

Quantum Information and the Black Hole Interior

Felix Haehl
(U. of Southampton)

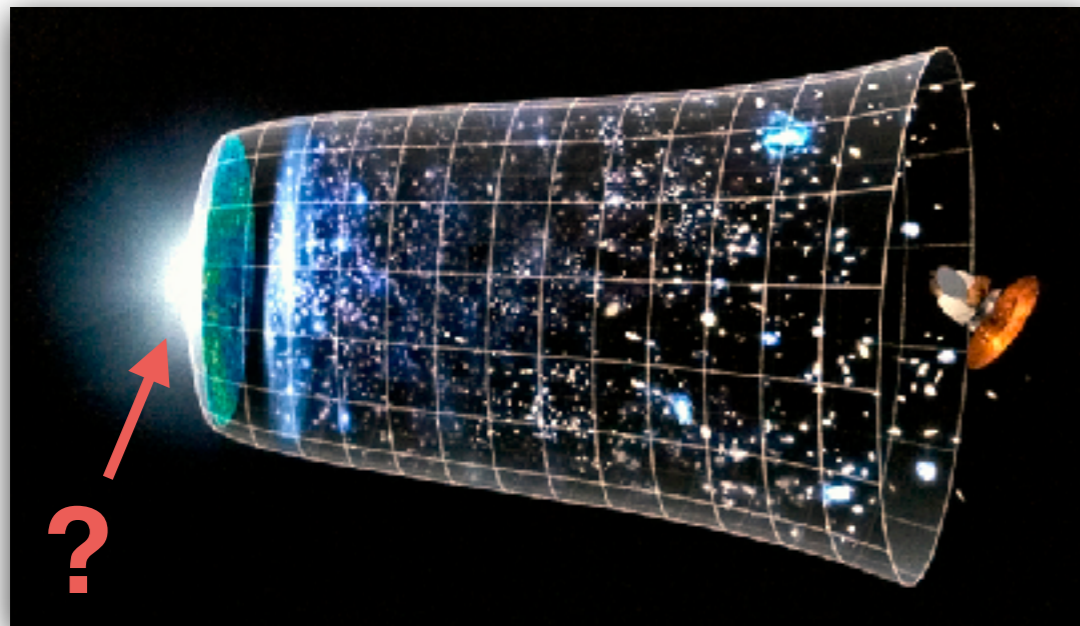
Mostly based on work with: Y. Zhao

Outline

1. Review: black holes and quantum information
2. Quantum circuit model of the interior
3. Collisions behind the horizon
4. Operator size
5. Conclusion

- **Dynamics of spacetime geometry** in extreme situations: quantum effects can be important or dominant

Cosmology



The beginning of the universe...?

Black holes



[Event Horizon Telescope '19]

Black hole interior?

Curvature singularity?

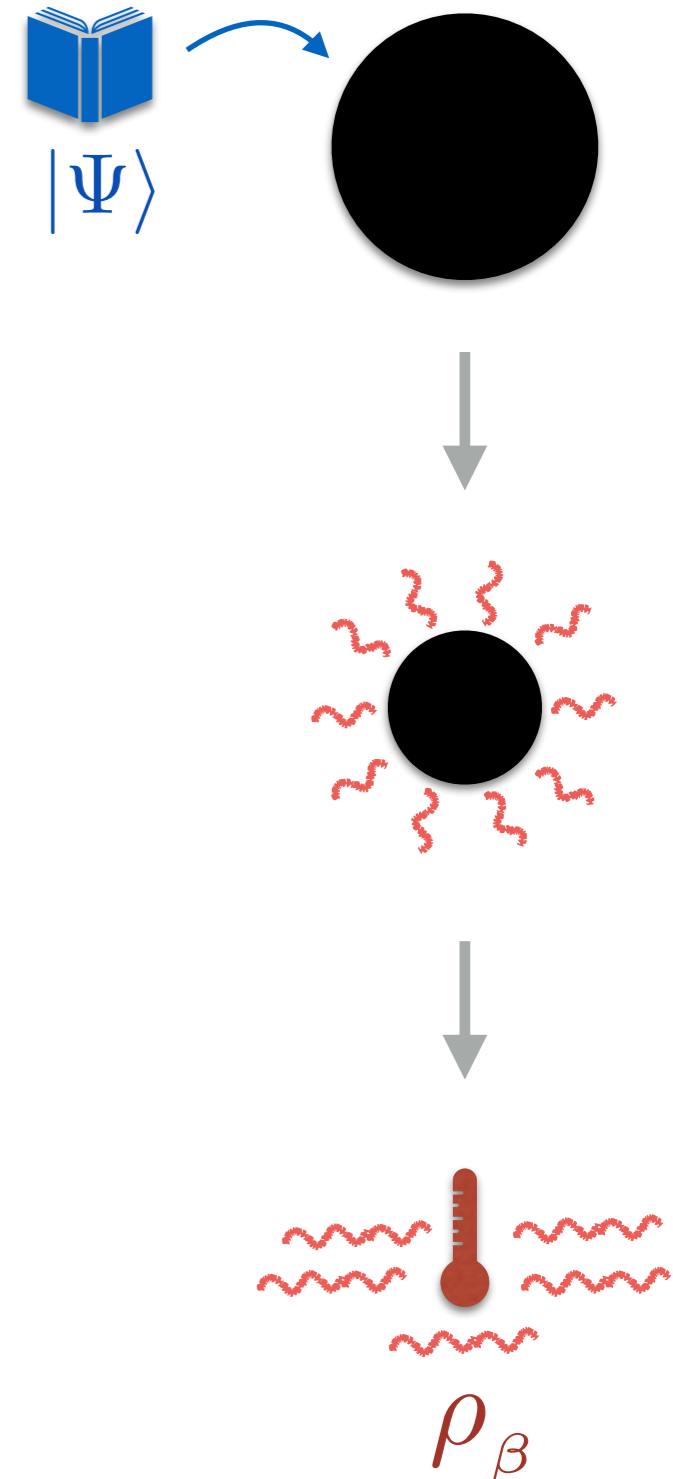
Thermodynamic properties?

The information problem

- Black holes look like **thermodynamic systems** with energy, entropy, temperature [Bekenstein '72] [Hawking '74]
- Emit Hawking radiation and eventually evaporate
- Hawking radiation is thermal (“scrambled”) —> information seems to be destroyed
- Essence of the information paradox:

Tension between **unitary quantum evolution** and **thermodynamic properties** of the horizon (general relativity + Hawking radiation).

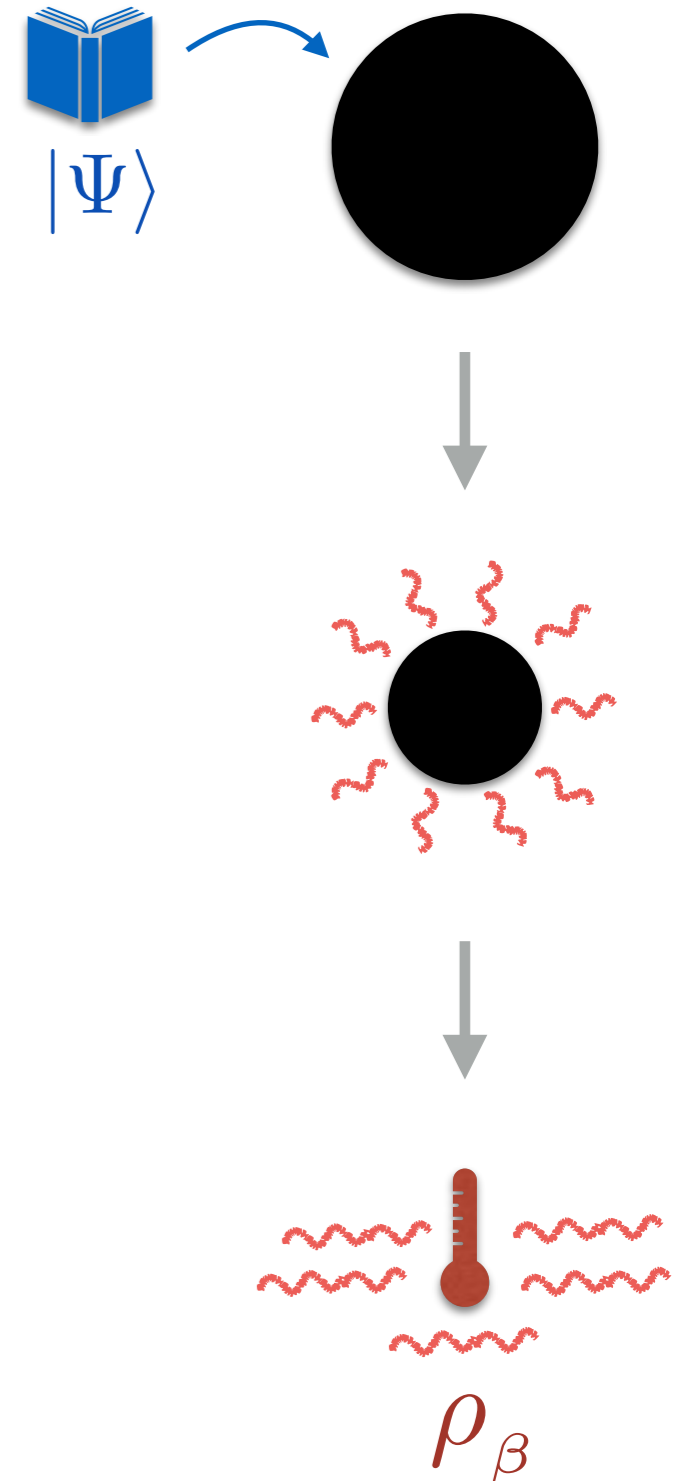
[Hawking '76] ... [Almheiri/Marolf/Polchinski/Sully '12] ...



The information problem

Tension between **unitary quantum evolution** and **thermodynamic properties** of the horizon (general relativity + Hawking radiation).

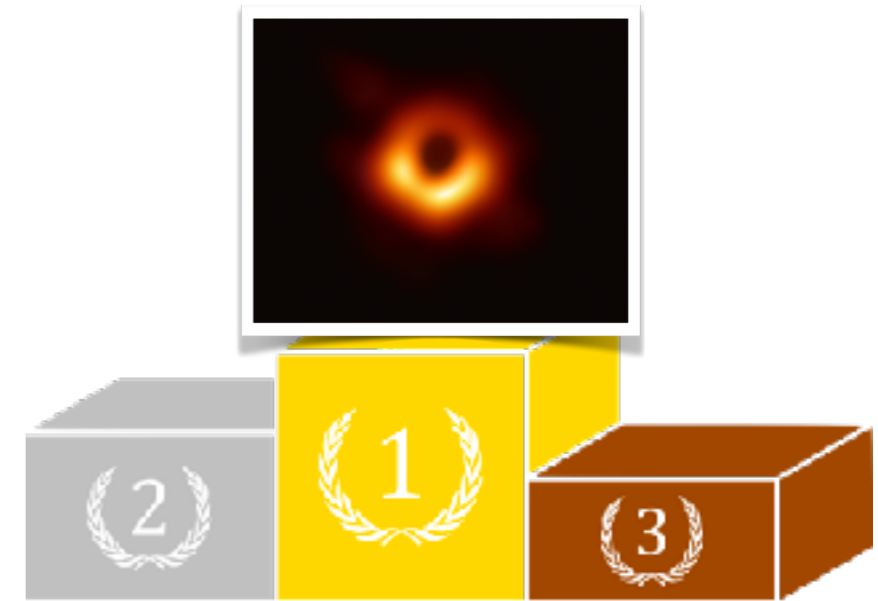
- Enormous progress...
[Penington '19] [Almheiri/Engelhardt/Marolf/Maxfield '19]
- ...and still many confusions
- Core theme: **thermodynamic, statistical, and quantum information theoretic properties** of black holes



Black Holes Excel at Information Theoretic Tasks

- ▶ The **“densest hard drives”**:
maximum entropy, given mass and size
[Bekenstein '81]
- ▶ The **“most ideal fluids”**:
saturate shear viscosity bound
[Kovtun/Son/Starinets '04]
- ▶ The **“fastest information scramblers”**:
take the shortest time to make information
inaccessible to local measurements [Sekino/Susskind '08]

(...)



Holographic duality

Quantum gravity in
asymptotic AdS_{d+1}



Conformal QFT_d

class. & qu. gravity

strongly coupled CFT

black holes

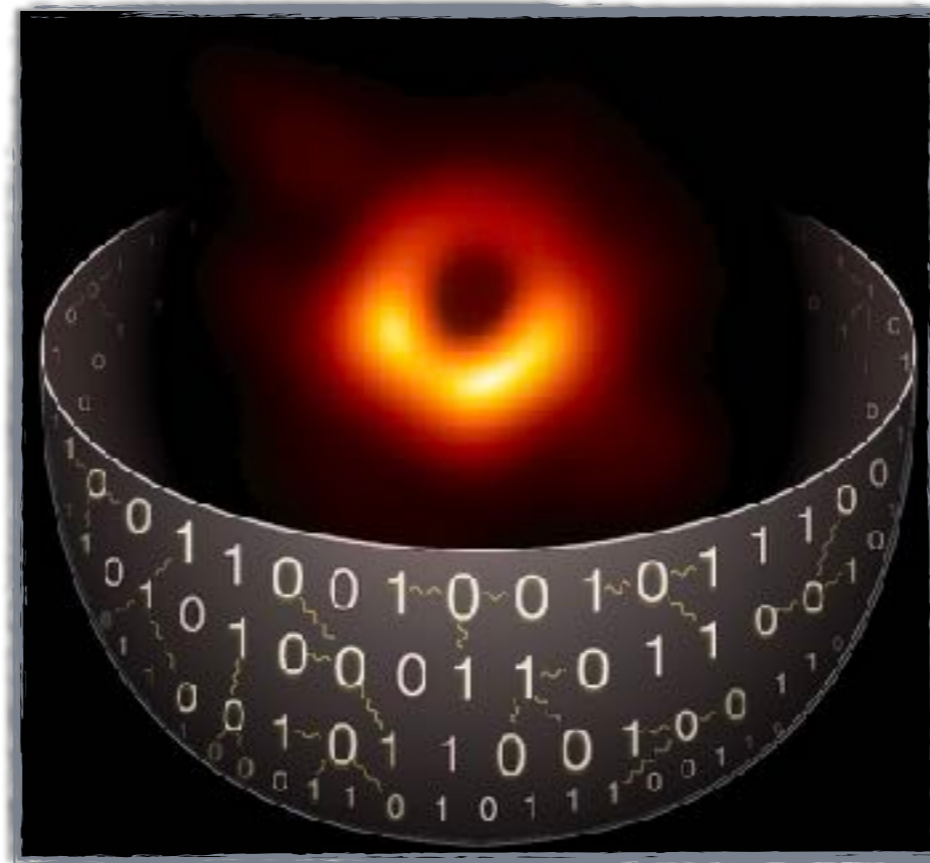
thermal states

grav. dynamics

entropy,
hydrodynamics,
chaos

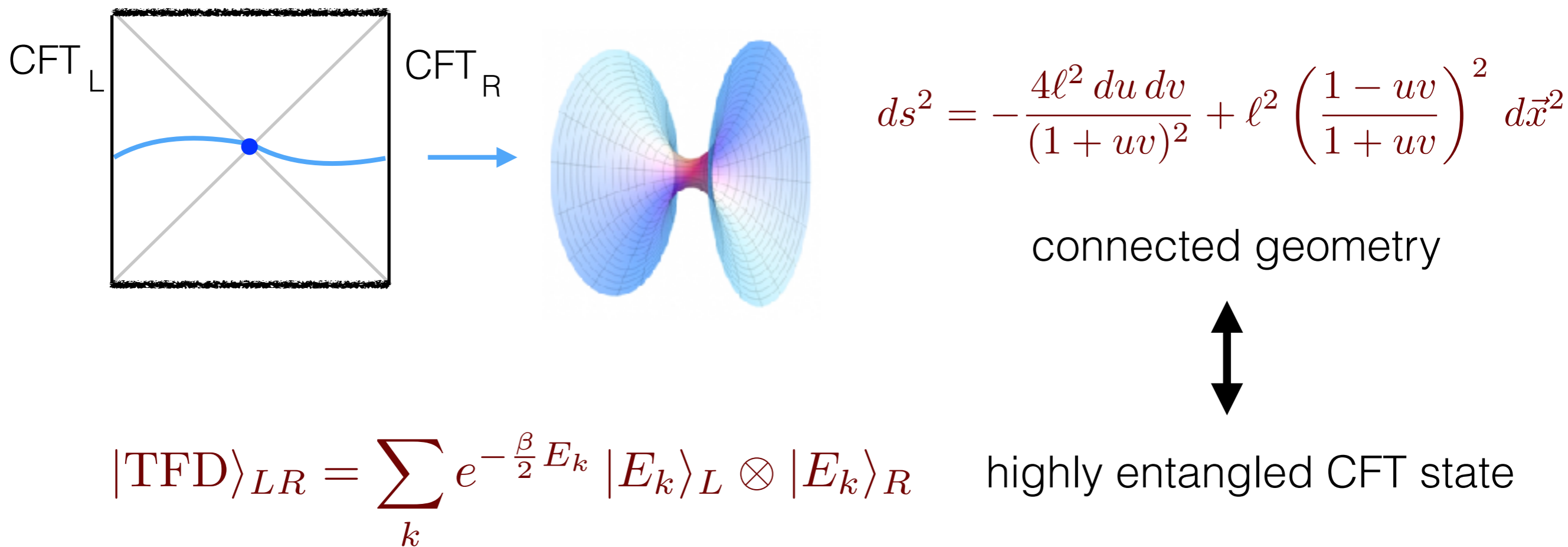
geometry, locality,
unitarity issues

entanglement,
quantum information



Geometry and Entanglement

- Use AdS/CFT to learn about the fate of an infalling observer, black hole interior, and the singularity
- Setup: **asymptotically AdS black hole**

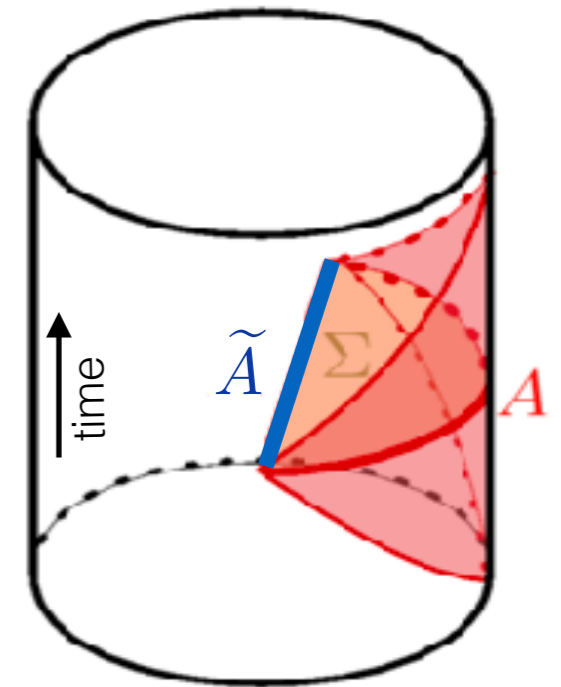


$$\langle \text{TFD} | \mathcal{O}_R | \text{TFD} \rangle = \text{tr} (\mathcal{O}_R e^{-\beta H_R})$$

[Maldacena '01]

[Van Raamsdonk '10]

- **Entanglement** of the dual CFT state is believed to be crucial for having a connected geometry.



