t_{13}) \, q^R_{r\star}(t_{24}) \, \left[q^{>}_{r\star}(t_{34} - i\beta/2) + u\right]^{p-2}$$

- Condition for exponential growth of the OTOC:

$$\mathcal{F}_{\mathrm{conn.}}(t_1, t_2; t_3, t_4) = \frac{1}{\beta^2} \int dt dt' \, \tilde{K}_{\mathrm{ret}}(t_1, t_2; t, t') \, \mathcal{F}_{\mathrm{conn.}}(t, t'; t_3, t_4)$$

  - Two ways to solve this eigenvalue problem:
    (1) perturbatively in the conformal limit
    (2) numerically

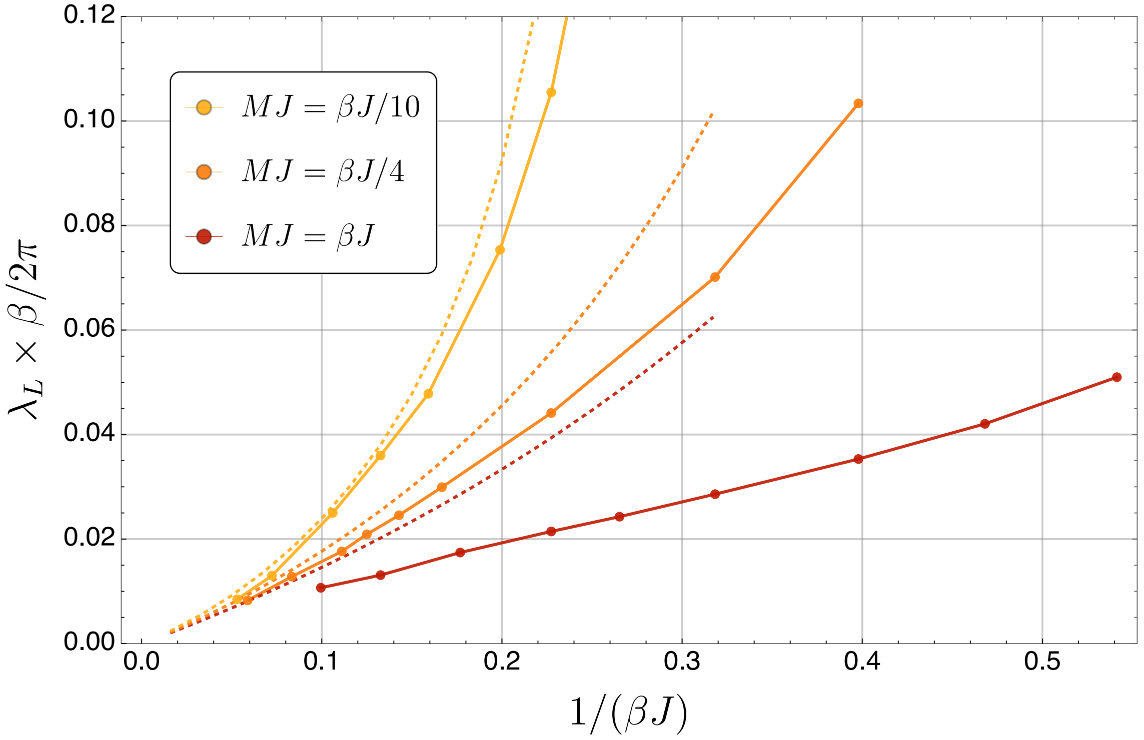
Perturbative analysis (conformal SG)

- Exponential growth ansatz: $\mathcal{F}_{\text{conn.}}(t_1, t_2, 0, 0) \sim f(t_1 - t_2)\, e^{\lambda_L (t_1+t_2)/2}$

- For $\beta J \sim MJ \gg 1$: conformal perturbation theory around $q^{\approx}$ turns exponential growth condition into a 'Pöschl-Teller' Schrödinger problem:

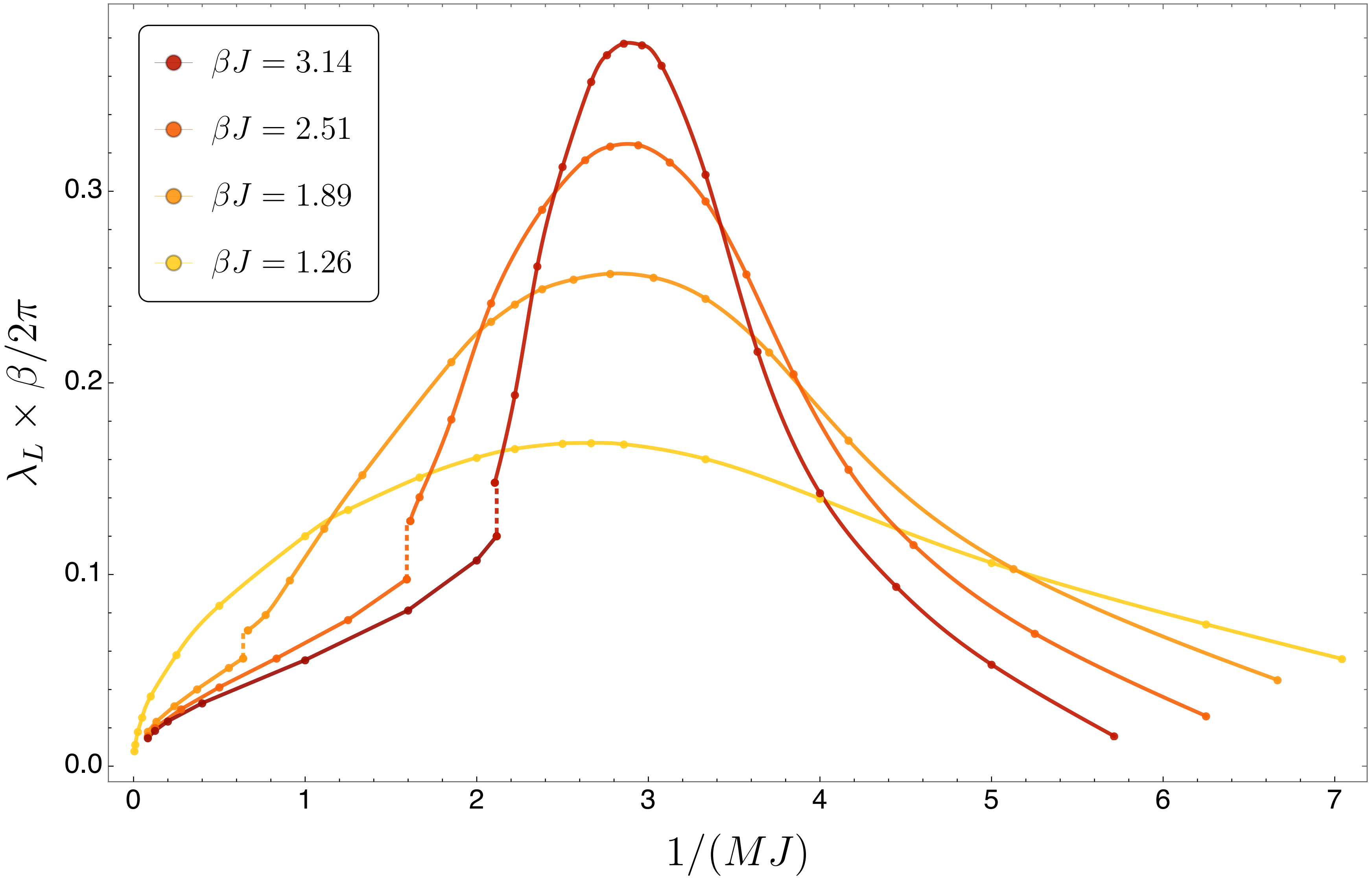
$$-\frac{1}{2}\, f''(x) - \frac{6}{\cosh^2 x}\, f(x) = -\left(2 + \frac{3Mu\beta^2\lambda_L}{2(p-2)\pi\gamma^2}\right) f(x)$$

