# A topological gauge theory for entropy and the emergence of hydrodynamics

Felix Haehl (Durham University & PI)

Perimeter Institute - 20 November 2015

FH, R. Loganayagam, M. Rangamani [1510.02494], [1511.xxxxx], ...

see also [1312.0610], [1412.1090], [1502.00636]

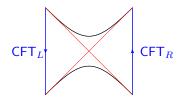
# Motivation

Big Q: How to do low energy effective field theory for **mixed states**?

- Important question for understanding QFT with dissipation etc.
- Density matrix!
  - $\Rightarrow$  path integral evolves both  $|\,\cdot\,\rangle$  and  $\langle\,\cdot\,|$
  - $\Rightarrow$  Schwinger-Keldysh doubling:  $\mathcal{H}_{phys} \subset \mathcal{H}_R \otimes \mathcal{H}_L$

Schwinger, Keldysh, Feynman-Vernon, '60s

- Many applications, e.g., black hole dynamics:
  - Double copy somehow encodes physics behind horizon
  - ► The two copies are coupled (→ entanglement, dissipation, complementarity, unitarity, ...)



### Motivation

- Preview: 'doubling' is powerful and dangerous
  - How to formulate unitarity etc. in the doubled theory and keep track of it along RG?
- In general these are hard non-equilibrium questions
- Start with a more tractable regime to learn about the general structure
  - ► Hydrodynamics: generic description of dynamics of mixed states near thermal equilibrium (length scales L ≫ lmfp)

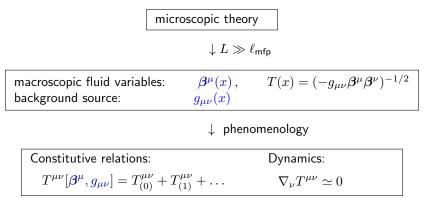
#### Part I: The structure of hydrodynamics

- The Eightfold Way
- Effective actions I: Landau-Ginzburg
- Preview: missing ingredients

- Doubling
- Topological limit
- KMS condition
- Part III: Gauge theory of entropy
  - Example: Langevin particle
  - · Effective actions II: Schwinger-Keldysh and fluids
  - $\circ$  Hydrodynamic gauged  $\sigma$ -model and gravity

# Phenomenology of hydrodynamics

• Hydrodynamics: near-equilibrium EFT for long wavelength fluctuations about Gibbsian density matrix



• E.g. ideal fluid:  $T^{\mu\nu}_{(0)} = \varepsilon(T) T^2 \beta^{\mu} \beta^{\nu} + p(T) \left(g^{\mu\nu} + T^2 \beta^{\mu} \beta^{\nu}\right)$ 

# Phenomenology of hydrodynamics

• On top of this, one imposes the following

Second law constraint:  $\exists J_S^{\mu} = s(T) T \beta^{\mu} + J_{S,(1)}^{\mu} + \dots \quad \text{with} \quad \nabla_{\mu} J_S^{\mu} \gtrsim 0 \quad \text{(on-shell)}$ 

- Gives interesting constraints on physically allowed constitutive relations, e.g.:
  - ★ Ideal fluid:  $\varepsilon + p = s T$
  - ★ 1<sup>st</sup> order: viscosities  $\eta, \zeta \ge 0$
  - ★ Many more at higher orders...

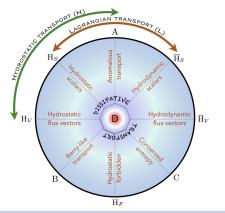
Bhattacharyya '12

Son-Surowka '09

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# Classification



FH-Loganayagam-Rangamani '14 '15

Theorem: The eightfold way of hydrodynamic transport

- ▷ There are eight classes of  $\{T^{\mu\nu}, J_S^{\mu}\}$  consistent with  $\nabla_{\mu} J_S^{\mu} \gtrsim 0$ .
- > A simple algorithm constructs these explicitly at any order in  $abla_{\mu}$ .
- $\triangleright$  Constitutive relations not produced by this algorithm, are forbidden by second law (Class  $H_F$ ).