- [Ryu/Takayanagi '06]: $S_{\text{vN}}(\rho_A) = \frac{\text{area}(\tilde{A})}{4G_N}$
 [Hubeny/Rangamani/Takayanagi '07]
 $\rho_A = \text{Tr}_{A^c}(\rho)$
 $S_{\text{vN}}(\rho) = -\text{Tr}(\rho \ln \rho)$

- Perturbing ρ_A constrains $\text{area}(\tilde{A})$
 => can derive **local Einstein equations** from entanglement

[Faulkner/Guica/Hartman/Myers/Van Raamsdonk '13]
 [Faulkner/**FH**/Hijano/Parrikar/Rabideau/Van Raamsdonk '17]

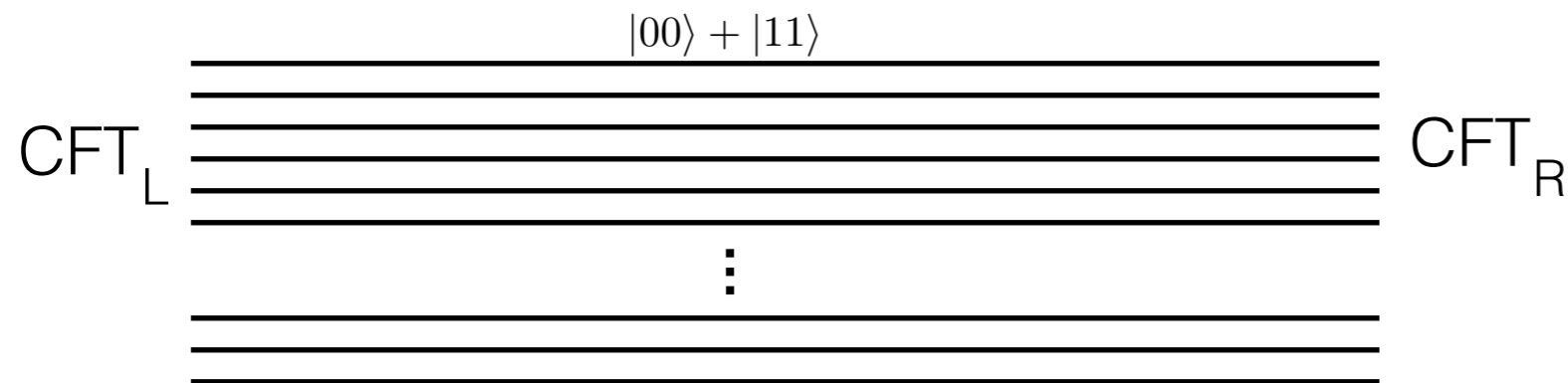
- Quantum corrections: $S_{\text{vN}}(\rho_A) = \frac{\text{area}(\tilde{A})}{4G_N} + \underbrace{\mathcal{O}(G_N^0) + \mathcal{O}(G_N) + \dots}_{S_{\text{bulk}}}$
 [Faulkner/Lewkowycz/Maldacena '13]
 [Engelhardt/Wall '14]
 (crucial for info. paradox!)

- [Maldacena/Susskind '13]: **ER** = **EPR**
 Einstein-Rosen bridge = entangled qubits

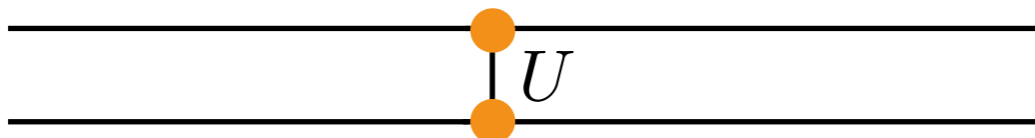
- ▶ ER = EPR Can we test this??

$$|\text{TFD}\rangle_{LR} = \sum_k e^{-\frac{\beta}{2} E_k} |E_k\rangle_L \otimes |E_k\rangle_R$$

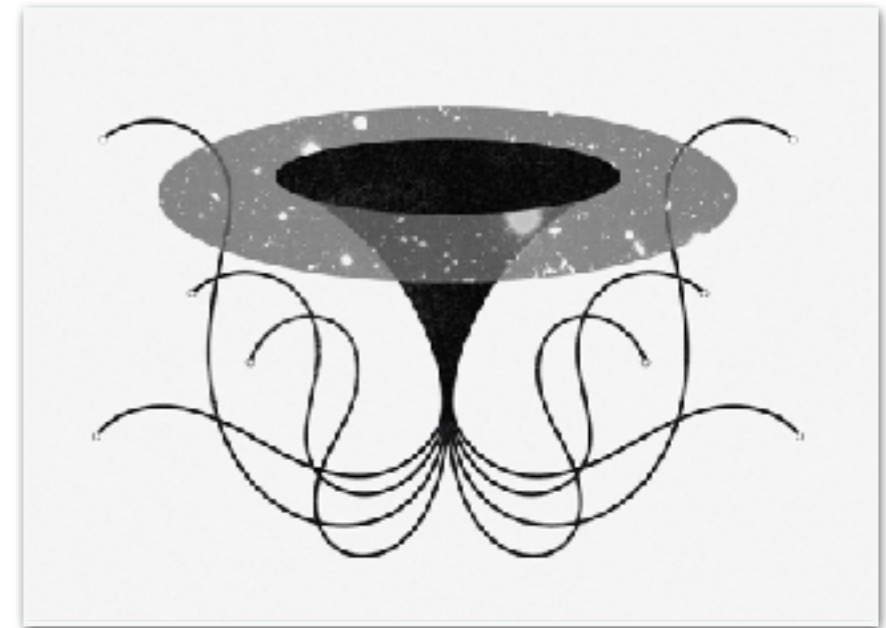
- Represent by EPR pairs (qubits):



- To model time evolution we perform operations on the qubits
 —> construct a **quantum circuit**
- 2-qubit **quantum gates**:



- ▶ U can be **undone** by either side alone: act with U^{-1} on R or L

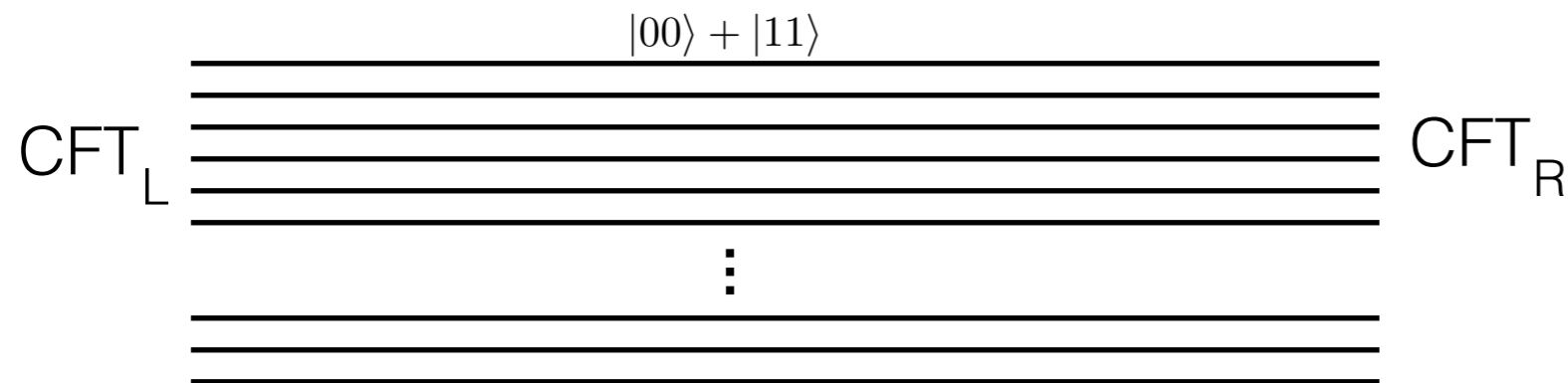


[Olena Shmahalo/Quanta Magazine]

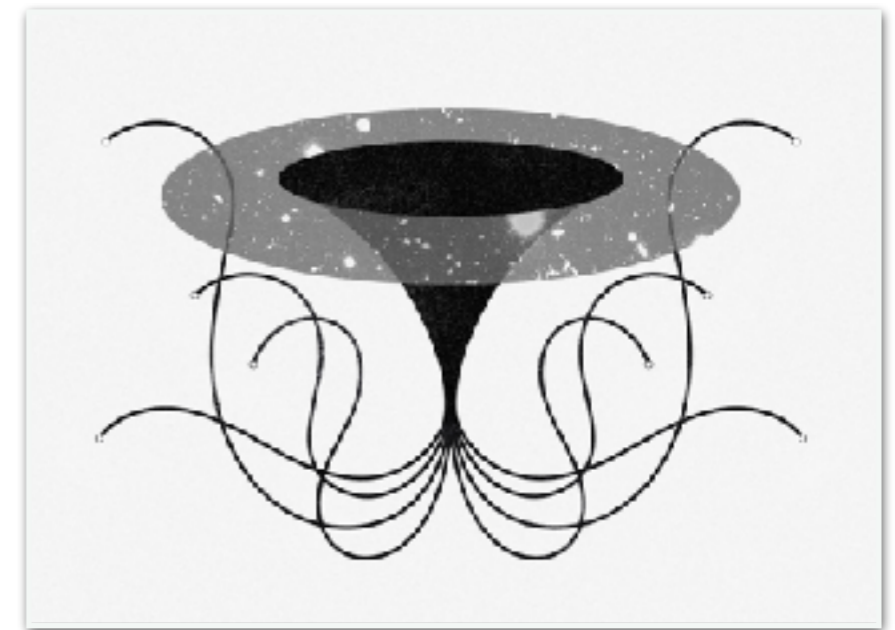
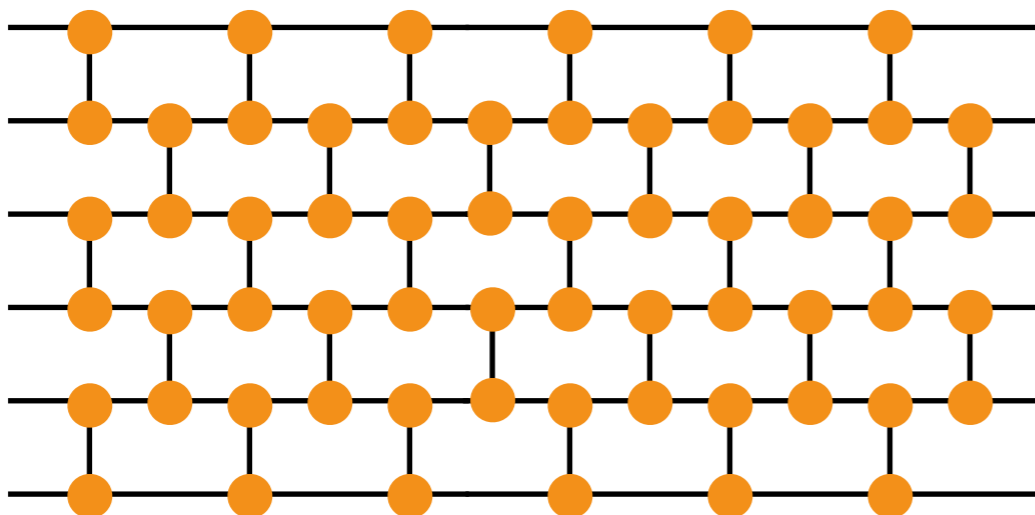
- ▶ ER = EPR Can we test this??

$$|\text{TFD}\rangle_{LR} = \sum_k e^{-\frac{\beta}{2} E_k} |E_k\rangle_L \otimes |E_k\rangle_R$$

- Represent by EPR pairs (qubits):

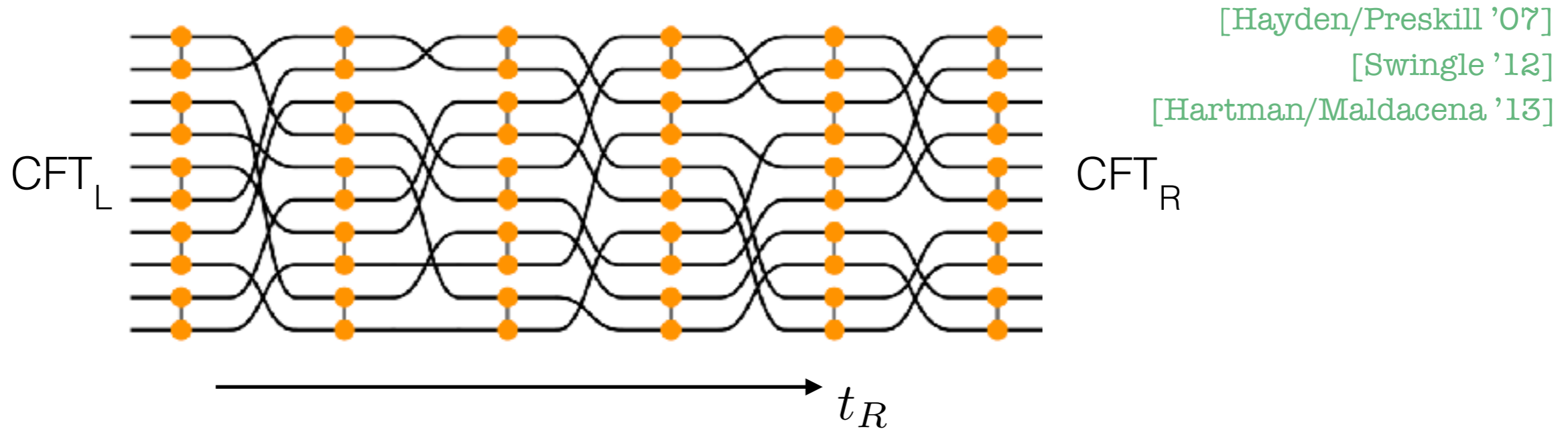


- To model time evolution we perform operations on the qubits
 —> construct a **quantum circuit**