—> unique positive solution: $\boxed{\frac{\beta}{2\pi}\,\lambda_L \approx \frac{5m}{24} = 5(p-2)\left(\frac{1}{24\beta\mathcal{J}} + \frac{p}{36\pi\beta\mathcal{J}\sqrt{M\mathcal{J}}} + \dots\right)}$



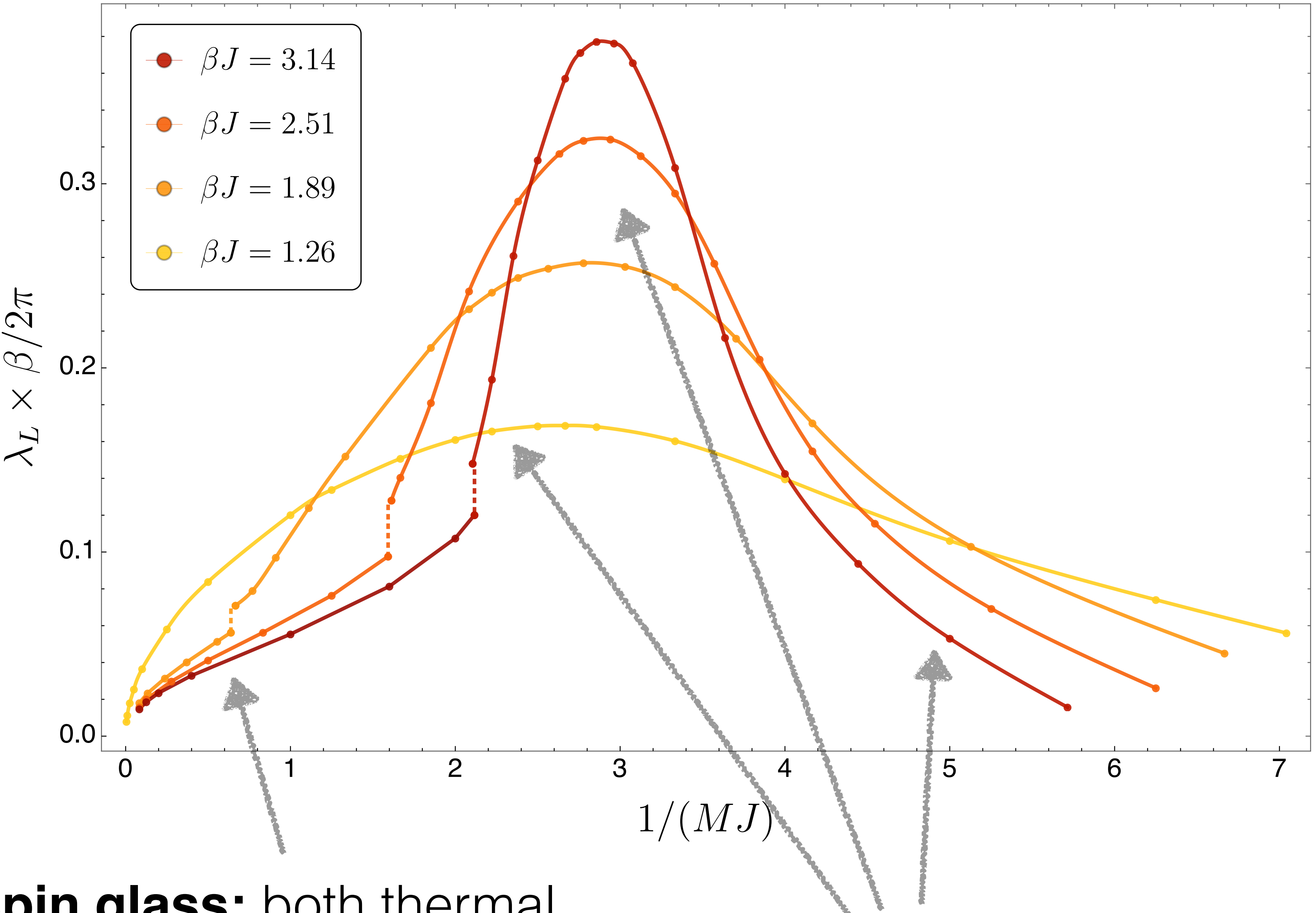
similar results: [Bera/Lokesh/Banerjee '21]