# Lesson from classification (see talk last week):

- We now have a very good understanding of this structure
- In particular:
  - Second Law constraint as organizing principle
  - This constraint has been solved in generality
- This structure is highly non-trivial, but yet tractable
  - Nice testing ground for general ideas about low-energy EFT

## So what's the problem?

• But: doesn't make much sense from point of view of Wilsonian field theory

Phenomenological hydrodynamics	Natural for field theorist
• ??	Schwinger-Keldysh path integral
• "current algebra": construct all	• fields & symmetries $\Rightarrow$ action $S_{\text{eff}}[\phi, g_{\mu\nu}]$
tensor structures $T^{\mu u}[meta^\mu,g_{\mu u}]$	• $T^{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta S_{\text{eff}}}{\delta g_{\mu\nu}}$
• impose second law constraint	• ??
by hand: $ abla_{\mu}J^{\mu}_{S}\gtrsim 0$	
dynamics: conservation law	• dynamics: $\delta_{\phi}S_{\text{eff}} = 0$
• ??	dual black hole description

Part I: The structure of hydrodynamics

The Eightfold Way

### • Effective actions I: Landau-Ginzburg

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### Effective actions I: Landau-Ginzburg

- Consider Landau-Ginzburg action:
  - Fields: fluid vector & background geometry = { $\beta^{\mu}, g_{\mu\nu}$ }
  - Symmetries: diffeomorphism invariance

$$S_{\rm eff} = \int \sqrt{-g} \; \mathcal{L}[\pmb{\beta}^\mu, g_{\mu\nu}]$$

Basic variation defines hydrodynamic currents:

$$\delta S_{\text{eff}} = \int \sqrt{-g} \left[ \frac{1}{2} T^{\mu\nu} \, \delta g_{\mu\nu} + T \, \mathfrak{h}_{\sigma} \, \delta \beta^{\sigma} + \underbrace{\nabla_{\mu} (\cdots)^{\mu}}_{\text{surface term}} \right]$$

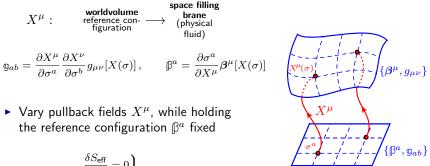
Further, define entropy current:

$$J_{S}^{\mu} = s T \beta^{\mu} \qquad \text{with} \qquad s \equiv \left[\frac{1}{\sqrt{-g}} \frac{\delta S_{\text{eff}}}{\delta T}\right]_{\{u^{\mu}, g_{\mu\nu}\} \text{ fixed}} = -\mathfrak{h}_{\sigma} \beta^{\sigma}$$

- Can show:  $\{T^{\mu\nu}, J^{\mu}_{S}\}$  solve the 2<sup>nd</sup> law constraint
- What about dynamics?

### **Dynamics**

• To get correct dynamics, formulate problem as a  $\sigma$ -model:



$$\frac{\overline{\delta X^{\mu}} = 0}{\delta X^{\mu}} \} \qquad \Rightarrow \qquad \nabla_{\mu} T^{\mu\nu} \simeq 0$$
  
+ diffeo Bianchi id.

Lesson: fluids are naturally  $\sigma$ -models with dynamical d.o.f. = pullback maps

(c.f. formulation of non-dissipative fluids in terms of Goldstone modes Dubovsky-Hui-Nicolis-Son '11) Felix Haehl (Durham University & PI), 9/27

- Is this a complete Lagrangian theory of hydrodynamics?
  - ▶ No. Out of 8 classes, this construction only covers 2.
  - The remaining 6 classes involve both dissipative and non-dissipative transport.

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### Preview: missing ingredients

- $\sigma\text{-model }S_{\mathrm{eff}}[\pmb{\beta}^{\mu}[X^{\mu}],g_{\mu\nu}]$  captures 2 out of 8 classes
- In these 2 classes,

 $\mathbf{N}^{\mu} \equiv J^{\mu}_{S} - (J^{\mu}_{S})_{\text{canonical}} \equiv J^{\mu}_{S} + \pmb{\beta}_{\nu}T^{\mu\nu}$ 

... is the Noether current for diffeomorphisms along  $eta^\mu$ 

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... is the Noether current for diffeomorphisms along  $eta^\mu$ 

Proposal for upgrading the  $\sigma$ -model to get 8 classes:

- Gauge 'thermal translations' along  ${\cal B}^{\mu} \longrightarrow U(1)_{\sf T}$  gauge symmetry
- Supersymmetrize the  $\sigma$ -model ( $\mathcal{N}_{T} = 2$ , à la  $\mathcal{V}_{afa}$ -Witten 94)
- $S_{\rm eff}$  with these symmetries will give precisely the 8 classes consistent with second law (and nothing else)

... let's understand this better

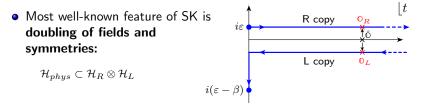
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# Schwinger-Keldysh I: doubling

• Non-equilibrium effective field theory in general described by **Schwinger-Keldysh** formalism



 Integrating out high energy modes from SK path integral leads to coupling between R and L ("influence functionals")

Feynman-Vernon, '60s

Just doubling everything gives too much freedom (easy to write influence functionals which violate microscopic unitarity)

 Important obstacle for systematic understanding of non-equilibrium physics (mixed states, dissipation, fluctuations, noise...)