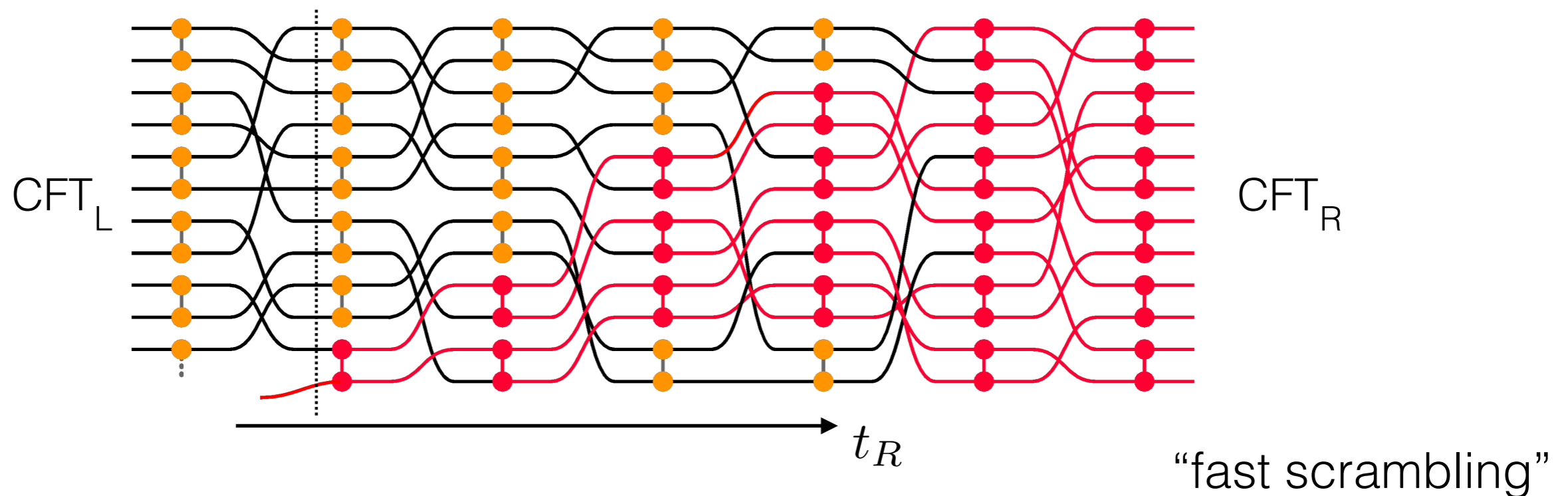


[Olena Shmahalo/Quanta Magazine]

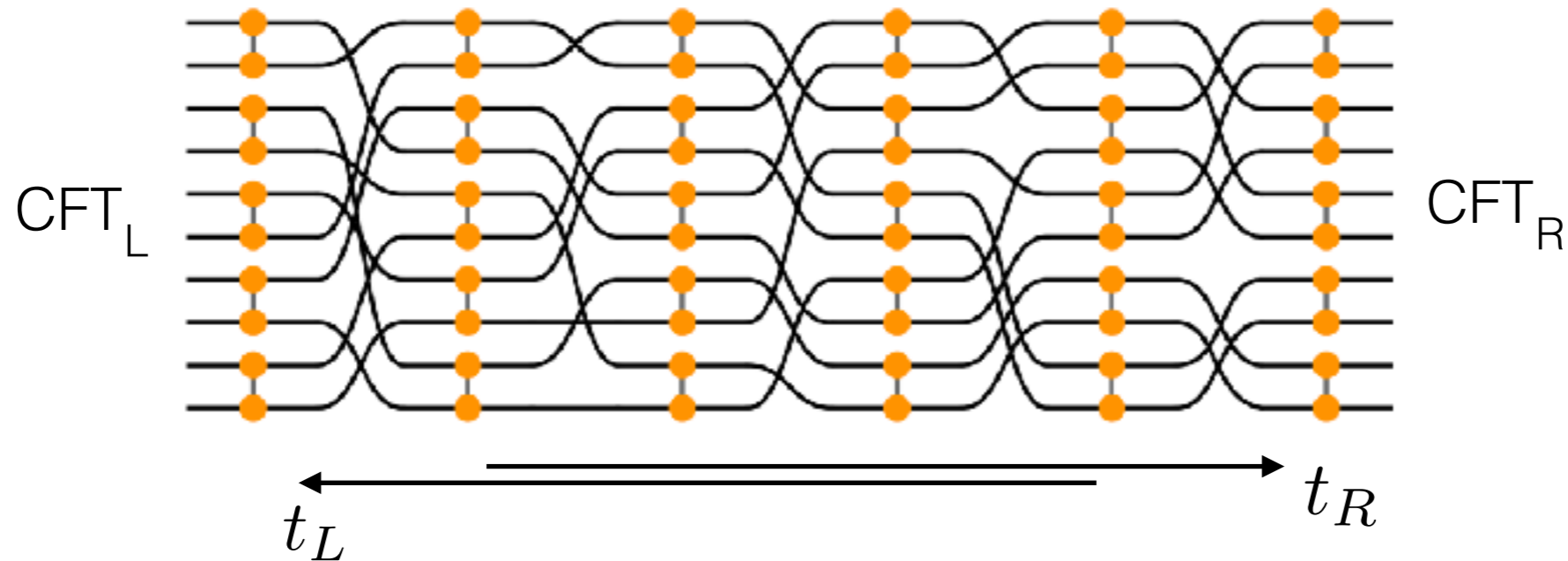
- Model for black hole time evolution: **random unitary all-to-all circuit**



- Information spreads exponentially fast through the circuit:



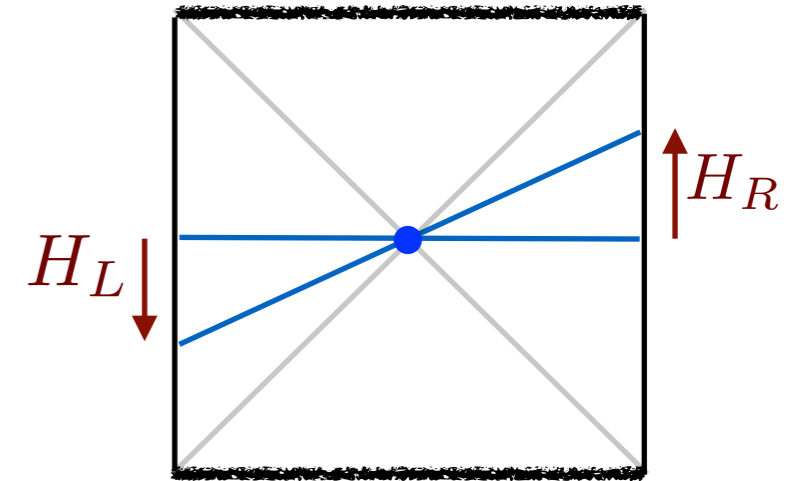
- Model for black hole time evolution: **random unitary all-to-all circuit**



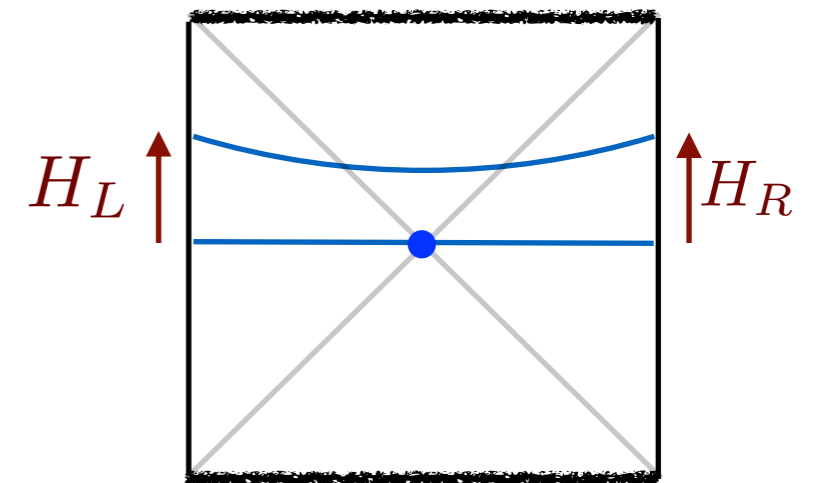
- ▶ Any gate could be implemented by either side
- ▶ Evolution in t_L can be **undone** by inverse evolution in t_R
- ▶ **Depth** of circuit is $t_R - t_L$
- ▶ Large depth means: the state of $\text{CFT}_L \cup \text{CFT}_R$ has a high **computational complexity** [Susskind '14]
- ▶ Note: the state of either CFT alone is always thermal
- ▶ Depth keeps increasing long after thermalisation

- Similar features in the TFD and black hole:

$$e^{it(H_R - H_L)} |\text{TFD}\rangle_{LR} = |\text{TFD}\rangle_{LR}$$



$$|\text{TFD}(t)\rangle = e^{it\frac{H_R + H_L}{2}} |\text{TFD}\rangle = \sum_k e^{-\left(\frac{\beta}{2} - it\right)E_k} |E_k\rangle_L \otimes |E_k\rangle_R$$

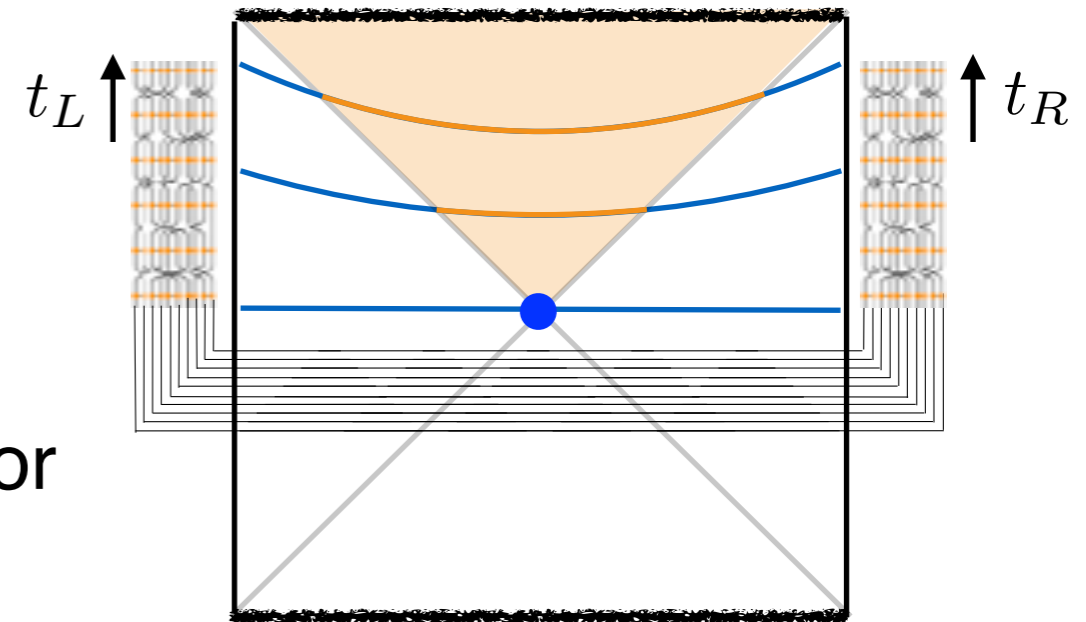


but still: $\text{Tr}_L (|\text{TFD}(t)\rangle\langle\text{TFD}(t)|) = e^{-\beta H_R}$

- Evolving TFD with $H_L + H_R$ explores more of the interior
- This is invisible from the point of view of either CFT alone. Need $\text{CFT}_L \cup \text{CFT}_R$ to probe into the interior

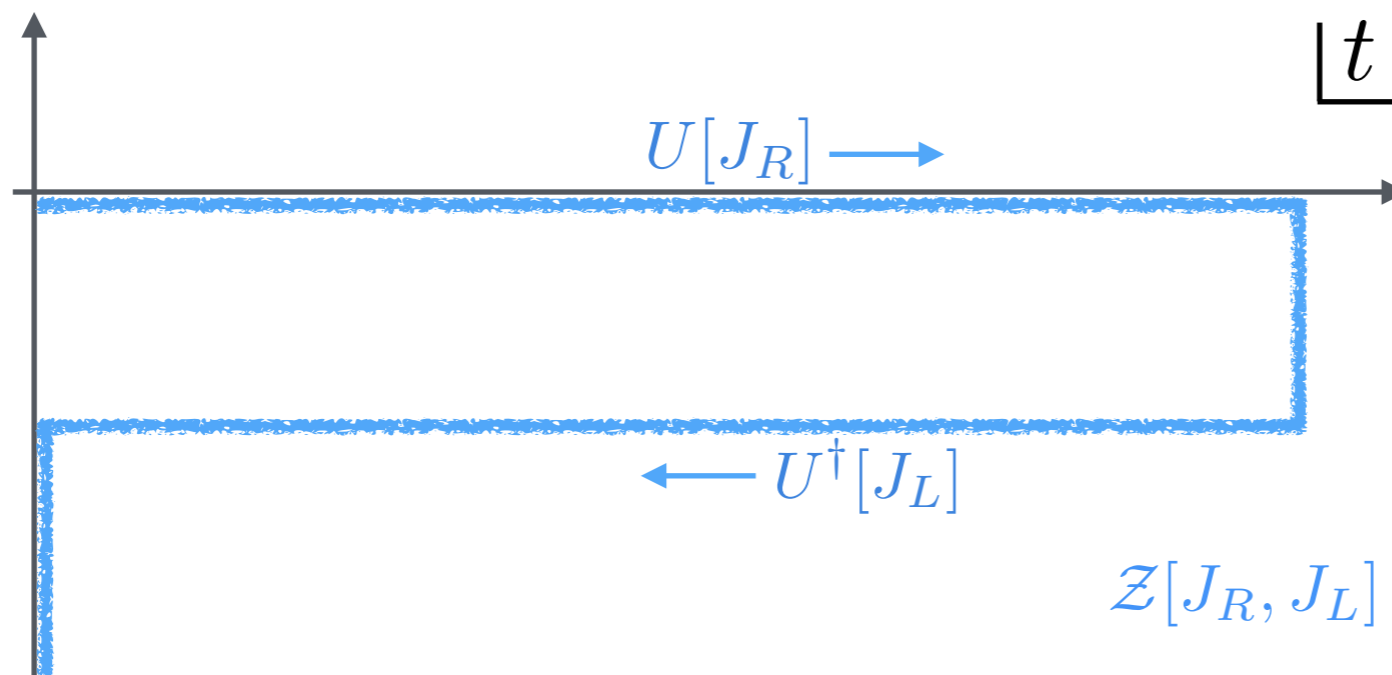
Circuit / black hole connection

- Proposal:
 - (1) quantum circuit represents evolution of black hole interior
 - (2) Depth of circuit (#gates) is proportional to spatial volume of interior



[Hayden/Preskill '07][Susskind '14]

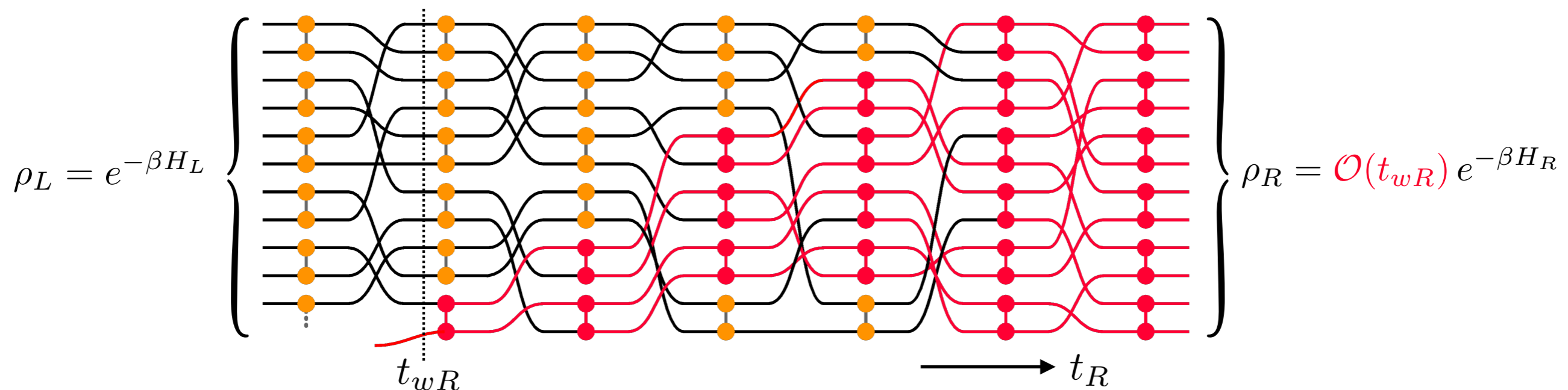
- N.b.: (1) is a 'quantum info' description of time evolution along the Schwinger-Keldysh contour:



$$\mathcal{Z}[J_R, J_L] = \text{tr} \left(U[J_R] \rho_\beta^{\frac{1}{2}} U^\dagger[J_L] \rho_\beta^{\frac{1}{2}} \right)$$

Quantum butterfly effect

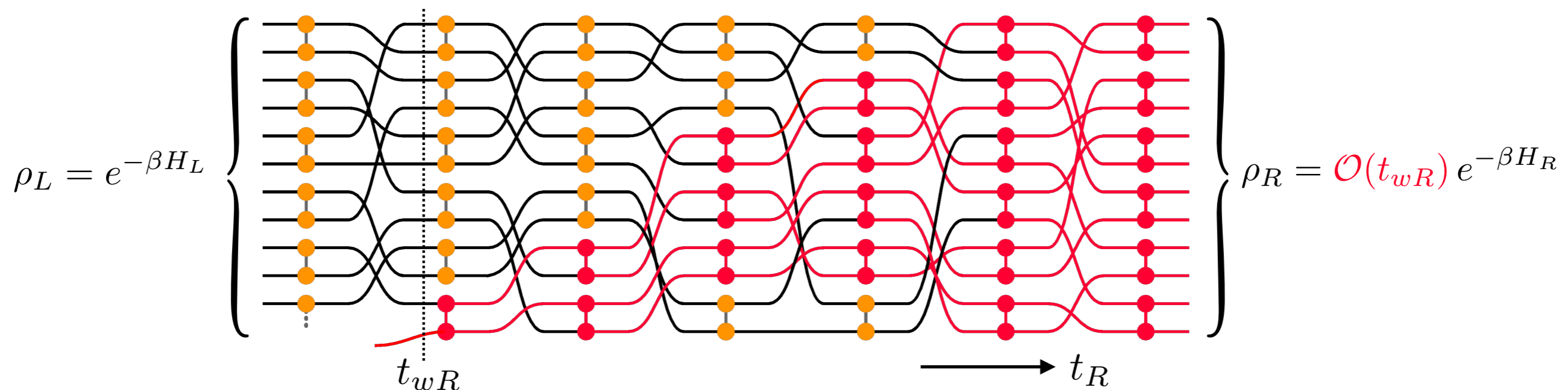
- ▶ Evolution in t_L can be **undone** by inverse evolution in t_R ...
- ▶ ...unless we change the state on one side (add a perturbation)!