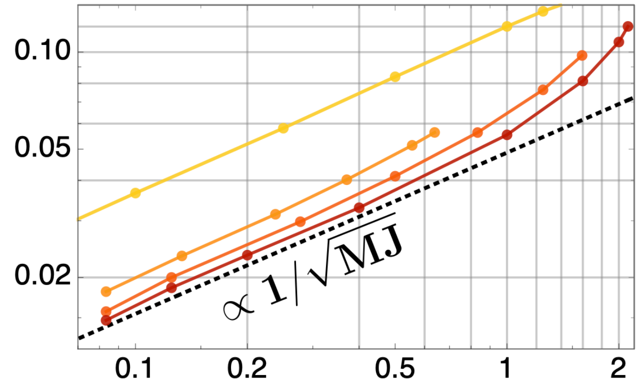
Lyapunov exponent: numerics



similar results: [Bera/Lokesh/Banerjee '21]

Lyapunov exponent: numerics





Spin glass: both thermal and quantum fluctuations increase λ_L

Paramagnet: non-monotonic dependence on MJ with a peak value

similar results: [Bera/Lokesh/Banerjee '21]

Holographic speculations

Signatures of gravity?

- p-spin glass has features reminiscent of gravity:
  - emergent conformal symmetry (appearance of $\Delta = 1$ mode)…

    … 'on top of' constant background value *u*
  - extensive number of (metastable) states…

    … contributing to the complexity
  - non-zero quantum Lyapunov exponent…

    … which vanishes at strong coupling

- Can we accommodate for these effects in $nAdS_2/nCFT_2$?

- At first sight: have learnt a lot in recent years about the replica trick in gravity to compute entropies, free energies, etc.

  [Lewkowycz/Maldacena] [Saad/Shenker/Stanford] [Penington/Shenker/Stanford/Yang] [Almheiri/Hartman/Maldacena/Shaghoulian/Tajdini] [Engelhardt/Fischetti/Maloney] …..

  …but it seems like we need a bit more to incorporate the above

AdS2 fragmentation

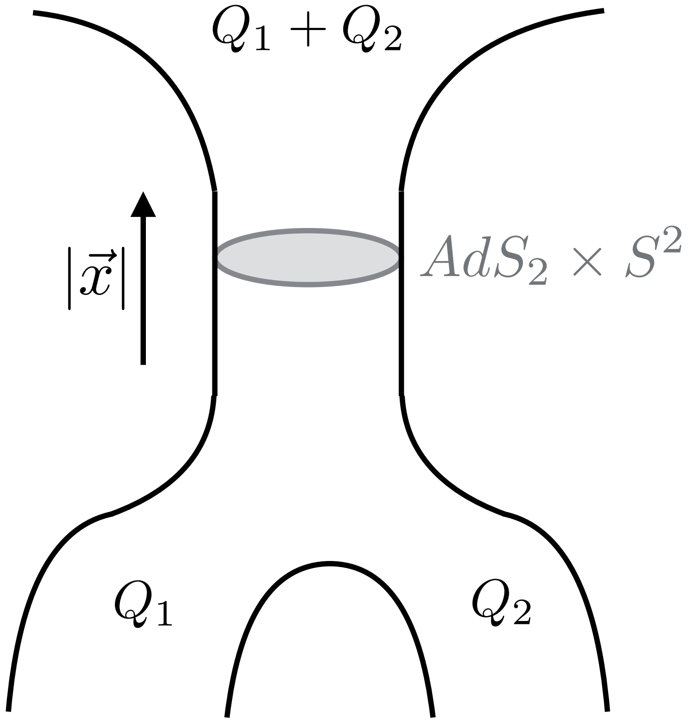
❖ Disclaimer: the following is a suggestion. We have not yet worked out any detailed picture.