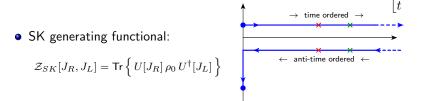
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### Schwinger-Keldysh II: topological limit

• Second defining feature of SK path integrals: time ordering prescription



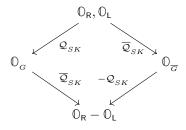
In particular:

$$\mathcal{Z}_{SK}[J_R = J_L \equiv J] = \operatorname{Tr} \rho_0 \quad \Rightarrow \quad \langle \mathcal{T}_{SK} \prod_i (\mathbb{O}_R(t_i) - \mathbb{O}_L(t_i)) \rangle = 0$$

⇒ The sector of difference operators  $(O_R - O_L)$  is protected for any SK path integral!

# Schwinger-Keldysh II: topological limit

- I.e.: If there are sources only for difference operators, any generic SK theory has a topological symmetry.
- This comes from **unitarity** of SK construction
- Most natural way to realize this: cohomological structure
  - ► Every operator Ô represented by a quadruplet {O<sub>R</sub>, O<sub>L</sub>, O<sub>G</sub>, O<sub>G</sub>}
  - ► SK supercharges  $Q_{SK}$ ,  $\overline{Q}_{SK}$  define topological sector (c.f. BRST):



FH-Loganayagam-Rangamani '15 Witten '82, Vafa-Witten '94

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# Schwinger-Keldysh II: topological limit

- A version of this topological symmetry should survive RG
- Hence, we need to have it also in the low-energy theory!
  - This then ensures that the low-energy effective theory comes from a unitary QFT
- In particular hydrodynamics should have a topological limit

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### Schwinger-Keldysh III: KMS condition

● In thermal equilibrium: Euclidean periodicity ⇒ KMS invariance

$$\tilde{\mathbb{O}}(t) \equiv e^{-i\delta_{\beta}}\mathbb{O}(t) \equiv \mathbb{O}(t-i\beta) \stackrel{\mathsf{KMS}}{\stackrel{\downarrow}{=}} \mathbb{O}(t)$$

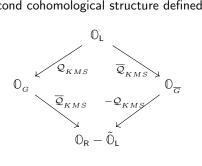
• Can replace  $\mathbb{O}_L \to \tilde{\mathbb{O}}_L$  in the previous time-ordering discussion:

$$\langle \mathcal{T}_{SK} \prod_{i} (\mathbb{O}_R(t_i) - \tilde{\mathbb{O}}_L(t_i)) \rangle = 0$$

⇒ The sector of **KMS rotated difference operators**  $(\mathbb{O}_R - \tilde{\mathbb{O}}_L)$  is also topological!

# Schwinger-Keldysh III: KMS condition

- In global thermal equilibrium: second topological sector  $\mathbb{O}_R \tilde{\mathbb{O}}_L$ 
  - Associated to a non-local symmetry (thermal translations)
  - Encode in second cohomological structure defined by  $Q_{KMS}$ ,  $\overline{Q}_{KMS}$ :



• Proposal for macroscopic description (local equilibrium,  $L \gg T^{-1}$ ):

▷ KMS becomes emergent local U(1)<sub>T</sub> gauge invariance
 ▷ Q<sub>KMS</sub>, Q
<sub>KMS</sub> → BRST charges of U(1)<sub>T</sub>

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### Toy model: Langevin particle

• Consider Brownian motion of Langevin particle at x(t):

$$-\mathrm{Eom} \equiv m\,\frac{d^2x}{dt^2} + \frac{\partial U}{\partial x} + \nu\,\frac{dx}{dt} = \mathbb{N}$$

• Martin-Siggia-Rose (MSR) construction:

Martin-Siggia-Rose '73

De Dominicis-Peliti '78

$$\begin{split} & [dx] \int [d\mathbb{N}] \,\delta(\mathsf{Eom} + \mathbb{N}) \,\mathsf{det}\left(\frac{\delta \mathsf{Eom}}{\delta x}\right) \,e^{i\,S_{\mathsf{Gaussian noise}}[\mathbb{N}]} \\ &= [dx] \int [df] [d\overline{\psi}] [d\psi] \,\exp\,i \int dt \left(f\,\mathsf{Eom} + i\,\nu\,f^2 + \overline{\psi}\left(\frac{\delta \mathsf{Eom}}{\delta x}\right)\psi\right) \end{split}$$

