Non-linear gravity from entanglement in CFTs

Felix Haehl (UBC, Vancouver)

Benasque, 26 July 2017

Based on **[1705.03026], [w.i.p.]**
T. Faulkner, FH, E. Hijano, C. Rabideau, O. Parrikar, M. v. Raamsdonk

Outline

- **Part I: Introduction and results**
- Part II: Schematic derivation
- Part III: Details
- Part IV: Generalizations

Some questions

How does AdS/CFT work?

$\downarrow$

What characterizes holographic CFTs?

$\downarrow$

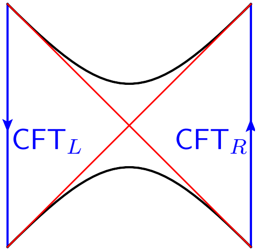
What characterizes states with a semi-classical gravity dual?

$\downarrow$

How does $\left\{ \begin{array}{c} \text{local geometry} \\ \text{gravitational dynamics} \end{array} \right\}$ emerge from CFT data?

Geometry from entanglement

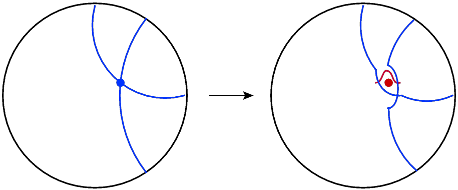
- AdS/CFT realizes **entanglement through connected spacetime**
- Example: Thermofield double *[Maldacena '01, van Raamsdonk '10]*
  - CFT_R and CFT_L non-interacting
  - ... but entangled!
  - connected bulk (with horizon)



- ER=EPR *[Maldacena-Susskind '13]*
- Good way to quantify this: entanglement entropy *[RT '06, HRT '07]*
  - Bulk extremal surfaces probe connectedness

Gravity from entanglement

- So far: purely geometrical picture
- Perturbations of geometry $\leftrightarrow$ perturbations of HRRT surfaces



- Consistent bulk perturbations satisfy Einstein equations
  - Can we **see the dynamics emerge** from consistency with the way entanglement entropy changes?

Setup

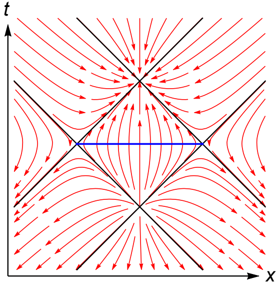
- Relative entropy:

$$\begin{aligned} S(\rho_A||\rho_A^{(0)}) &= \mathsf{Tr}\left(\rho_A \log \rho_A - \rho_A \log \rho_A^{(0)}\right) \\ &= \Delta\langle H_A^{(0)}\rangle - \Delta S_A \\ &\geq 0 \end{aligned}$$

- First law of entanglement: $\delta\langle H_A^{(0)}\rangle - \delta S_A = 0$
- In AdS/CFT:

$$S_A = \frac{\mathsf{area}(\widetilde{A})}{4G_N} \qquad \text{(HRRT)}$$

$$\delta\langle H_A^{(0)}\rangle = \int_A d\Sigma^\mu \, \delta\langle T_{\mu\nu}\rangle \, \zeta_A^\nu \sim \int_A d\Sigma^\mu \, (\delta g_{\mu\nu}^{(\mathsf{FG})}) \, \zeta_A^\nu$$



Gravity from entanglement

- First law of entanglement: $\delta S_A = \delta \langle H_A \rangle$
- linearized Einstein equations $\Leftrightarrow$ first law of entanglement
  [Faulkner-Guica-Hartman-Myers-van Raamsdonk '13]
  - Entanglement is realized geometrically
  - Small changes in entanglement structure are reflected by correct dynamics of the geometry
- Very nice, but: linearized Einstein equations are somewhat limited in really probing dynamics
  - No matter couplings/backreaction

Goal: Derive second order Einstein equations (incl. matter coupling) from perturbations of entanglement structure