❖ Near horizon of 4d extremal two-RN black holes with fixed charge:

$$ds^2 = -V^{-2}dt^2 + V^2 d\vec{x}^2$$

$$\star F = dt \wedge dV^{-1}$$

$$V = \frac{Q_1}{|\vec{x}-\vec{x}_1|} + \frac{Q_2}{|\vec{x}-\vec{x}_2|}$$



[Majumdar/Papapetrou '47]

[Brill '92]

[Maldacena/Michelson/Strominger '98]

- Large $|\vec{x}|$: same as geometry with a single throat with charge $Q_1 + Q_2$
- For $\vec{x} \to \vec{x}_{1,2}$: fragmentation into two (or more) *AdS2* regions

AdS$_2$ fragmentation

- Some features of fragmentation are reminiscent of the spin glass phase we discussed:
  - Large moduli space of geometries which locally minimize free energy (variational parameters $\vec{x}_i,\ Q_i$)

    —> counted by something like the complexity Σ ?

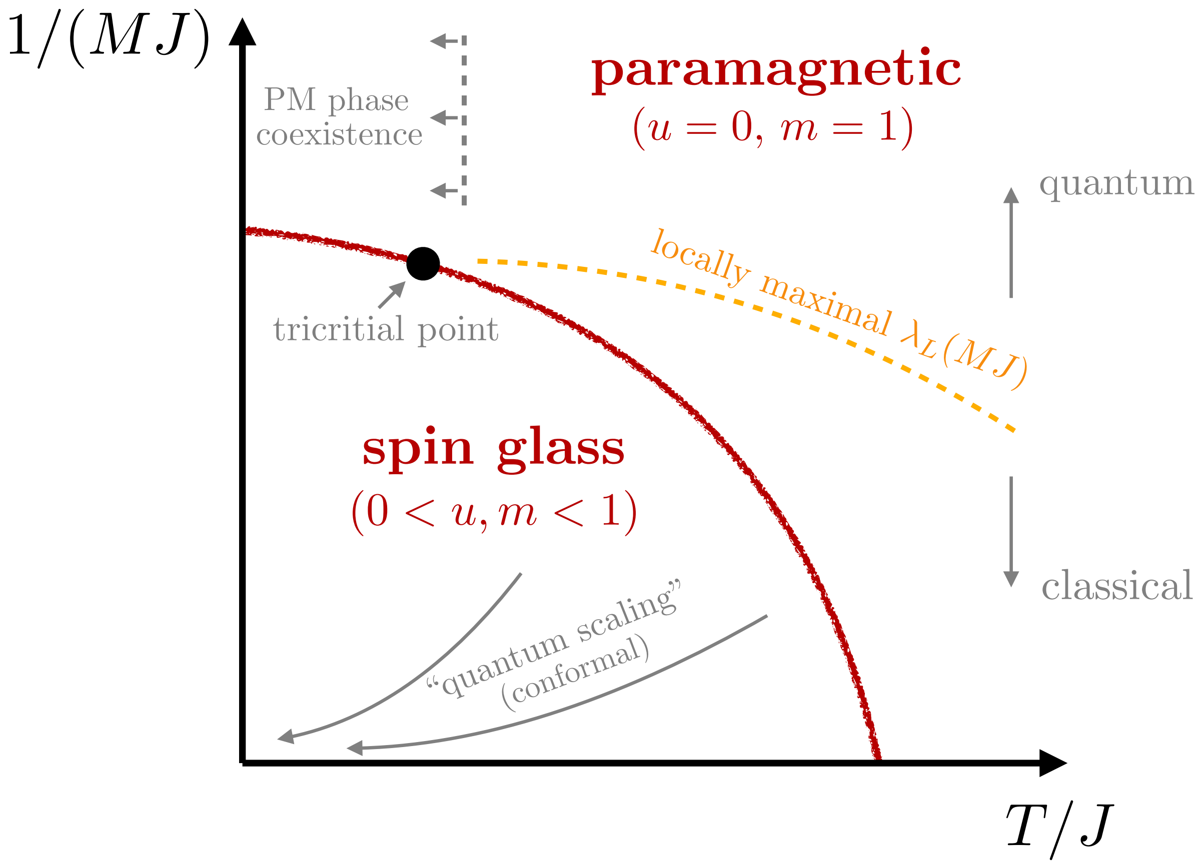
    —> action for $|\vec{x}_1 - \vec{x}_2|$ has no potential. Marginal operator?
  - Nonzero average dipole moment