• Can write this in terms of  $\mathcal{N}_{T} = 2$  supercharges  $\overline{\mathcal{Q}}$ ,  $\mathcal{Q}$ , implementing the algebras of before:

$$= [dx] \int [df] [d\overline{\psi}] [d\psi] \exp i \int dt \left\{ \overline{\mathcal{Q}}, \left[ \mathcal{Q}, \frac{m}{2} \left( \frac{dx}{dt} \right)^2 - U(x) - i \nu \overline{\psi} \psi \right] \right\} \bigg|_{\substack{\text{gauge fixed}}}$$

Witten '82 Dijkgraaf-Moore '97 FH-Loganayagam-Rangamani '15

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### Toy model: Langevin particle

• Can make susy manifest by working in superspace:

$$\begin{aligned} x(S) &= x \underbrace{\overline{\mathcal{Q}}}_{\substack{\psi \\ Q}} + \theta \overline{\psi} + \theta \overline{\psi} + \theta \overline{\theta} f \\ \int dt \left\{ \overline{\mathcal{Q}}, \left[ \mathcal{Q}, \frac{m}{2} \left( \frac{dx}{dt} \right)^2 - U(x) - i \nu \overline{\psi} \psi \right] \right\} \Big|_{\substack{\text{gauge} \\ \text{fixed}}} \\ &= \int dt \, d\theta \, d\overline{\theta} \, \left( \frac{m}{2} \left( \frac{dx(S)}{dt} \right)^2 - U(x_{(S)}) - i \nu \, \mathcal{D}_{\theta} x_{(S)} \, \mathcal{D}_{\overline{\theta}} x_{(S)} \right) \Big|_{\substack{\text{gauge} \\ \text{fixed}}} \end{aligned}$$

• Invariance under CPT  $\Rightarrow$  Jarzynski relation:

$$\left\langle e^{-\boldsymbol{\beta}\,\Delta W}\right\rangle = e^{-\boldsymbol{\beta}\,\Delta F} \quad \Rightarrow \quad \left\langle \Delta W\right\rangle \geq \Delta F$$

Jarzynski '97

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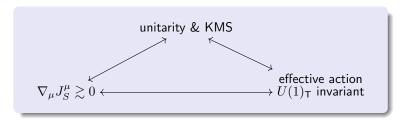
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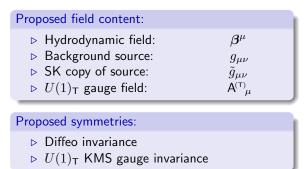
# Gauge theory of entropy in hydrodynamics

- Remember two features of fluids:
  - (1)  $\nabla_{\mu}J^{\mu}_{S} \gtrsim 0$  was mysterious from Wilsonian point of view
  - (2) For 'Lagrangian' classes of transport,  $J_S^{\mu}$  was roughly Noether current for translations along  $\beta^{\mu}$
- 'State-dependent' thermal translations of this type are precisely what implements KMS invariance of SK path integrals near equilibrium!
  - $J^{\mu}_{S}$  is the macroscopic current of emergent  $U(1)_{\mathsf{T}}$  gauge symmetry



# Effective actions II: Schwinger-Keldysh and fluids

• There already exists a SK framework for non-dissipative hydrodynamics:



• Theorem: any constitutive relations  $\{T^{\mu\nu}, \mathcal{G}^{\sigma}\}$  which satisfy adiabaticity equation can be obtained from a diffeo and  $U(1)_{\mathsf{T}}$  invariant Lagrangian (and vice versa):  $\mathcal{TH-Loganayagam-Rangamani '14-'15$ 

$$\mathcal{L}_{\mathsf{T}}[\boldsymbol{\beta}^{\mu}, g_{\mu\nu}, \tilde{g}_{\mu\nu}, \mathsf{A}^{(\mathsf{T})}_{\mu}] = \frac{1}{2} T^{\mu\nu}[\boldsymbol{\beta}^{\mu}, g_{\mu\nu}] \ \tilde{g}_{\mu\nu} - \frac{\mathcal{G}^{\sigma}[\boldsymbol{\beta}^{\mu}, g_{\mu\nu}]}{T} \ \mathsf{A}^{(\mathsf{T})}_{\sigma}$$