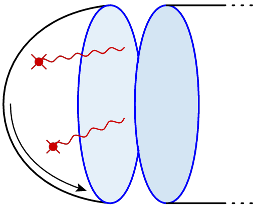
Setup

- Consider CFT states $|\psi_\lambda(\varepsilon)\rangle$ created by Euclidean path integrals:

$$\langle\varphi_{(0)}(\mathbf{x})|\psi_\lambda(\varepsilon)\rangle = \int^{\varphi(x_E^0=0)=\varphi_{(0)}} [D\varphi]\, e^{-\int_{-\infty}^{0} dx_E^0 \int d^{d-1}\mathbf{x}\, (\mathcal{L}_{CFT}+\lambda(x;\varepsilon)\,\mathcal{O}(x))}$$

with $\lambda(x,\varepsilon) = \varepsilon\,\lambda(x) + \mathcal{O}(\varepsilon^2)$

  - Euclidean sources $\lambda \;\Rightarrow$ initial state for Lorentzian evolution
  - Sources vanish as $x_E^0 \to 0$
  - Coherent excitations of bulk fields



- Leads to perturbations of entanglement entropy:

$$S_A = S_A^{(0)} + \varepsilon\,\delta S_A^{(1)} + \varepsilon^2\,\delta S_A^{(2)} + \dots$$

- In holography, there will be a dual bulk perturbation theory:

$$g = g_{AdS}^{(0)} + \varepsilon\,\delta g^{(1)} + \varepsilon^2\,\delta g^{(2)} + \dots$$
$$\phi = \varepsilon\,\delta\phi^{(1)} + \varepsilon^2\,\delta\phi^{(2)} + \dots$$

Results

(1) For any CFT in an EPI state $|\psi_\lambda(\varepsilon)\rangle$ there exists a bulk

$$\begin{aligned} g &= g^{(0)}_{AdS} + \varepsilon\, \delta g^{(1)} + \varepsilon^2\, \delta g^{(2)} + \dots \\ \phi &= \qquad\quad \varepsilon\, \delta\phi^{(1)} + \varepsilon^2\, \delta\phi^{(2)} + \dots \end{aligned}$$

that computes S_A correctly up to $\mathcal{O}(\varepsilon^2)$.

(2) This geometry satisfies gravitational equations of motion up to $\mathcal{O}(\varepsilon^2)$:

$$E^{ab}_{(2)} = \frac{1}{2}\, T^{ab}_{(2)}\,, \qquad T^{ab}_{(2)} \equiv T^{ab}(\delta\phi^{(1)}, \delta\phi^{(1)})$$

(3) For CFTs with "$c = a$" $E^{ab}_{(2)}$ is the (2^{nd} order) Einstein tensor, otherwise the equation of motion tensor for an appropriate higher curvature theory of gravity.

Remarks

- What does

$$g = g^{(0)}_{AdS} + \varepsilon\,\delta g^{(1)} + \varepsilon^2\,\delta g^{(2)} + \ldots\,, \qquad \phi = \varepsilon\,\delta\phi^{(1)} + \ldots$$

have to do with CFT data? Roughly:

  - $g^{(0)}_{AdS}(x,z) = \frac{\ell^2}{z^2}\left(dz^2 + dx^\mu\,dx_\mu\right)$ with $\frac{\pi^{d/2}}{\Gamma(d/2)}\frac{\ell^{d-1}}{8\pi G_N} = a$
  - $\delta g^{(1)}_{ab}(x,z) \sim \int_{\mathsf{bdry}(y)} K^{\mu\nu}_{ab}(x,z|y)\,\langle T_{\mu\nu}(y)\rangle$
  - $\delta\phi^{(1)}(x,z) \sim \int_{\mathsf{bdry}(y)} K(x,z|y)\,\langle\mathcal{O}(y)\rangle$

Remarks

- What does

$$g = g^{(0)}_{AdS} + \varepsilon\,\delta g^{(1)} + \varepsilon^2\,\delta g^{(2)} + \dots\,, \qquad \phi = \varepsilon\,\delta\phi^{(1)} + \dots$$

have to do with CFT data? Roughly:

  - $g^{(0)}_{AdS}(x,z) = \frac{\ell^2}{z^2}\left(dz^2 + dx^\mu\,dx_\mu\right)$ with $\frac{\pi^{d/2}}{\Gamma(d/2)}\frac{\ell^{d-1}}{8\pi G_N} = a$
  - $\delta g^{(1)}_{ab}(x,z) \sim \int_{\mathsf{bdry}(y)} K^{\mu\nu}_{ab}(x,z|y)\,\langle T_{\mu\nu}(y)\rangle$
  - $\delta\phi^{(1)}(x,z) \sim \int_{\mathsf{bdry}(y)} K(x,z|y)\,\langle \mathcal{O}(y)\rangle$

- If $d \neq 4$, by "a" and "c" I mean:

$$S^{(0)}_A = a^* \times \text{(universal)}$$
$$\langle T_{ab}(x)T_{cd}(y)\rangle = C_T \times \text{(universal)}_{abcd}$$

Remarks (2)

- The result is very universal: **CFT need not be holographic**
- The bulk geometry is therefore **completely auxiliary** (c.f., *[Faulkner '14]*)
- *If* the CFT is holographic, the construction will give an explicit mechanism for the emergence of local bulk dynamics
- How can this be true?
  - $\delta^{(2)} S(\rho_A || \rho_A^{(0)})$ only depends on very little CFT data: c and a
    - ★ Reason: ball-shaped A and $\rho =$ vacuum are very simple/symmetric
  - In this sense we are only probing/deriving a rather universal "sector" of AdS/CFT
  - Nevertheless interesting to see *how* bulk dynamics emerges: **entanglement is not just geometry, but also dynamics**

Outline

- Part I: Introduction and results
- **Part II: Schematic derivation**
- Part III: Details
- Part IV: Generalizations

Wald formalism

[Wald '93, Iyer-Wald '94]

- Consider (gravitational) Lagrangian $\mathbf{L}[\Phi] = \mathcal{L}[\Phi]\,\boldsymbol{\epsilon}$:

$$\delta\mathbf{L} = -\underbrace{\mathbf{E}_\Phi}_{\substack{\text{equations}\\\text{of motion}}}\delta\Phi + d\underbrace{\boldsymbol{\theta}(\Phi,\delta\Phi)}_{\substack{\text{presymplectic}\\\text{potential}}}$$

$$\underbrace{\boldsymbol{\omega}(\delta_1\Phi,\delta_2\Phi)}_{\substack{\text{symplectic}\\\text{current}}} = \delta_1\boldsymbol{\theta}(\Phi,\delta_2\Phi) - \delta_2\boldsymbol{\theta}(\Phi,\delta_1\Phi)$$

- In gravity, *any* vector $\boldsymbol{X}$ generates a symmetry $\rightarrow$ Noether currents:

$$\mathbf{J}_X = \boldsymbol{\theta}(\Phi,\mathcal{L}_X\Phi) - \mathbf{X}\cdot\mathbf{L}$$

$$d\mathbf{J}_X = 0 \ \ \text{(on shell)} \qquad \Rightarrow \qquad \mathbf{J}_X = d\mathbf{Q}_X + \mathbf{C}$$

- Variation of $\mathbf{J}_X$ yields:

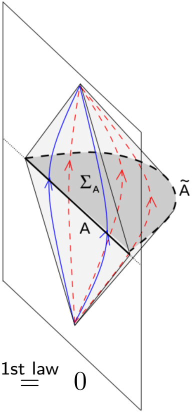
$$\boldsymbol{\omega}(\delta\Phi,\mathcal{L}_X\Phi) = d\underbrace{\left[\delta\boldsymbol{Q}_X - \mathbf{X}\cdot\boldsymbol{\theta}(\Phi,\delta\Phi)\right]}_{\equiv\boldsymbol{\chi}(\delta\Phi,\mathbf{X})} + \underbrace{\mathcal{G}(\Phi,\delta\Phi,\mathbf{X})}_{=0 \text{ if } \mathbf{E}_\Phi=\delta\mathbf{E}_\Phi=0}$$