- ▶ Epidemic model for “**operator growth**” [Susskind/Zhao '14]
[Roberts/Stanford/Streicher '18]...
- ▶ **Quantum butterfly effect:**
small perturbation has exponential effect on the future system
- ▶ R's p.o.v.:
go back in time \rightarrow insert tiny perturbation \rightarrow evolve forward \rightarrow get very different state

Quantum butterfly effect

- ▶ Evolution in t_L can be **undone** by inverse evolution in t_R ...
- ▶ ...unless we change the state on one side (add a perturbation)!



- ▶ Epidemic model for **“operator growth”**

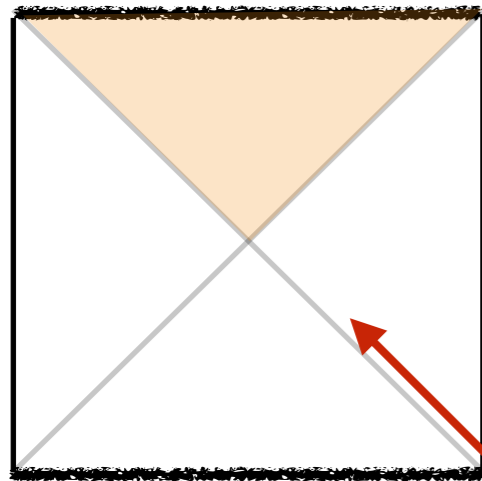
$$\frac{n_{\text{perturbed}}(t_R)}{n_{\text{tot}}} \sim \frac{1}{1 + e^{-(t_R - t_{wR} - t_*)}} \approx \Theta(t_R - t_{wR} - t_*)$$

$$t_* = \frac{\beta}{2\pi} \log \left(\frac{S}{\delta S} \right)$$

“scrambling time”

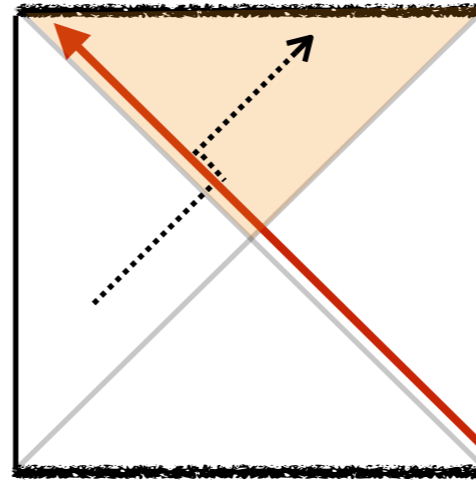
Shockwave geometry

- ▶ In gravity: send a signal from one side into the black hole

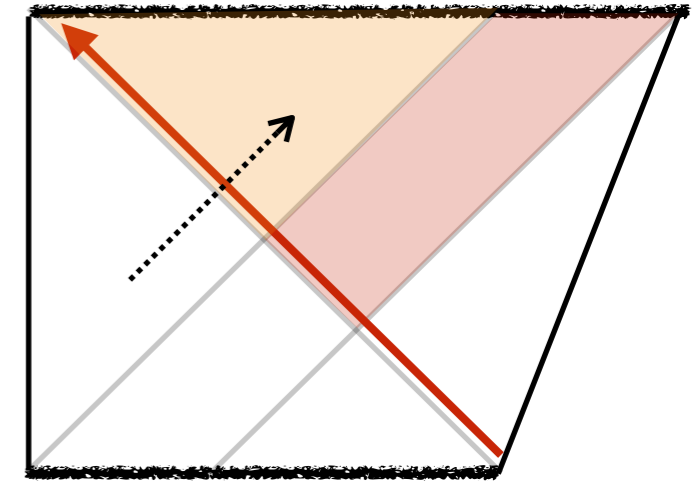


$$|\Psi\rangle = \mathcal{O}_R(t_w R) |\text{TFD}\rangle$$

Result is a shockwave geometry:

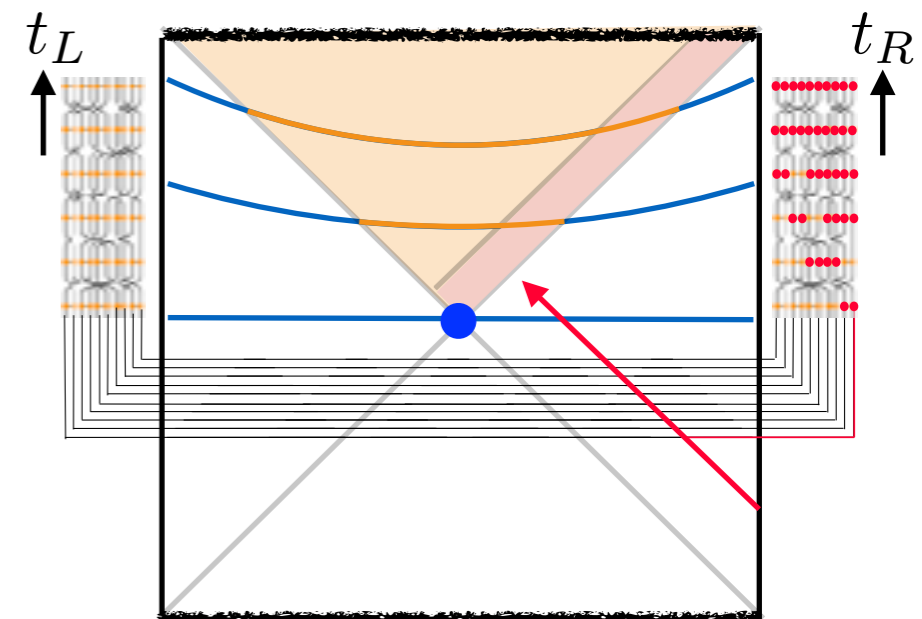


\mathbb{R}



[Dray/'t Hooft '85]...[Shenker/Stanford '13]
[Stanford/Susskind '14]

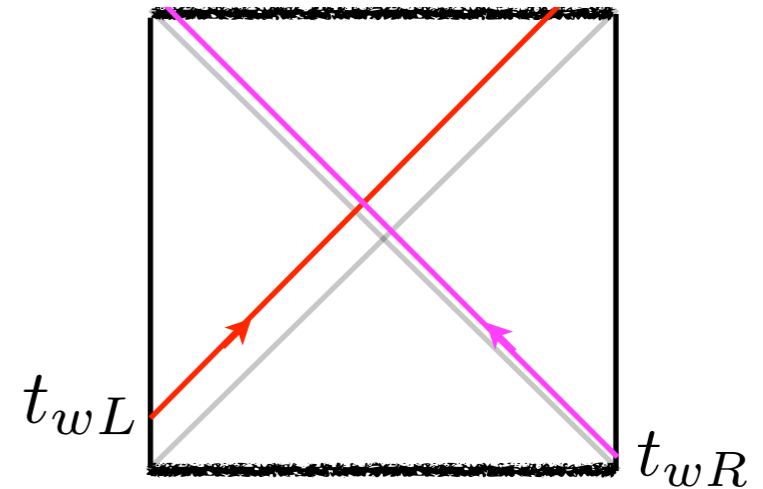
- ▶ Proposal:
(3) Gates affected by perturbation represent the part of the interior geometry accessible by CFT_R only.



Collisions in the interior

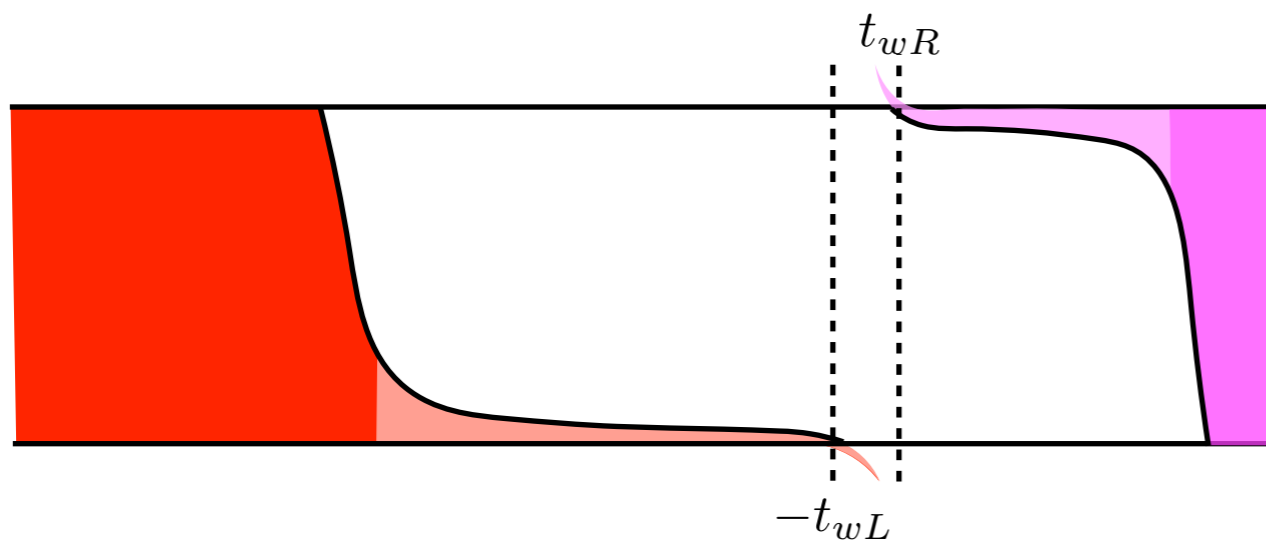
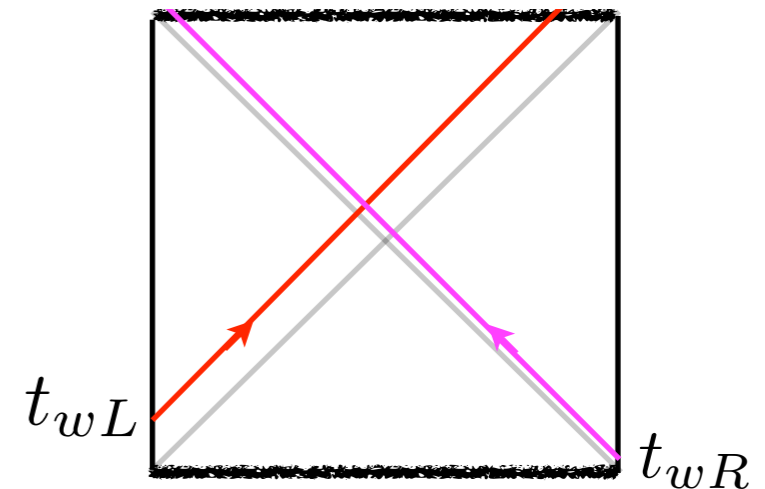
Collision in the interior

- ▶ Puzzle: two signals sent into black hole can meet, even though the dual CFTs don't interact.
 - ▶ Send too late: no collision
 - ▶ Send earlier: collision in the interior
 - ▶ Send very early: highly boosted, expect strong backreaction
- ▶ Correspondingly, overlapping perturbations in circuit:



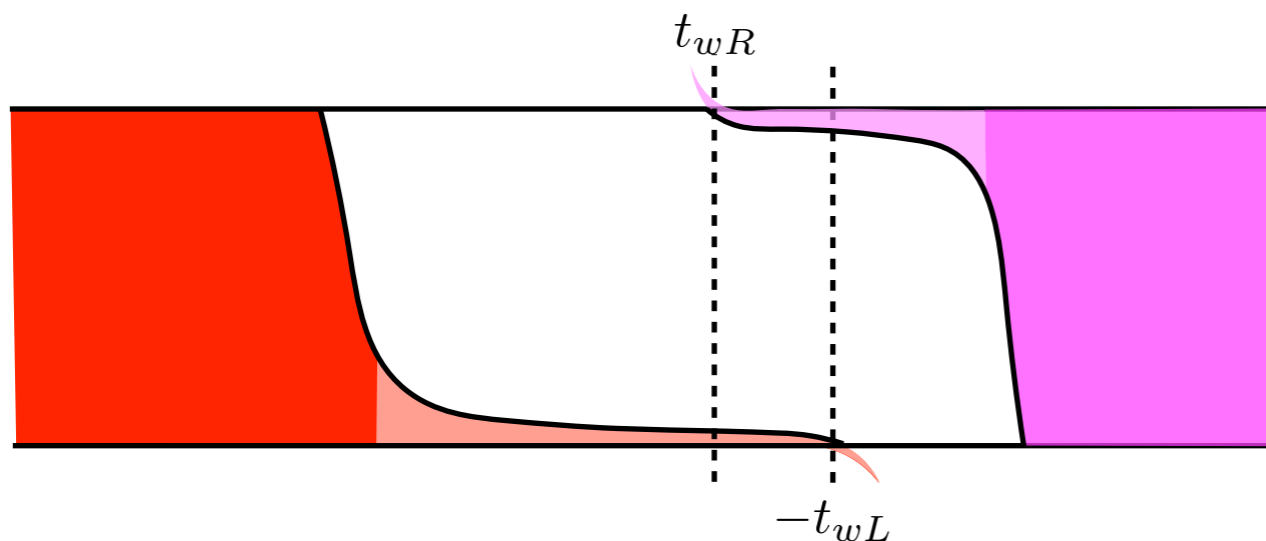
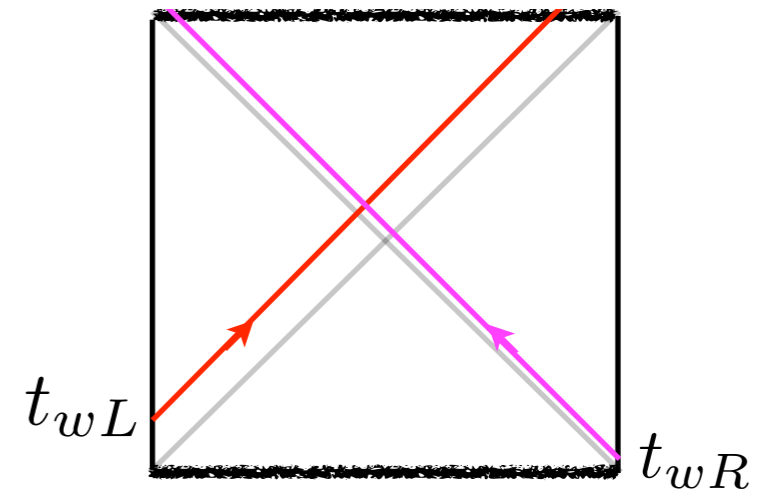
Collision in the interior

- ▶ Puzzle: two signals sent into black hole can meet, even though the dual CFTs don't interact.
 - ▶ Send too late: no collision
 - ▶ Send earlier: collision in the interior
 - ▶ Send very early: highly boosted, expect strong backreaction
- ▶ Correspondingly, overlapping perturbations in circuit:



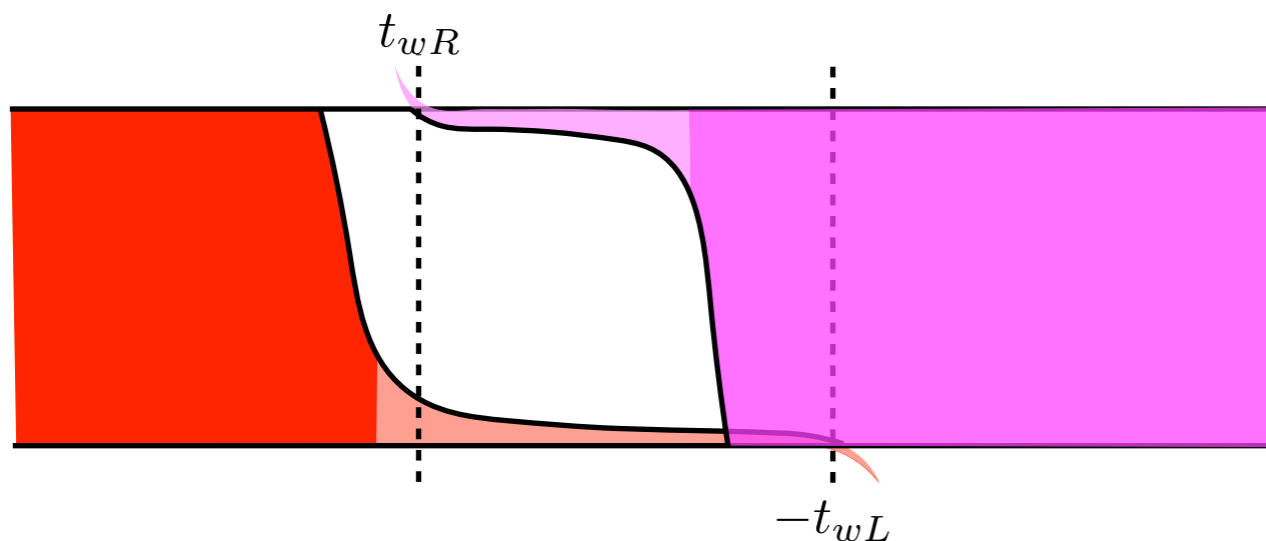
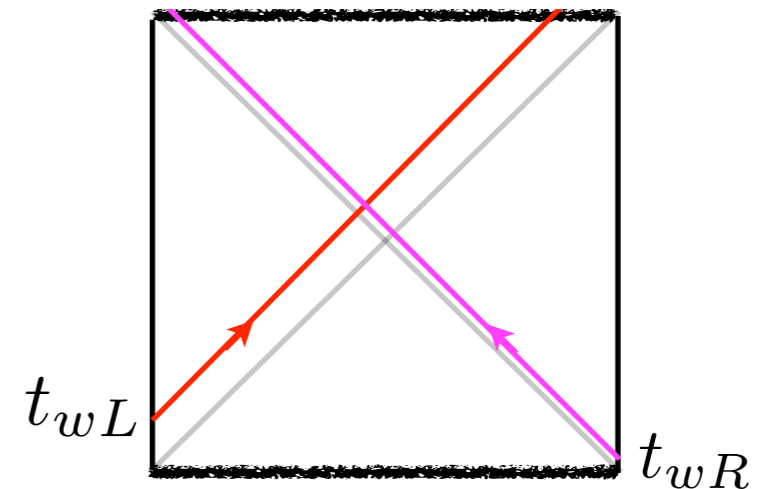
Collision in the interior

- ▶ Puzzle: two signals sent into black hole can meet, even though the dual CFTs don't interact.
 - ▶ Send too late: no collision
 - ▶ Send earlier: collision in the interior
 - ▶ Send very early: highly boosted, expect strong backreaction
- ▶ Correspondingly, overlapping perturbations in circuit:



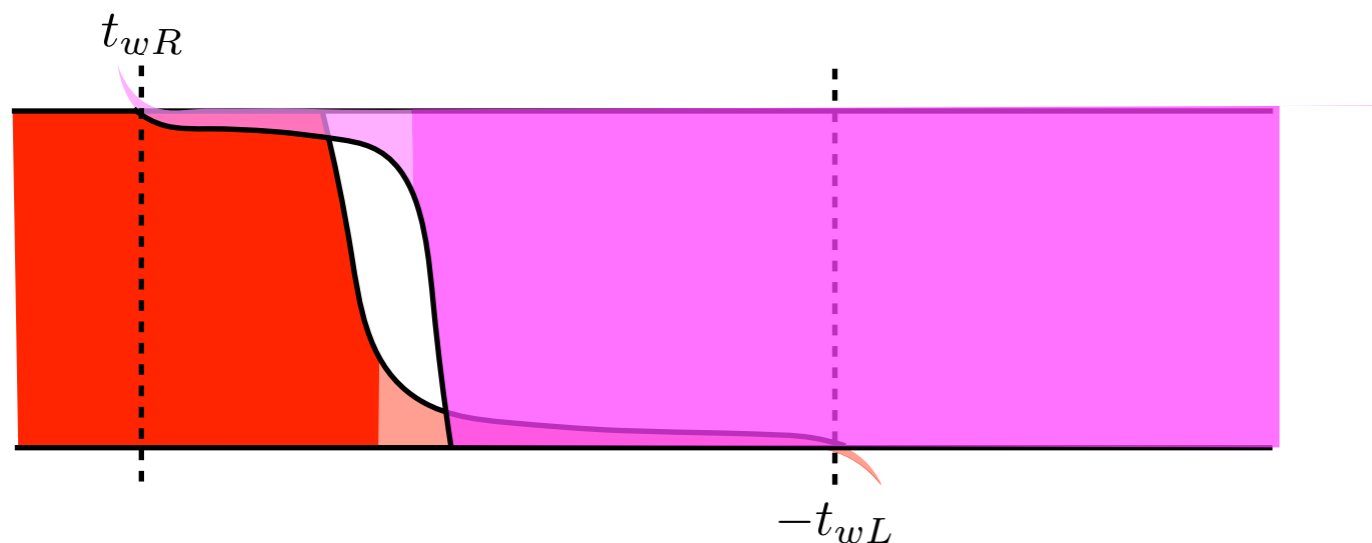
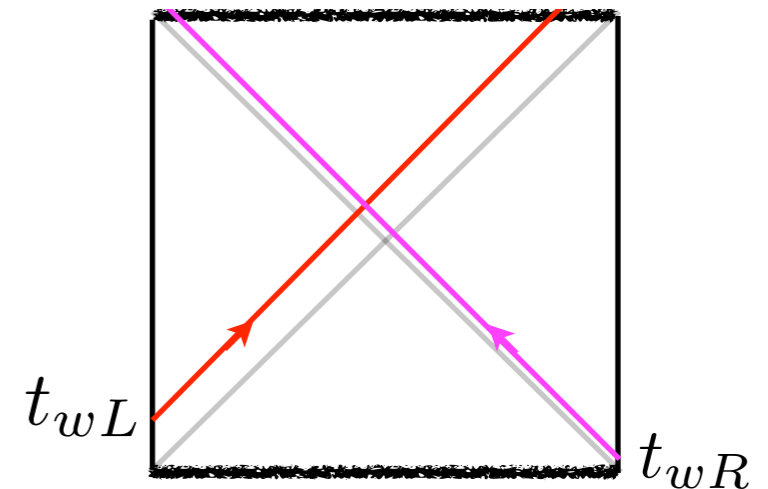
Collision in the interior

- ▶ Puzzle: two signals sent into black hole can meet, even though the dual CFTs don't interact.
 - ▶ Send too late: no collision
 - ▶ Send earlier: collision in the interior
 - ▶ Send very early: highly boosted, expect strong backreaction
- ▶ Correspondingly, overlapping perturbations in circuit:



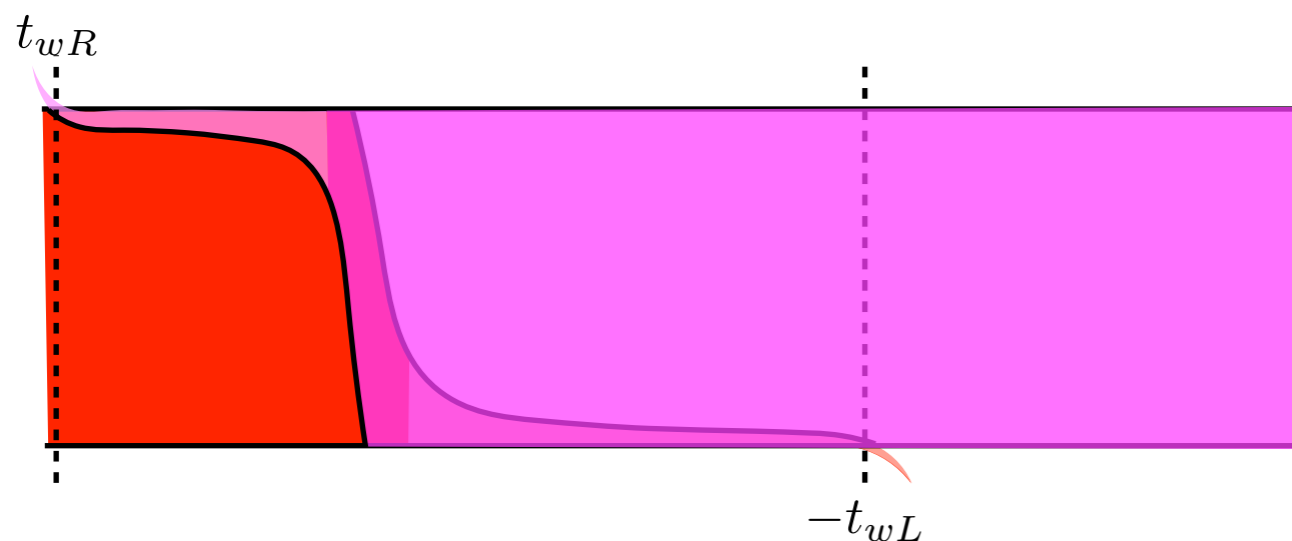
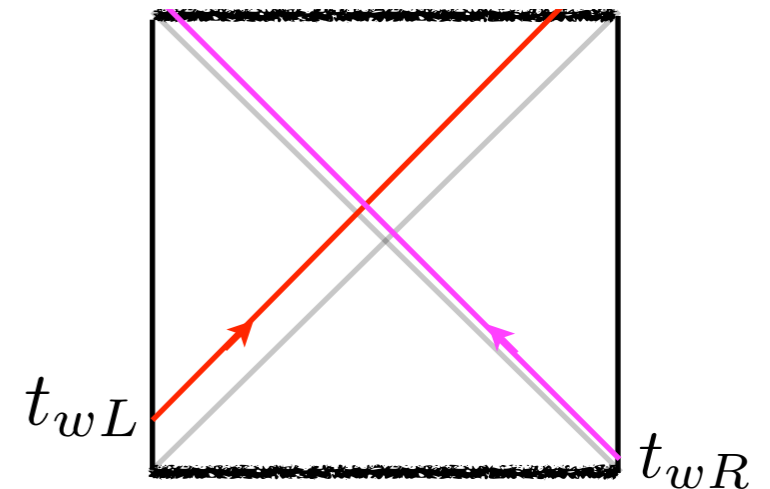
Collision in the interior

- ▶ Puzzle: two signals sent into black hole can meet, even though the dual CFTs don't interact.
 - ▶ Send too late: no collision
 - ▶ Send earlier: collision in the interior
 - ▶ Send very early: highly boosted, expect strong backreaction
- ▶ Correspondingly, overlapping perturbations in circuit:



Collision in the interior

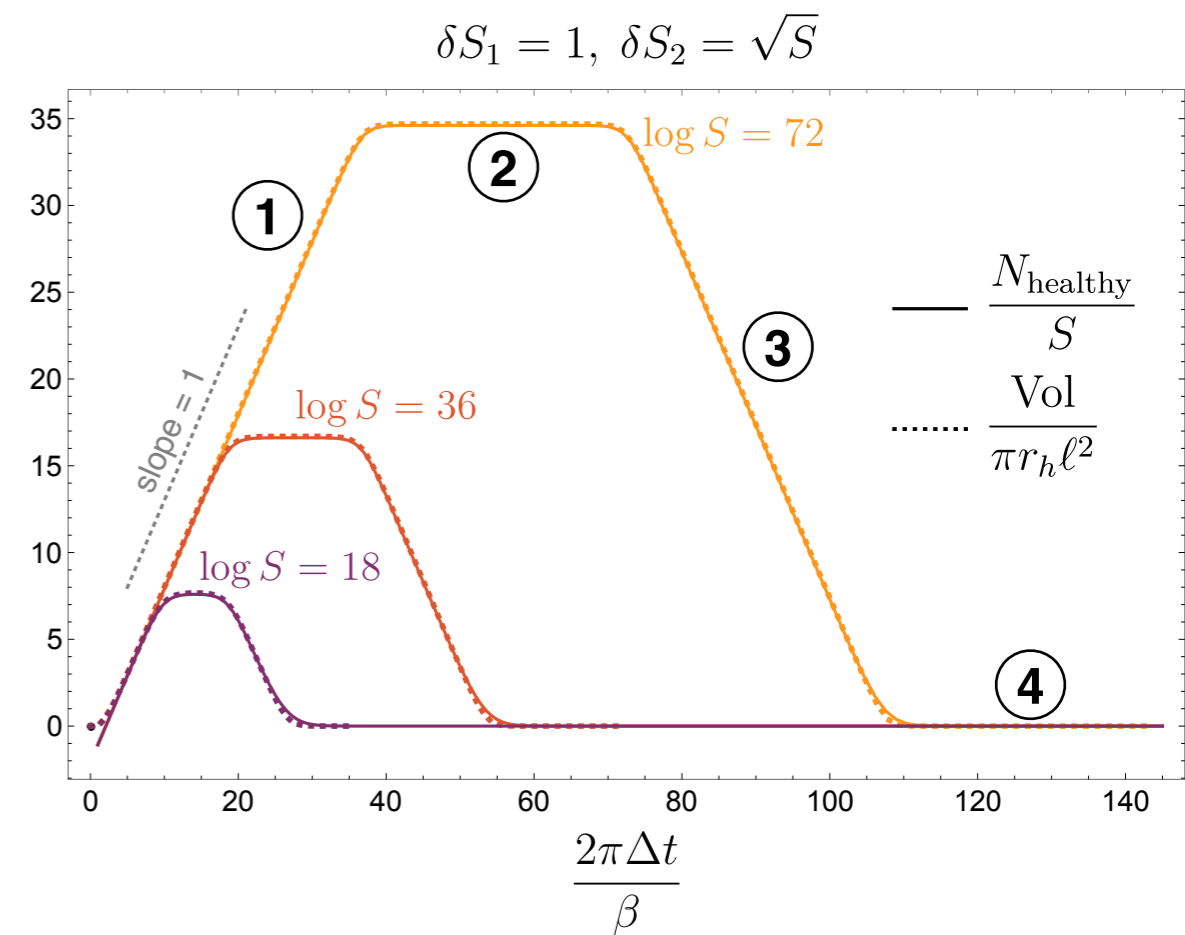
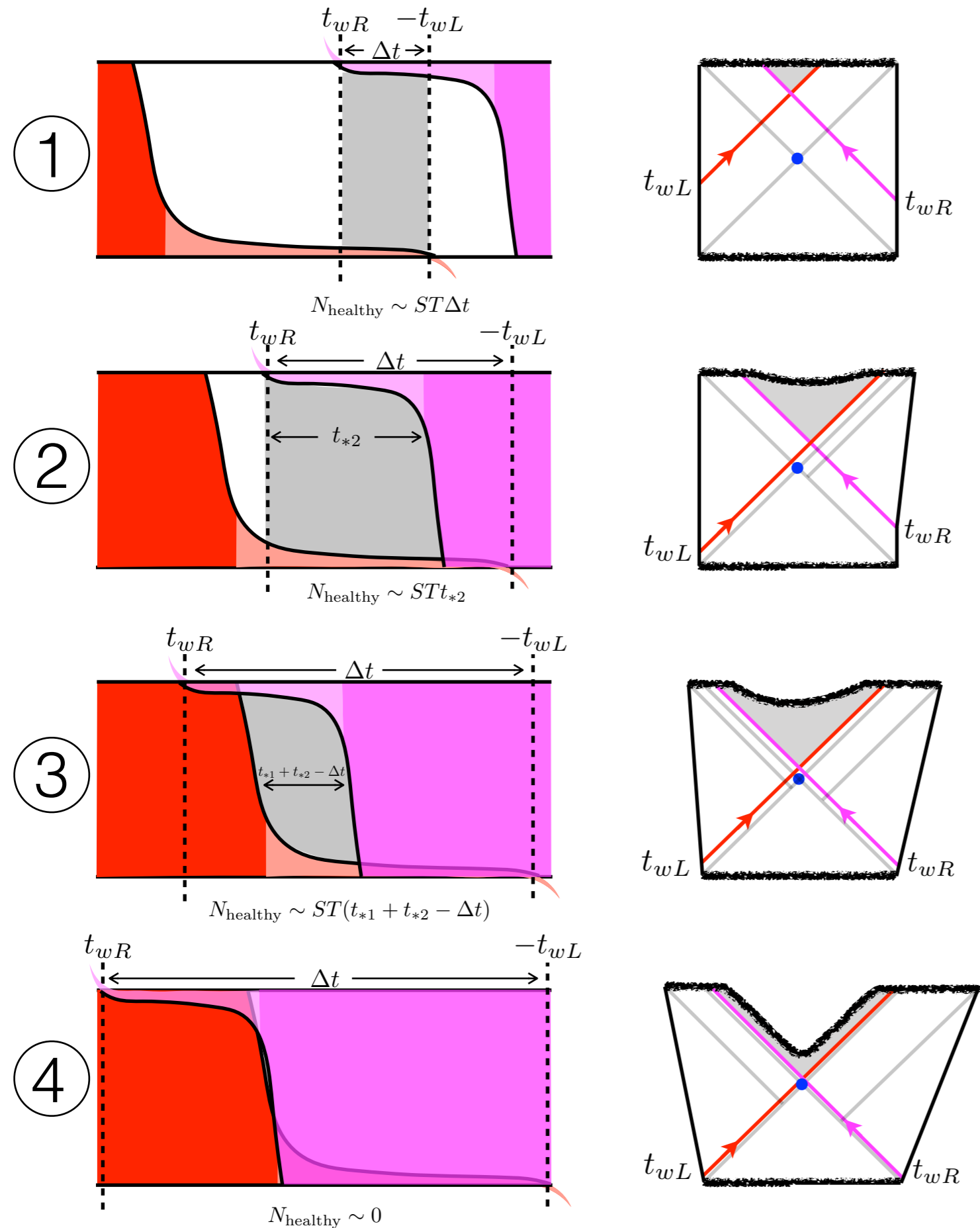
- ▶ Puzzle: two signals sent into black hole can meet, even though the dual CFTs don't interact.
 - ▶ Send too late: no collision
 - ▶ Send earlier: collision in the interior
 - ▶ Send very early: highly boosted, expect strong backreaction
- ▶ Correspondingly, overlapping perturbations in circuit:



Proposal:

(4) Overlap in circuit describes gravitational interaction. It represents post-collision spacetime.

$$\#(\text{healthy gates}) \propto \text{Vol}(\text{post-coll.})$$



► **Circuit model predicts:**
 post-collision volume becomes exponentially small after high-energy collision

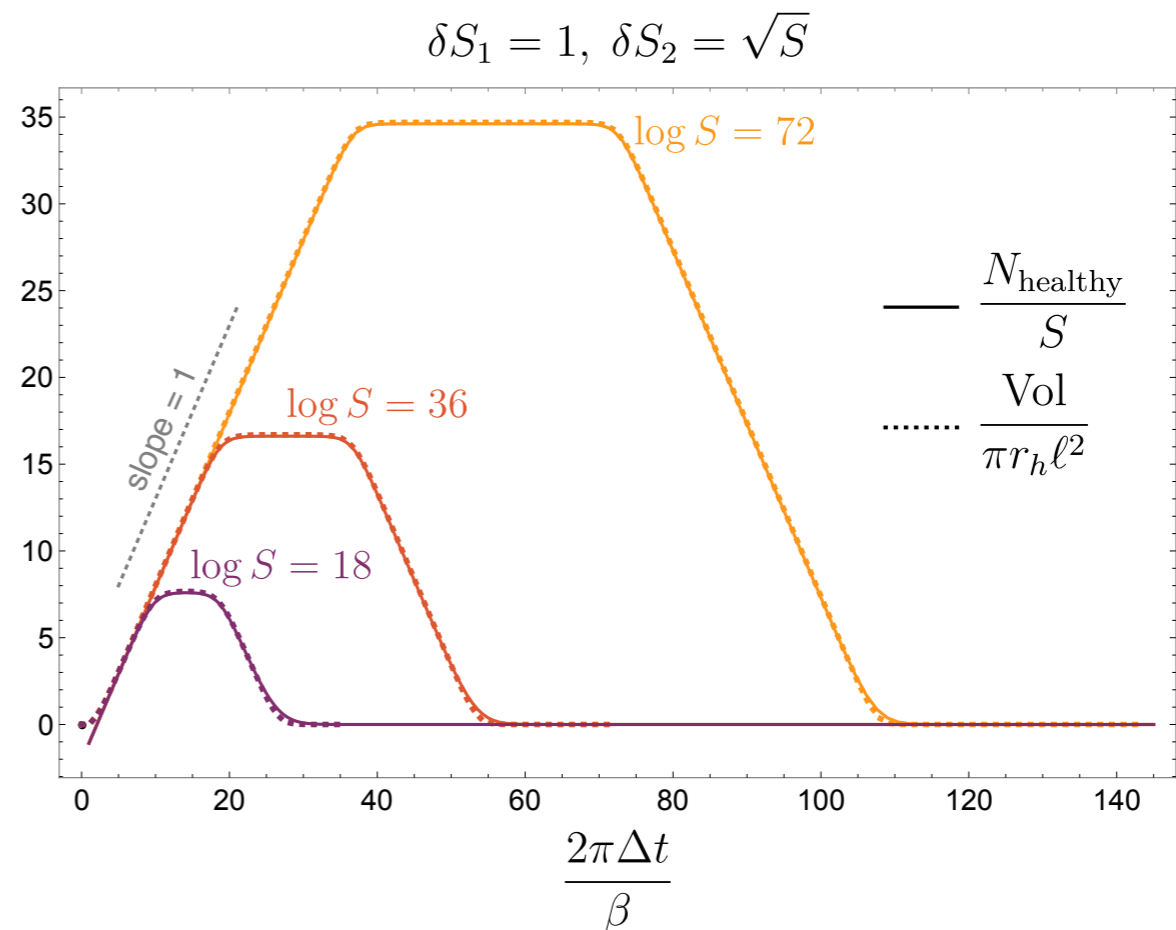
Checks

- ▶ Simplest check: spherically symmetric shocks in BTZ

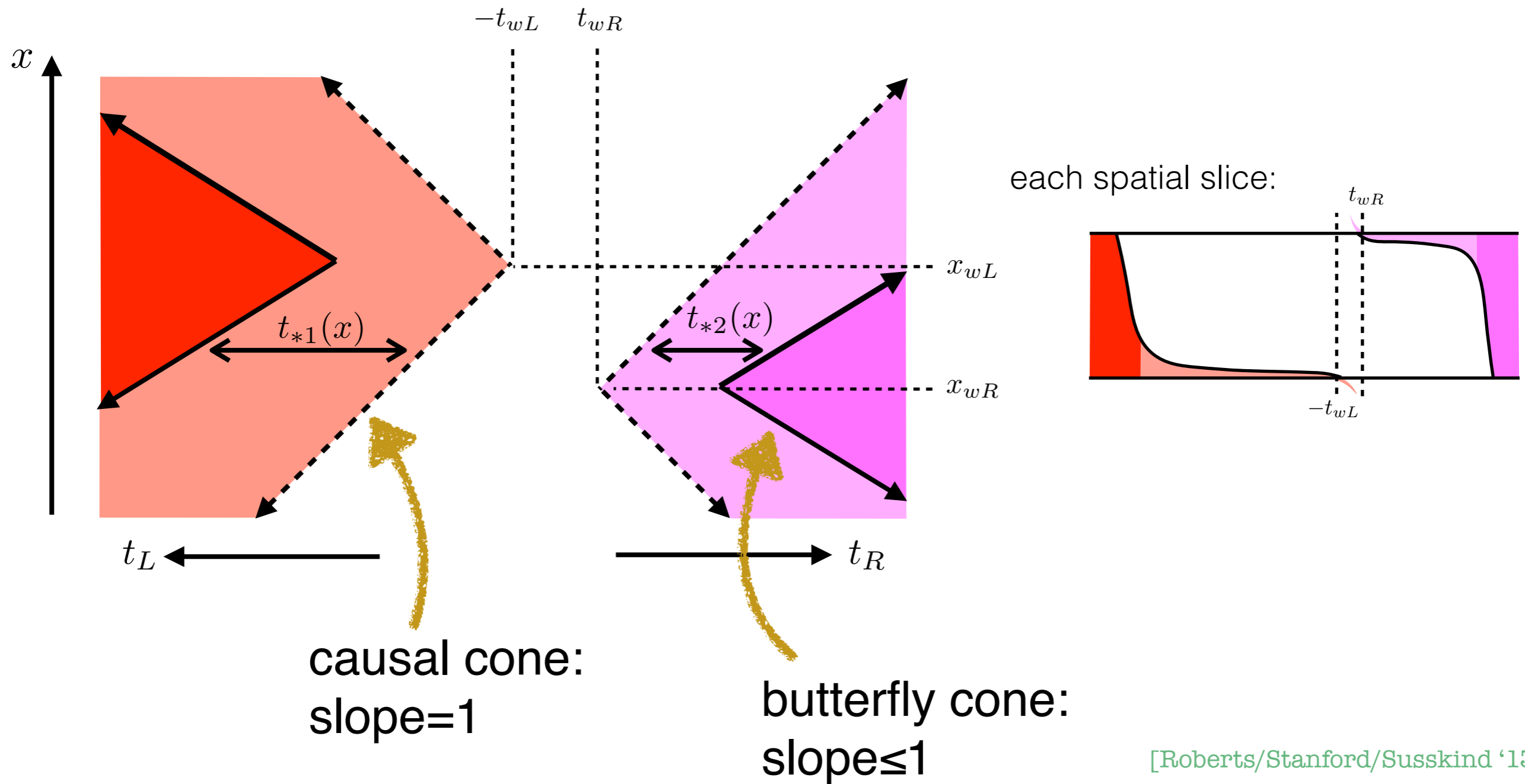
$$ds^2 = -\frac{4\ell^2}{(1+uv)^2} dudv + r_H^2 \frac{(1-uv)^2}{(1+uv)^2} d\phi^2$$

- ▶ Post-collision geometry remains locally AdS₃
- ▶ Glue BTZ patches along shock [Dray/'t Hooft '85]...[Shenker/Stanford '13]

- ▶ Very good match:



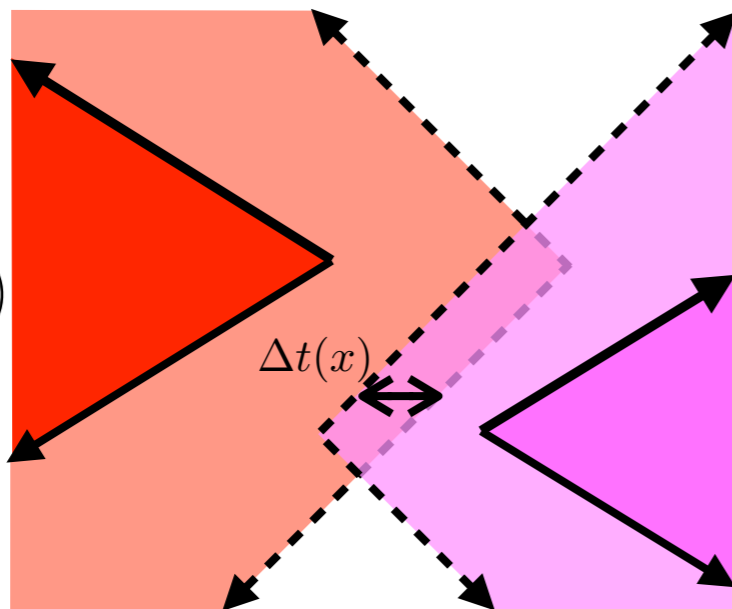
- ▶ More non-trivial checks: **localized shocks** (and higher dimensions)
- ▶ Add spatial direction(s) to the quantum circuit:



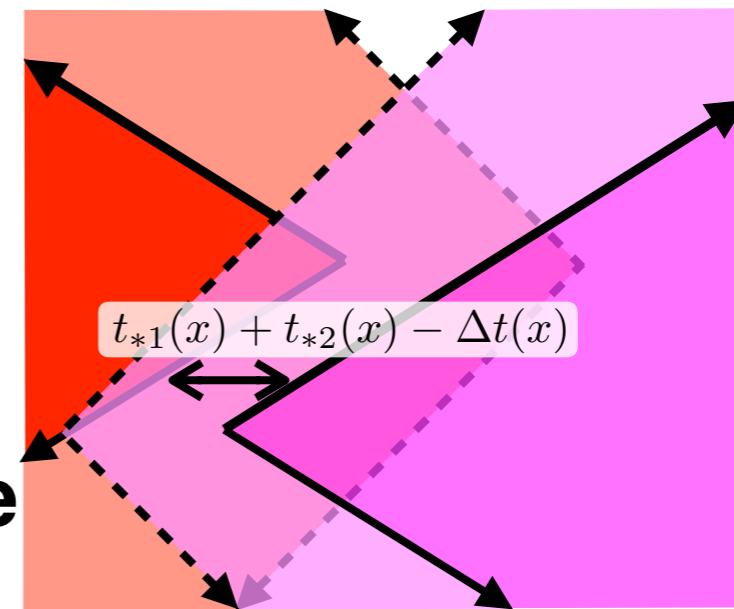
- ▶ Quantum chaos spreads ballistically with **butterfly velocity** v_B

- We have the same 4 regimes as before:

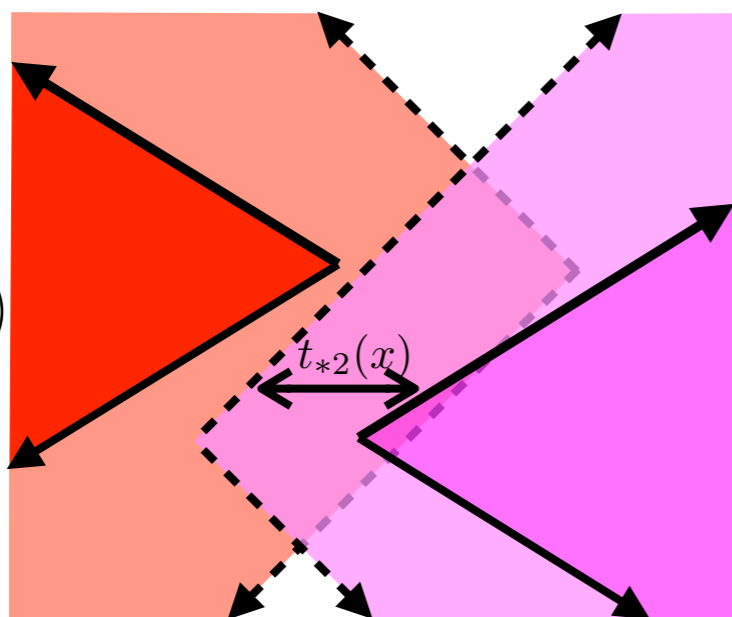
①
linear
growth



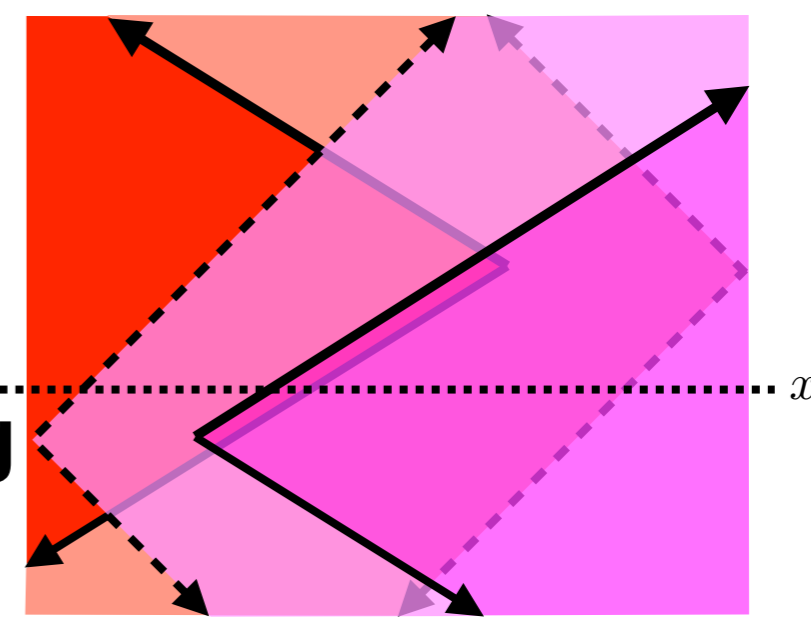
③
linear
decrease



②
constant

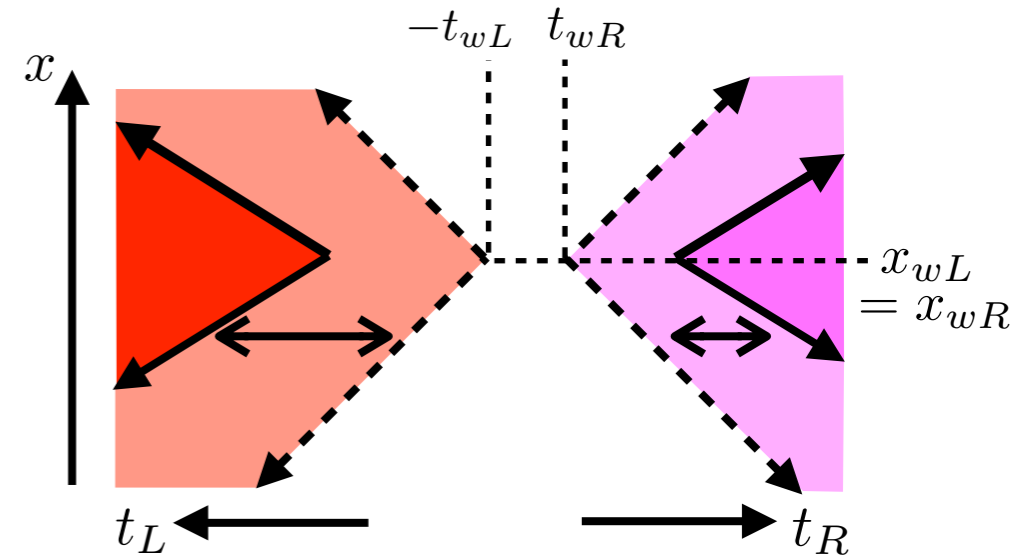
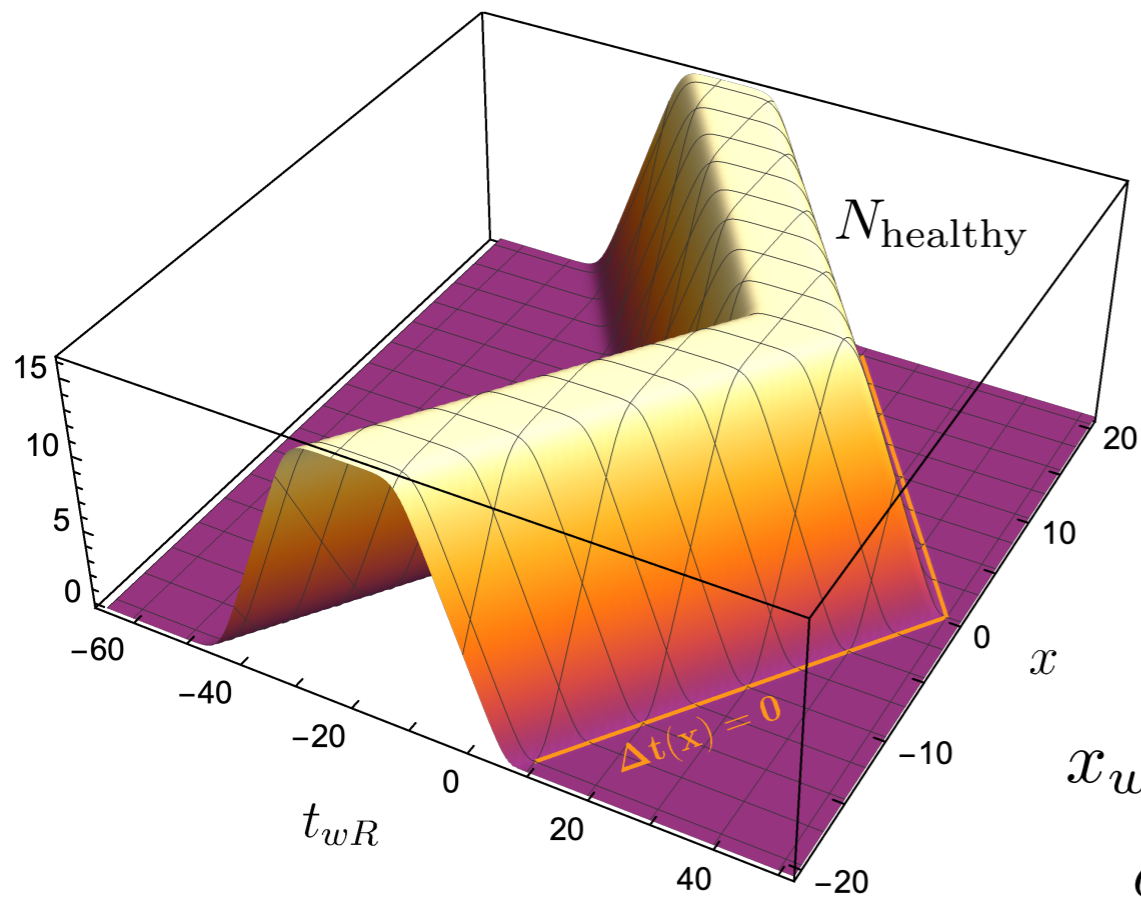


④
≈ vanishing



Prediction

(head-on, $d=2$)



$$x_{wL} = x_{wR} = 0, \quad t_{wL} = -50$$

$$\delta S_1 = 1, \quad \delta S_2 = \sqrt{S}$$

- ▶ Small Δt : checked linear growth of $\text{Vol}(x)$ numerically for localised shocks in planar/hyperbolic black holes
- ▶ Large Δt : ?

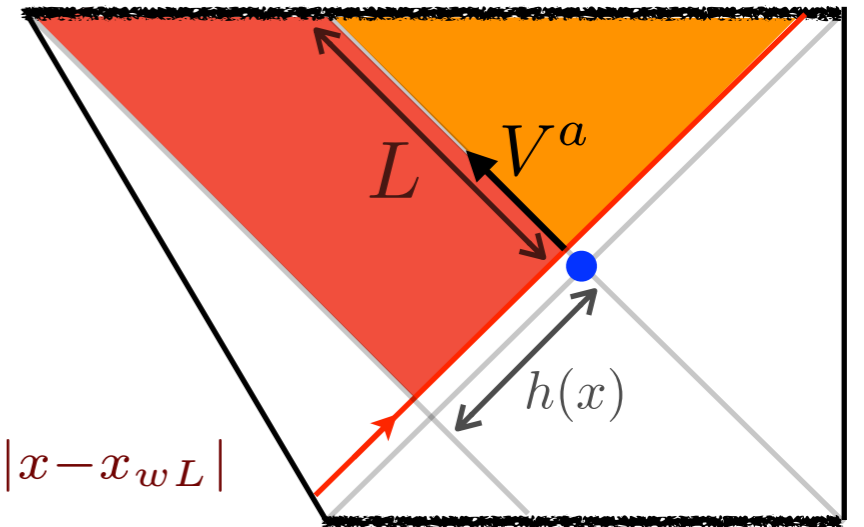
Late time regime in gravity

► At late times: derive upper bound on volume using Raychaudhuri eq.

• Single shock:

$$ds^2 = -\frac{4\ell^2}{(1+u\bar{v})^2} dudv + r_H^2 \left(\frac{1-u\bar{v}}{1+u\bar{v}} \right)^2 d\vec{x}^2$$

$$\bar{v} = v + \Theta(u)h(x) \quad h(x) \sim \frac{\delta S_1}{S} e^{-t_{wL} - \frac{1}{v_B}|x-x_{wL}|}$$



$$\text{Raychaudhuri:} \quad \dot{\theta} = -\frac{\theta^2}{d-1} - \sigma^2 + \omega^2 - R_{ab}K^a K^b + \dot{K}^a{}_{;a}$$

$$\text{across shock:} \quad T_{uu} \sim \delta(u)h(x) \quad \Rightarrow \quad \Delta\theta \sim -h(x)$$

$$\text{after shock:} \quad \dot{\theta} \leq -\frac{\theta^2}{d-1} \quad \Rightarrow \quad L \leq -\frac{d-1}{\theta} \sim \frac{1}{h(x)}$$

Late time regime in gravity

► At late times: derive upper bound on volume using Raychaudhuri eq.

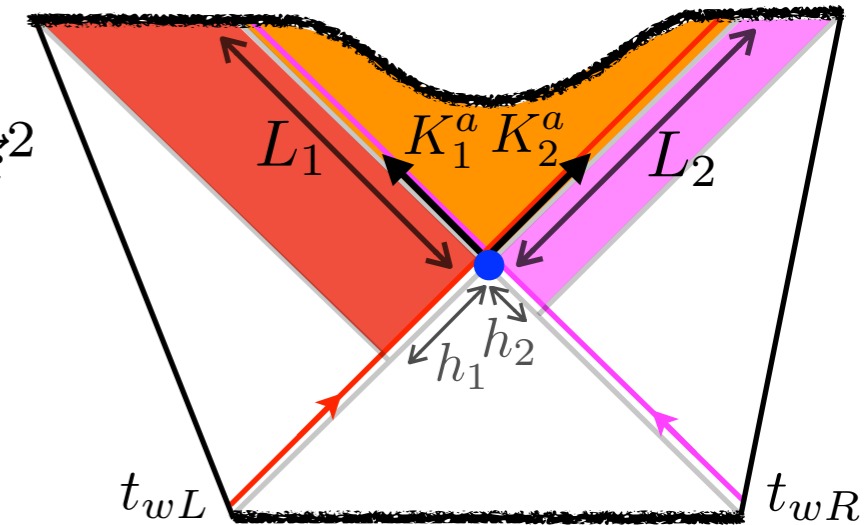
• Two shocks, ansatz:

$$ds_{\text{post-coll.}}^2 \approx -\frac{4\ell^2}{(1+uv)^2} dudv + \tilde{r}_H^2(x) \left(\frac{1-uv}{1+uv}\right)^2 d\vec{x}^2$$

strength of collision: $h_1(x)h_2(x) \sim \frac{\delta S_1 \delta S_2}{S^2} e^{\Delta t(x)}$

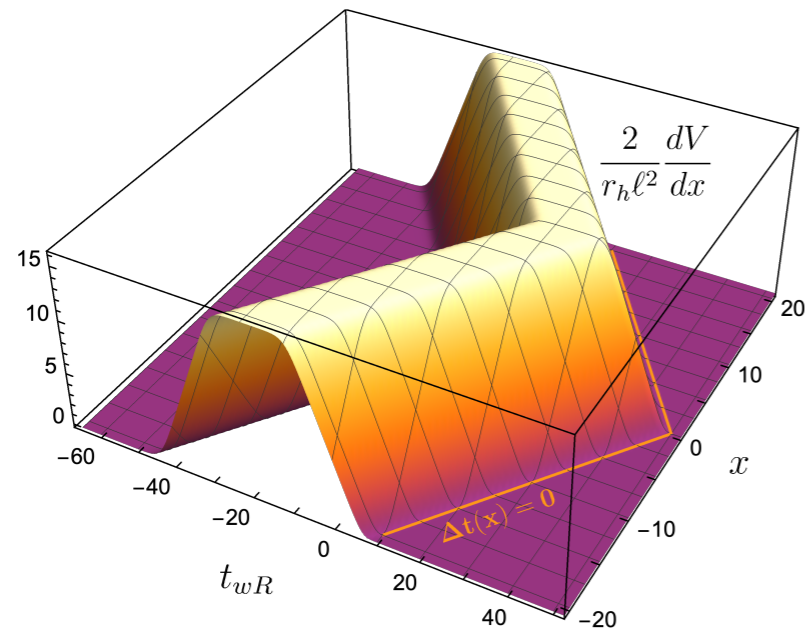
Raychaudhuri $\Rightarrow L_1 L_2 \leq \frac{\ell^2}{h_1(x)h_2(x)}$

\Rightarrow analytical upper bound on **Vol(post-collision)**

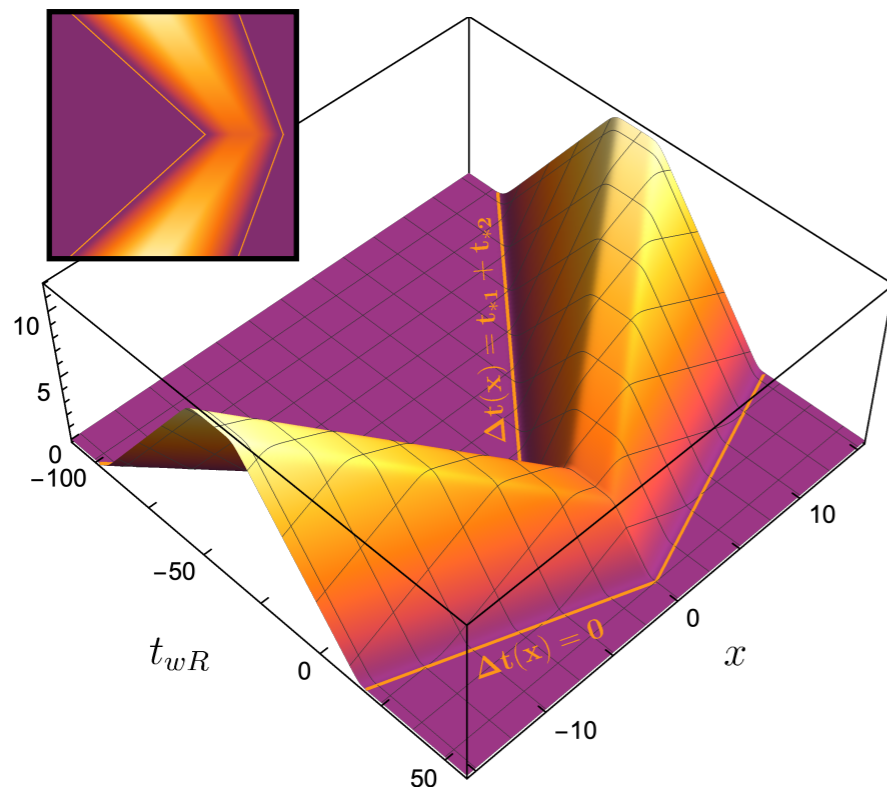
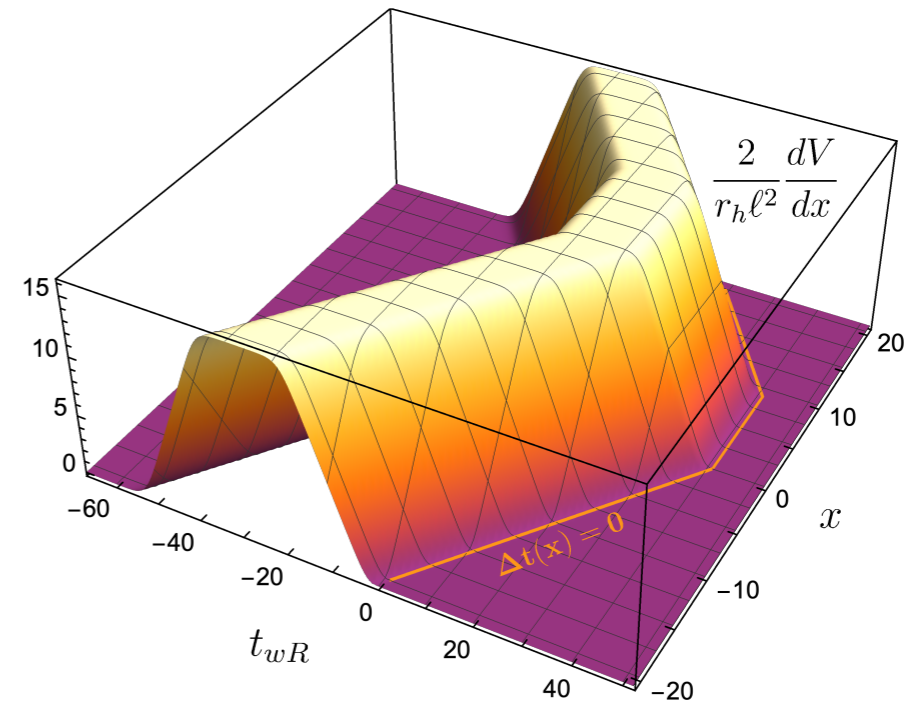


Gravity upper bound vs. circuit prediction

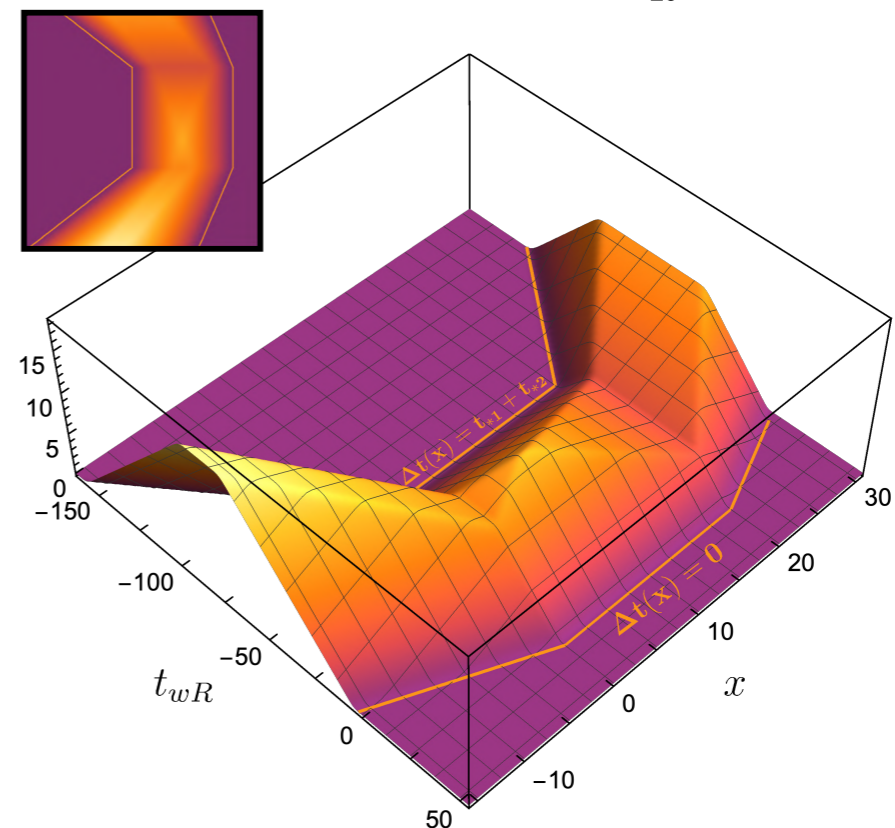
(head-on, AdS₃)