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## Effective actions II: some compelling features

- Field content and symmetries are such that we get precisely the 7 adiabatic classes and nothing more (no Class H<sub>F</sub>)
  - ▶  $U(1)_{\mathsf{T}}$  keeps Schwinger-Keldysh doubling under control
  - Adiabaticity equation  $\simeq U(1)_{\rm T}$  Bianchi identity
  - ► Conserved entropy current is gauge current of emergent U(1)<sub>T</sub> symmetry
- Upshot: we already have a very good guess for the bosonic part of non-dissipative Lagrangian
  - It nicely unifies the classification
  - ▶ It gives a natural explanation for the phenomenological framework

 $\Rightarrow$  justification for the proposal of emergent symmetries

Part I: The structure of hydrodynamics

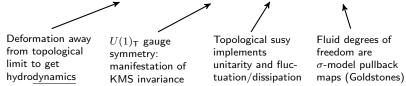
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# Hydrodynamics from field theorists' point of view

• Proposal summary:

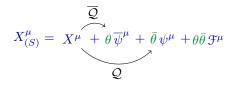
Hydrodynamics = deformation of a gauged topological  $\sigma$ -model



- Work in progress:
  - Write down this theory explicitly: fields, symmetries, actions
  - Check that it reproduces all of Second Law consistent hydrodynamics and no more

### Fluid $\sigma$ -model in superspace

- First step: make quadrupling and SK susy manifest, using superspace
  - **•** E.g. *σ*-model **pullback multiplet:**



- Similarly, a metric superfield  $g^{(S)}_{ab}$
- ▶ Plus a (super-)connection for emergent  $U(1)_{T}$  gauge symmetry:

$$\mathcal{A} = \mathcal{A}_a \, d\sigma^a + \mathcal{A}_\theta \, d\theta + \mathcal{A}_{\bar{\theta}} \, d\bar{\theta}$$

- Symmetries to impose are now very natural:
  - ▶ Super-diffeos, U(1)<sub>T</sub> invariance, CPT, ghost number conservation

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### Fluid $\sigma$ -model in superspace

• Formulate  $U(1)_{\mathsf{T}}$  gauged  $\sigma$ -model with these fields and symmetries:

$$S_{\rm eff}^{\rm (hydro)} = \int_{\substack{\text{world}\\\text{volume}}} d^4 \sigma \, d\theta \, d\bar{\theta} \, (\cdots)$$

• Most interestingly, we get dissipation (all of it, at any order in  $\nabla_{\mu}$ ):

$$S_{\text{eff}}^{(\text{dissipation})} \sim \int_{\substack{\text{world} \\ \text{volume}}} d^4 \sigma \, d\theta \, d\bar{\theta} \, \sqrt{-g^{(S)}} \left( i \, \boldsymbol{\eta}^{((ab)(cd))} \, \mathcal{D}_{\theta} \, g_{ab}^{(S)} \, \mathcal{D}_{\bar{\theta}} \, g_{cd}^{(S)} \right)$$

- Ghost bilinears responsible for dissipation
- ► Jarzynski holds (⇒ Second Law)
- Variation w.r.t.  $A_a$  gives entropy current

**Conserved**  $U(1)_{\mathsf{T}}$  current = standard entropy current + ghost terms

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# Outlook: gravity

AdS/CFT:

dissipating fluids <---> large AdS black holes

- Conjecture: long-wavelength, near-horizon AdS dynamics can be systematically characterized using our eightfold classification scheme
- In fluids: Second Law  $\leftrightarrow U(1)_T$  invariance  $\leftrightarrow$  microscopic consistency
  - ► SK doubling & ghosts: crucial (!) for field theoretic understanding
  - What will this teach us about dissipation, complementarity etc. in gravity?
- Gauge theory of fluid entropy  $\stackrel{?}{\rightsquigarrow}$  BH entropy

o ....

# Summary

- We found a complete classification and explicit solution of hydrodynamic transport [1412.1090 and 1502.00636]
- For full understanding from **field theorists' point of view**, need more ingredients: [1510.02494 and w.i.p.]
  - SK formalism
  - Hidden susy behind every relativistic fluid
  - SK path integral localizes on initial time thermal partition function if only difference operators are sourced
  - Ghosts account for dissipation
  - KMS conditions  $\Rightarrow U(1)_T$  gauge invariance in hydrodynamics
  - $U(1)_T$  symmetry current = entropy current + ghosts
- All this is dual to fundamental questions about gravity with horizons