Linearized Einstein equations from entanglement

[Faulkner et.al. '12]

$$\boldsymbol{\omega}(\delta\Phi,\mathcal{L}_X\Phi) = d\underbrace{\left[\delta\boldsymbol{Q}_X - \mathbf{X}\cdot\boldsymbol{\theta}(\Phi,\delta\Phi)\right]}_{\equiv\boldsymbol{\chi}(\delta\Phi,\mathbf{X})} + \underbrace{\mathcal{G}(\mathbf{E}_\Phi,\delta\mathbf{E}_\Phi,\mathbf{X})}_{=0 \text{ if } \mathbf{E}_\Phi=\delta\mathbf{E}_\Phi=0} \qquad (\star)$$

- Consider setup for calculating S_A
  - Rindler boost generator: Killing vector ξ_A
  - Integrate $(\star)$ over Σ_A, using $\Phi = g$ and $\mathbf{X} = \xi_A$:



$$\int_{\Sigma_A}\mathcal{G}(\mathbf{E}_g,\delta\mathbf{E}_g,\xi_A) = \int_{\Sigma_A}\left[\underbrace{\boldsymbol{\omega}(\delta g,\mathcal{L}_{\xi_A}g)}_{=0} - d\boldsymbol{\chi}(\delta g,\xi_A)\right]$$

$$= \int_{\widetilde{A}}\boldsymbol{\chi}(\delta g,\xi_A) - \int_A\boldsymbol{\chi}(\delta g,\xi_A)$$

$$= \frac{\delta[\text{area}(\widetilde{A})]}{4G_N} - \delta E \overset{HRRT}{=} \delta S_{EE} - \delta\langle H_A\rangle \overset{\text{1st law}}{=} 0$$

- Since this holds for any slice Σ_A: $\mathcal{G}(\mathbf{E}_g,\delta\mathbf{E}_g,\xi_A) = 0 \quad \Rightarrow \quad \delta\mathbf{E}_g = 0$

Non-linear perturbations

- This formalism was good for linearized perturbations
  - How to go beyond?
  - Non-linear version of first law seems not useful: $S(\rho_A||\rho_A^{(0)}) \geq 0$

Non-linear perturbations

- This formalism was good for linearized perturbations
  - How to go beyond?
  - Non-linear version of first law seems not useful: $S(\rho_A || \rho_A^{(0)}) \geq 0$
- *[Hollands-Wald '12]* : Can choose a gauge s.t. $(\star)$ holds beyond $\mathcal{O}(\varepsilon)$
- Ensure that geometry near $\widetilde{A}$ "looks the same" at $\mathcal{O}(\varepsilon^2)$:
  - GNCs such that $\widetilde{A}$ has fixed location: $K\big|_{\widetilde{A}} = 0$
  - ξ_A remains Killing near $\widetilde{A}$: $\mathcal{L}_{\xi_A} g(\varepsilon)\big|_{\widetilde{A}} = 0$
- Can now take another ε-derivative of $(\star)$:

$$\int_{\Sigma_A} \mathcal{G}(\mathbf{E}_g, \frac{d^2\mathbf{E}_g}{d\varepsilon^2}) = \int_{\Sigma_A} \boldsymbol{\omega}(\frac{dg}{d\varepsilon}, \mathcal{L}_{\xi_A}\frac{dg}{d\varepsilon}) - \frac{d^2}{d\varepsilon^2}\Big[E_A^{grav} - \frac{\text{area}(\widetilde{A})}{4G_N}\Big]$$

Non-linear Einstein equations

$$\int_{\Sigma_A} \mathcal{G}(\mathbf{E}_g, \frac{d^2\mathbf{E}_g}{d\varepsilon^2}) = \int_{\Sigma_A} \boldsymbol{\omega}(\frac{dg}{d\varepsilon}, \mathcal{L}_{\xi_A}\frac{dg}{d\varepsilon}) - \frac{d^2}{d\varepsilon^2}\left[E_A^{grav} - \frac{\text{area}(\widetilde{A})}{4G_N}\right]$$