    —> similar to *u* in the p-spin model?

Summary

Summary

- Paramagnetic phase (similar to SYK) vs. spin glass phase
- Emergent reparametrization symmetry, gapless, nonzero quantum Lyapunov exponent, …
  - More systematic conformal perturbation theory?
  - Holographic interpretation in terms of AdS_2 fragmentation?
  - Connect to string theory constructions?



Backup

Equations of motion

- After some rewritings, the equations of motion are:

$$\frac{1}{\hat{q}_r(k)} - \frac{1}{\hat{q}_r(0)} = M\left(\frac{2\pi k}{\beta}\right)^2 - J^2\left(\hat{\Lambda}_r(k) - \hat{\Lambda}_r(0)\right)$$

(e.o.m. for. $\hat{q}_r(k \neq 0)$)

$$\frac{1}{\beta}\sum_{k=-\infty}^{\infty} \hat{q}_r(k) = 1 - u$$

(spherical constraint for $\hat{q}_r(0)$)

$$\frac{m-1}{2}\left[\frac{(\beta J)^2}{2} p\, u^{p-1} - \frac{u}{\frac{\hat{q}_r(0)}{\beta}\left(\frac{\hat{q}_r(0)}{\beta} + mu\right)}\right] = 0$$

(equation for *u*)

- What about *m?*

(a) $\frac{\delta S_{eff}}{\delta m} = 0 \quad \Rightarrow \quad \frac{(\beta J)^2}{2} u^p + \frac{1}{m}\frac{u}{\frac{\hat{q}_r(0)}{\beta} + mu} + \frac{1}{m^2}\log\left[\frac{\frac{\hat{q}_r(0)}{\beta}}{\frac{\hat{q}_r(0)}{\beta} + mu}\right] = 0$

(b) marginal stability $\quad \Rightarrow \quad \beta^2 \mathcal{J}^2 u^{p-2}\left(\frac{\hat{q}_r(0)}{\beta}\right)^2 = 1$

String theory constructions

- *N=2* SUGRA in *d=4* has multi-horizon (“fragmented”) black hole solutions:
  - Exponentially many bound states of black holes
  - Exhibit slow relaxation etc.

[Denef ’00]
[Cardoso et al. ’00]

- Certain brane constructions in string theory lead to quiver quantum mechanics with a sector similar to the p-spin model:
  - Chiral and vector multiplets
  - SUSY fixes much of the Lagrangian —> bosonic potential:

$$V = \sum_{i,a} \left| \frac{\partial W(\phi)}{\partial \phi_i^a} \right|^2 + \frac{1}{g_{\mathrm{YM}}^2} \sum_a \left(\theta_a - \sum_i |\phi_i^a|^2 \right)^2$$

superpotential

$W(\phi) = \Omega_{ijk}\phi_i^1\phi_j^2\phi_k^3 + \dots$

FY parameters

[Douglas/Moore ’96]
[Denef/Moore ’07]