- Via HRRT, we have

$$\frac{d^2}{d\varepsilon^2}\left[E_A^{grav} - \frac{\text{area}(\widetilde{A})}{4G_N}\right] \overset{HRRT}{=} \frac{d^2}{d\varepsilon^2}\left[\langle H_A\rangle - S_A^{EE}\right]_{\varepsilon=0} \equiv \delta^{(2)}S(\rho_A||\rho_A^{(0)})$$

- Goal: do a **CFT calculation** to show that

$$\delta^{(2)}S(\rho_A||\rho_A^{(0)}) = \int_{\Sigma_A} \boldsymbol{\omega}(\frac{dg}{d\varepsilon}, \mathcal{L}_{\xi_A}\frac{dg}{d\varepsilon})$$

  - Nonlinear equations of motion $\frac{d^2\mathbf{E}_g}{d\varepsilon^2}\Big|_{\varepsilon=0} = 0$ then follow
  - I've only written metric perturbations. Scalar fields work exactly the same way. So we get $E^{ab}_{(2)} - \frac{1}{2}T^{ab}_{(2)} = 0$

Outline

- Part I: Introduction and results
- Part II: Schematic derivation
- **Part III: Details**
- Part IV: Generalizations

Conformal perturbation theory

- Second order relative entropy is a quadratic functional in $\delta\rho_A = \varepsilon \int d^d x \, \lambda(x) \rho_A^{(0)} \, \mathcal{O}(x) + O(\varepsilon^2)$, i.e., a **2-pt. function**:

$$\delta^{(2)} S(\rho_A || \rho_A^{(0)}) \equiv \frac{d^2}{d\varepsilon^2} \mathrm{Tr} \left[\rho_A \log \rho_A - \rho_A \log \rho_A^{(0)} \right]_{\varepsilon=0}$$

$$= - \int_{-\infty}^{\infty} \frac{ds}{4 \sinh^2\left(\frac{s \pm i\epsilon}{2}\right)} \mathrm{Tr} \left[(\rho_A^{(0)})^{-1} \delta\rho_A \, (\rho_A^{(0)})^{\pm \frac{is}{2\pi}} \, \delta\rho_A \, (\rho_A^{(0)})^{\mp \frac{is}{2\pi}} \right]$$

$$\sim \underbrace{\int d^d x_a \, d^d x_b \, \lambda(x_a) \lambda(x_b)}_{\text{smear Eucl. sources}} \int_{-\infty}^{\infty} \underbrace{\frac{ds^2 \, \langle \mathcal{O}(\tau_a, \vec{x}_a) \mathcal{O}(\tau_b + is, \vec{x}_b) \rangle}{\sinh^2\left(\frac{s + i\,\epsilon\, \mathrm{sgn}(\tau_a - \tau_b)}{2}\right)}}_{\text{Eucl. 2-pt. function pushed into real time}}$$

  - Modular evolution gives relative boost by "Schwinger parameter" s
  - Offsets Euclidean correlator into real time: is

Two-point function from embedding space

- Introduce **auxiliary AdS-Rindler wedge**:

$$ds^2 = -(r_B^2 - 1)ds_B^2 + \frac{dr_B^2}{(r_B^2 - 1)} + r_B^2\, dY_B^2$$

- Can write 2-pt. function as **asymptotic symplectic flux** evaluated on AdS$_{d+1}$ bulk-boundary propagators:

$$\langle \mathcal{O}(\tau, Y_a)\mathcal{O}(is, Y_b)\rangle = \int_{r_B\to\infty} ds_B\, dY_B\, \boldsymbol{\omega}\big(K_E(is_B, r_B, Y_B|\tau, Y_a),\, K_R(s_B, r_B, Y_B|s, Y_b)\big)$$

$$K_E(is_B, r_B, Y_B|\tau, Y_a) \sim \frac{1}{2\Delta - d}\langle \mathcal{O}(\tau, Y_a)\mathcal{O}(is_B, Y_B)\rangle\, r_B^{-\Delta} + \ldots \quad (r_B \to \infty)$$

$$K_R(s_B, r_B, Y_B|s, Y_b) \sim \delta(s_B - s)\delta^{d-1}(Y_B - Y_b)\, r_B^{-d+\Delta} + \ldots$$
$$+ \frac{1}{2\Delta - d} G_R(s_B, Y_B|s, Y_b)\, r_B^{-\Delta} + \ldots \quad (r_B \to \infty)$$

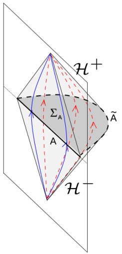
  - Intuition: space of asymptotic solutions parametrized as $\boldsymbol{\omega} \sim \delta q \wedge \delta p \sim \delta\phi_{(\Delta-d)} \wedge \delta\phi_{(-\Delta)}$

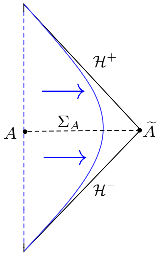
Symplectic flux

- Summary: relative entropy is

$$\delta^{(2)} S(\rho_A || \rho_A^{(0)}) \sim \int_{\text{bdry}(x_a)} \int_{\text{bdry}(x_b)} \lambda(x_a)\lambda(x_b) \int_{-\infty}^{\infty} \frac{ds^2}{\sinh^2\left(\frac{s + i\,\epsilon\,\text{sgn}(\tau_a - \tau_b)}{2}\right)} \int_{\partial \text{AdS}} \boldsymbol{\omega}(K_E, K_R)$$

- $\boldsymbol{\omega}$ is conserved $\Rightarrow$ can push the ∂AdS-integral to the horizon:





- Perform s-integral and get

$$\delta^{(2)} S(\rho_A || \rho_A^{(0)}) = \int_{\mathcal{H}^+} \boldsymbol{\omega}\left(\delta\phi, \mathcal{L}_{\xi_A} \delta\phi\right)$$

$$\text{with} \quad \delta\phi(y) \sim \int_{\text{bdry}(x)} \lambda(x)\, K_E(y|x)$$

Summary

- Gravitons ($\delta\phi \to \delta g_{ab}$) work similarly (a bit more subtle because of gauge choices...)
- CFT result:

$$\delta^{(2)} S(\rho_A || \rho_A^{(0)}) = \int_{\Sigma_A} \boldsymbol{\omega}(\delta\phi, \mathcal{L}_{\xi_A}\delta\phi) + \int_{\Sigma_A} \boldsymbol{\omega}(\delta g, \mathcal{L}_{\xi_A}\delta g)$$

$$\text{with} \quad \delta\phi(y) \sim \int_{\mathsf{bdry}(x)} \lambda(x)\, K_E(y|x) \quad \text{etc.}$$

- Hollands-Wald and HRRT tell us:

$$\delta^{(2)} S(\rho_A || \rho_A^{(0)}) = \int_{\Sigma_A} \boldsymbol{\omega}(\delta\phi, \mathcal{L}_{\xi_A}\delta\phi) + \int_{\Sigma_A} \boldsymbol{\omega}(\delta g, \mathcal{L}_{\xi_A}\delta g) + \int_{\Sigma_A} \mathcal{G}(\mathbf{E}, \frac{d^2\mathbf{E}}{d\varepsilon^2})$$

- Independence of $\Sigma_A \quad \Rightarrow \quad \frac{d^2\mathbf{E}}{d\varepsilon^2} = 0$

Outline

- Part I: Introduction and results
- Part II: Schematic derivation
- Part III: Details
- **Part IV: Generalizations**

Generalizations

- I have implicitly assumed "$c = a$": consistent with Einstein gravity
- If "$c \neq a$", consistency of AdS/CFT demands that our derivation shouldn't work
  - Indeed, using δg solving $\mathcal{O}(\varepsilon)$ Einstein equations $\Rightarrow$ CFT result ends up with wrong normalization
- **Higher curvature theories** of gravity can give $c \neq a$
- Idea: Use any $\mathbf{L} = \boldsymbol{\epsilon}\, f(\text{Riem})$ with same c and a as CFT
  - Can now similarly derive equations of motion of $\mathbf{L}$

    [FH-Hijano-Parrikar-Rabideau, w.i.p.]

  - Subtlety: need to work with appropriate entanglement functional

$$\delta^{(2)} S^{(\text{EE})}_{\widetilde{A}} = \delta^{(2)} S^{Wald}_{A} + \delta^{(2)} S^{extr.}_{A}$$

$$\delta^{(2)} S^{extr.}_{A} = \int_{\widetilde{A}} \sqrt{\bar{g}}\, \frac{\partial^2 f}{\partial R_{+\alpha+\beta} \partial R_{-\gamma-\delta}}\, \delta K^{+}_{\alpha\beta}\, \delta K^{-}_{\gamma\delta}$$

[Dong, Camps '13]

Deriving entanglement functional

- Gravity *[Hollands-Wald '12]* :

$$\left[\delta^{(2)} E_A^{grav,(L)} - \delta^{(2)} S_A^{Wald,(L)}\right] = \int_{\Sigma_A} \boldsymbol{\omega}^{(L)}\left(\frac{dg}{d\varepsilon}, \mathcal{L}_{\xi_A}\frac{dg}{d\varepsilon}\right) - \int_{\Sigma_A} \mathcal{G}\left(\mathbf{E}_g^{(L)}, \frac{d^2\mathbf{E}_g^{(L)}}{d\varepsilon^2}\right)$$

- CFT *[FH-Hijano-Parrikar-Rabideau, w.i.p.]* :

$$\delta^{(2)} S(\rho_A || \rho_A^{(0)}) = \int_{\Sigma_A} \boldsymbol{\omega}^{(L)}\left(\frac{dg}{d\varepsilon}, \mathcal{L}_{\xi_A}\frac{dg}{d\varepsilon}\right) + \delta^{(2)} S_A^{extr.,(L)}$$

- HRRT:

$$\delta^{(2)} S(\rho_A || \rho_A^{(0)}) = \left[\delta^{(2)} E_A^{grav,(L)} - \delta^{(2)} S_{\widetilde{A}}^{(\mathrm{EE})}\right]$$

- Can **assume** $\frac{d^2\mathbf{E}_g^{(L)}}{d\varepsilon^2} = 0$ and solve these equations for $\delta^{(2)} S_{\widetilde{A}}^{(\mathrm{EE})}$
  - New derivation of (perturbative) entanglement functional
  - No replica trick (as in *[Dong, Camps '13]*)

Other generalizations

- Perturbations of other states
  - Done for linearized equations by *[Dong-Lewkowycz '17]*
- Higher orders: full Einstein equations encoded in CFT entanglement?
  - $\mathcal{O}(\varepsilon^3)$: dependence on
    $\langle TTT \rangle = a \times (\text{univ.})_1 + b \times (\text{univ.})_2 + c \times (\text{univ.})_3$
  - $\mathcal{O}(\varepsilon^4)$: dependence on $\langle TTTT \rangle \to$ lots of OPE data! Should be very constraining
  - Note: basic gravitational identity of *[Hollands-Wald '12]* is already valid beyond 2$^{\text{nd}}$ order
- Quantum corrections?

Results

(1) For any CFT in an EPI state $|\psi_\lambda(\varepsilon)\rangle$ there exists a bulk

$$g = g^{(0)}_{AdS} + \varepsilon\,\delta g^{(1)} + \varepsilon^2\,\delta g^{(2)} + \dots$$
$$\phi = \qquad \varepsilon\,\delta\phi^{(1)} + \varepsilon^2\,\delta\phi^{(2)} + \dots$$

that computes S_A correctly up to $\mathcal{O}(\varepsilon^2)$.

(2) This geometry satisfies gravitational equations of motion up to $\mathcal{O}(\varepsilon^2)$:

$$E^{ab}_{(2)} = \frac{1}{2}\,T^{ab}_{(2)}\,, \qquad T^{ab}_{(2)} \equiv T^{ab}(\delta\phi^{(1)}, \delta\phi^{(1)})$$

(3) For CFTs with "$c = a$" $E^{ab}_{(2)}$ is the (2nd order) Einstein tensor, otherwise the equation of motion tensor for an appropriate higher curvature theory of gravity