(finite impact parameter, AdS₃)



(head-on, AdS₅)



(finite impact parameter, AdS₅)

Operator size

Operator size

- ▶ Spread of perturbation through circuit: “growth” of an operator
- ▶ Make this more precise
- ▶ Toy model: **Sachdev-Ye-Kitaev quantum mechanics**

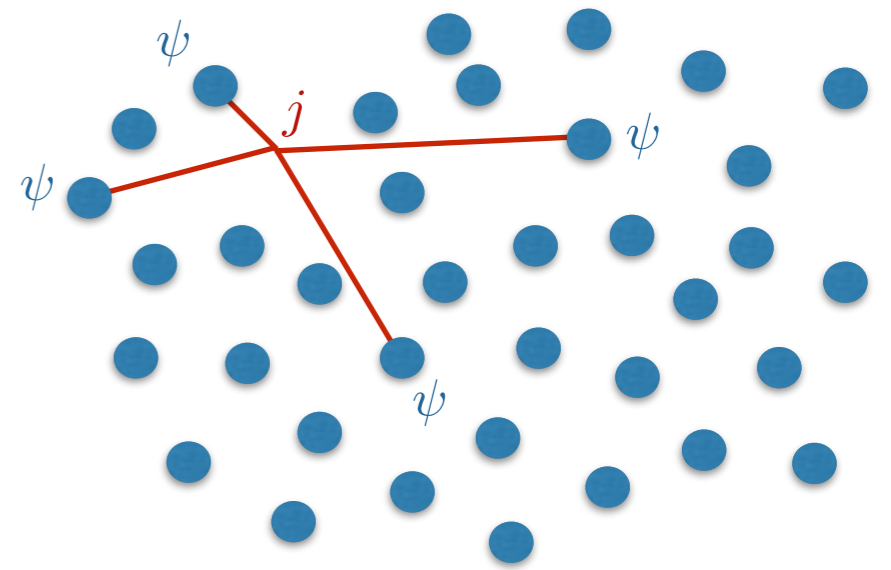
- N Majorana fermions with random, Gaussian all-to-all couplings
- Emergent conformal symmetry (time reparametrizations $\tau \rightarrow f(\tau)$) in the IR
- Symmetry is spontaneously & explicitly broken. Effective action for Goldstone:

$$I_{\text{eff.}}[f] \propto -\frac{N}{J} \int d\tau \text{Schw}(f(\tau), \tau)$$

- Same action describes AdS₂ dilaton gravity

$$H = - \sum_{ijkl}^N j_{ijkl} \psi_i \psi_j \psi_k \psi_l$$

$$\overline{j_{ijkl}} = 0, \quad \overline{j_{ijkl}^2} = J^2 / N^3$$



[Sachdev/Ye '93] [Kitaev '15]
[Maldacena/Stanford '16] ...

Operator size

- ▶ Spread of perturbation through circuit: “growth” of an operator
- ▶ Make this more precise
- ▶ Toy model: **Sachdev-Ye-Kitaev quantum mechanics**

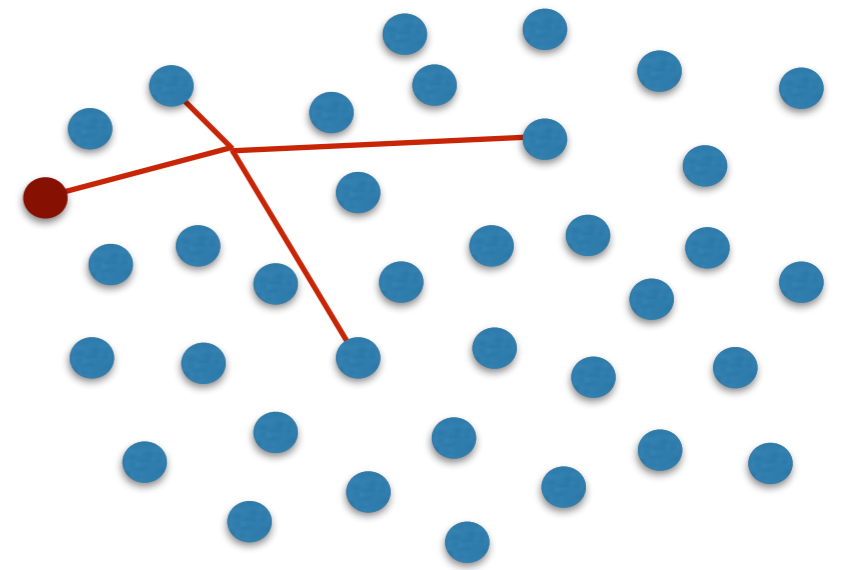
- N Majorana fermions with random, Gaussian all-to-all couplings
- Emergent conformal symmetry (time reparametrizations $\tau \rightarrow f(\tau)$) in the IR
- Symmetry is spontaneously & explicitly broken. Effective action for Goldstone:

$$I_{\text{eff.}}[f] \propto -\frac{N}{J} \int d\tau \text{Schw}(f(\tau), \tau)$$

- Same action describes AdS₂ dilaton gravity

$$H = - \sum_{ijkl}^N j_{ijkl} \psi_i \psi_j \psi_k \psi_l$$

$$\overline{j_{ijkl}} = 0, \quad \overline{j_{ijkl}^2} = J^2 / N^3$$



[Sachdev/Ye '93] [Kitaev '15]
[Maldacena/Stanford '16] ...

Operator size

- ▶ Spread of perturbation through circuit: “growth” of an operator
- ▶ Make this more precise
- ▶ Toy model: **Sachdev-Ye-Kitaev quantum mechanics**

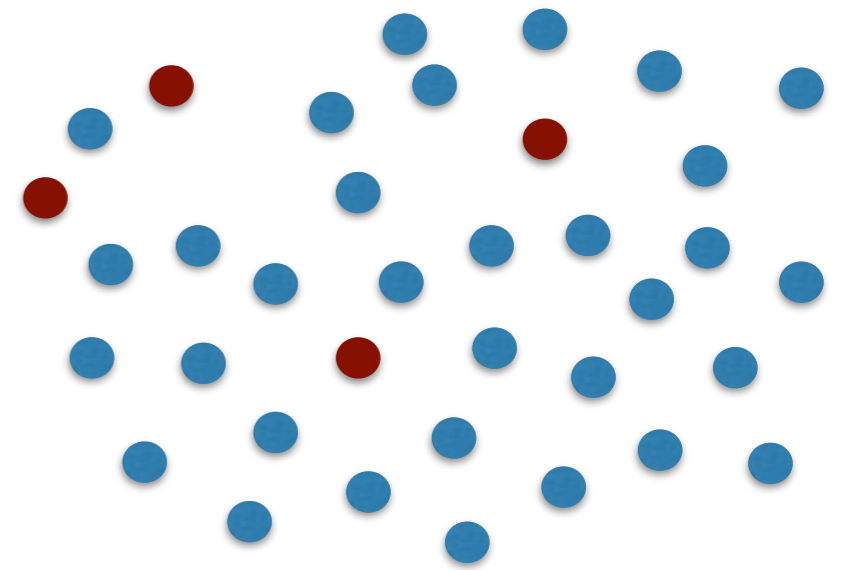
- N Majorana fermions with random, Gaussian all-to-all couplings
- Emergent conformal symmetry (time reparametrizations $\tau \rightarrow f(\tau)$) in the IR
- Symmetry is spontaneously & explicitly broken. Effective action for Goldstone:

$$I_{\text{eff.}}[f] \propto -\frac{N}{J} \int d\tau \text{Schw}(f(\tau), \tau)$$

- Same action describes AdS₂ dilaton gravity

$$H = - \sum_{ijkl}^N j_{ijkl} \psi_i \psi_j \psi_k \psi_l$$

$$\overline{j_{ijkl}} = 0, \quad \overline{j_{ijkl}^2} = J^2 / N^3$$



[Sachdev/Ye '93] [Kitaev '15]

[Maldacena/Stanford '16] ...

Operator size

- ▶ Spread of perturbation through circuit: “growth” of an operator
- ▶ Make this more precise
- ▶ Toy model: **Sachdev-Ye-Kitaev quantum mechanics**

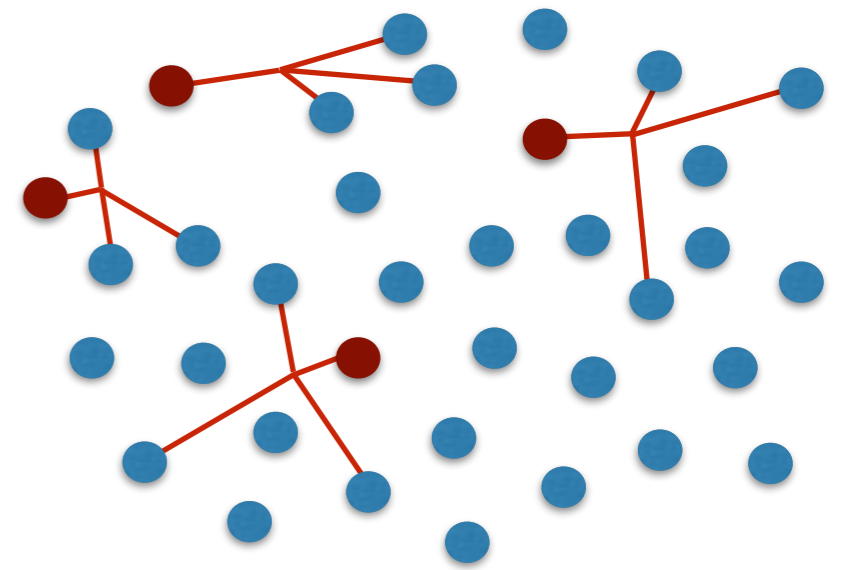
- N Majorana fermions with random, Gaussian all-to-all couplings
- Emergent conformal symmetry (time reparametrizations $\tau \rightarrow f(\tau)$) in the IR
- Symmetry is spontaneously & explicitly broken. Effective action for Goldstone:

$$I_{\text{eff.}}[f] \propto -\frac{N}{J} \int d\tau \text{Schw}(f(\tau), \tau)$$

- Same action describes AdS₂ dilaton gravity

$$H = - \sum_{ijkl}^N j_{ijkl} \psi_i \psi_j \psi_k \psi_l$$

$$\overline{j_{ijkl}} = 0, \quad \overline{j_{ijkl}^2} = J^2 / N^3$$



[Sachdev/Ye '93] [Kitaev '15]
[Maldacena/Stanford '16] ...

Operator size

- ▶ Spread of perturbation through circuit: “growth” of an operator
- ▶ Make this more precise
- ▶ Toy model: **Sachdev-Ye-Kitaev quantum mechanics**

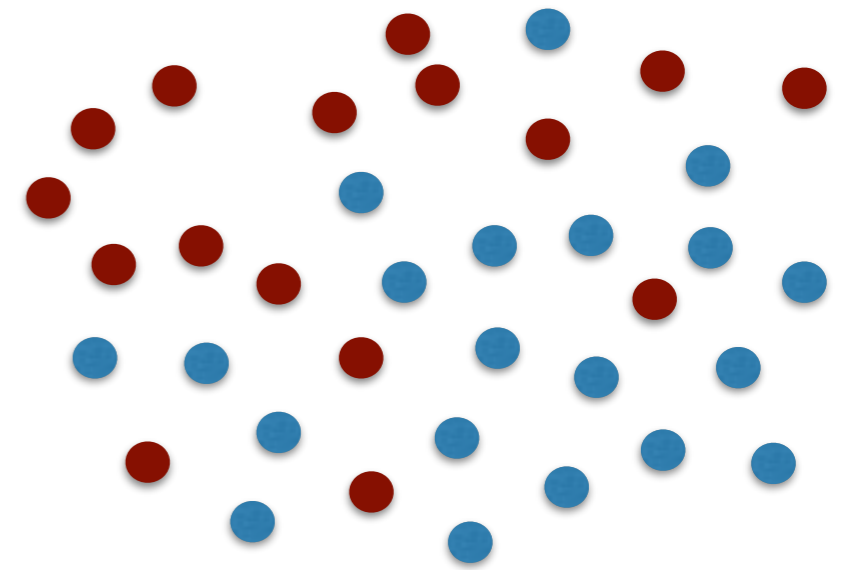
- N Majorana fermions with random, Gaussian all-to-all couplings
- Emergent conformal symmetry (time reparametrizations $\tau \rightarrow f(\tau)$) in the IR
- Symmetry is spontaneously & explicitly broken. Effective action for Goldstone:

$$I_{\text{eff.}}[f] \propto -\frac{N}{J} \int d\tau \text{Schw}(f(\tau), \tau)$$

- Same action describes AdS₂ dilaton gravity

$$H = - \sum_{ijkl}^N j_{ijkl} \psi_i \psi_j \psi_k \psi_l$$

$$\overline{j_{ijkl}} = 0, \quad \overline{j_{ijkl}^2} = J^2 / N^3$$



[Sachdev/Ye '93] [Kitaev '15]
[Maldacena/Stanford '16] ...

Operator size in SYK

▶ Orthogonal operator basis: $\Gamma_{i_1 \dots i_k} = i^{\frac{k(k-1)}{2}} \psi_{i_1} \dots \psi_{i_k}$ ($i_1 < \dots < i_k$, $\{\psi_i, \psi_j\} = 2\delta_{ij}$)

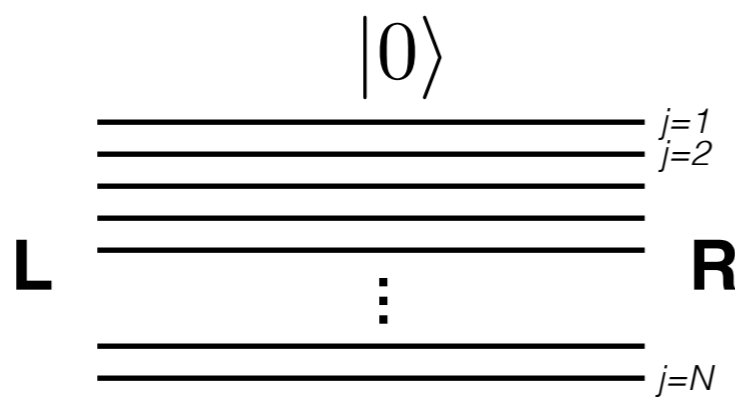
▶ Canonically purify the system: $\psi_i \rightarrow \psi_i^R, \psi_i^L$

“qubit” basis: $c_j = \frac{1}{2}(\psi_j^L + i\psi_j^R)$ $\{c_j, c_k\} = \{c_j^\dagger, c_k^\dagger\} = 0$, $\{c_j, c_k^\dagger\} = \delta_{jk}$

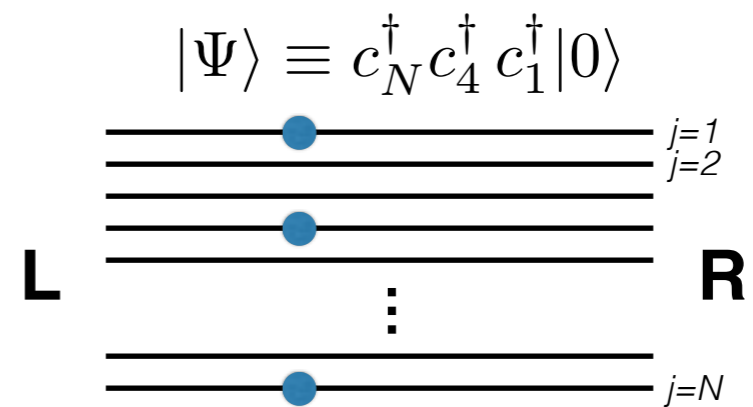
maximally entangled state: $c_j|0\rangle = 0 \quad \forall j$

▶ **“Size operator”**: $\hat{n}_\infty \equiv \sum_{j=1}^N c_j^\dagger c_j$
 [Qi/Streicher '18]

$$\langle \mathcal{O} | \hat{n}_\infty | \mathcal{O} \rangle = \# \text{ flavors contained in } \mathcal{O}$$



$$\Rightarrow \langle 0 | \hat{n}_\infty | 0 \rangle = 0$$



$$\Rightarrow \langle \Psi | \hat{n}_\infty | \Psi \rangle = 3$$

▶ Similar construction exists in more general models [Gu/Kitaev/Zhang '21]

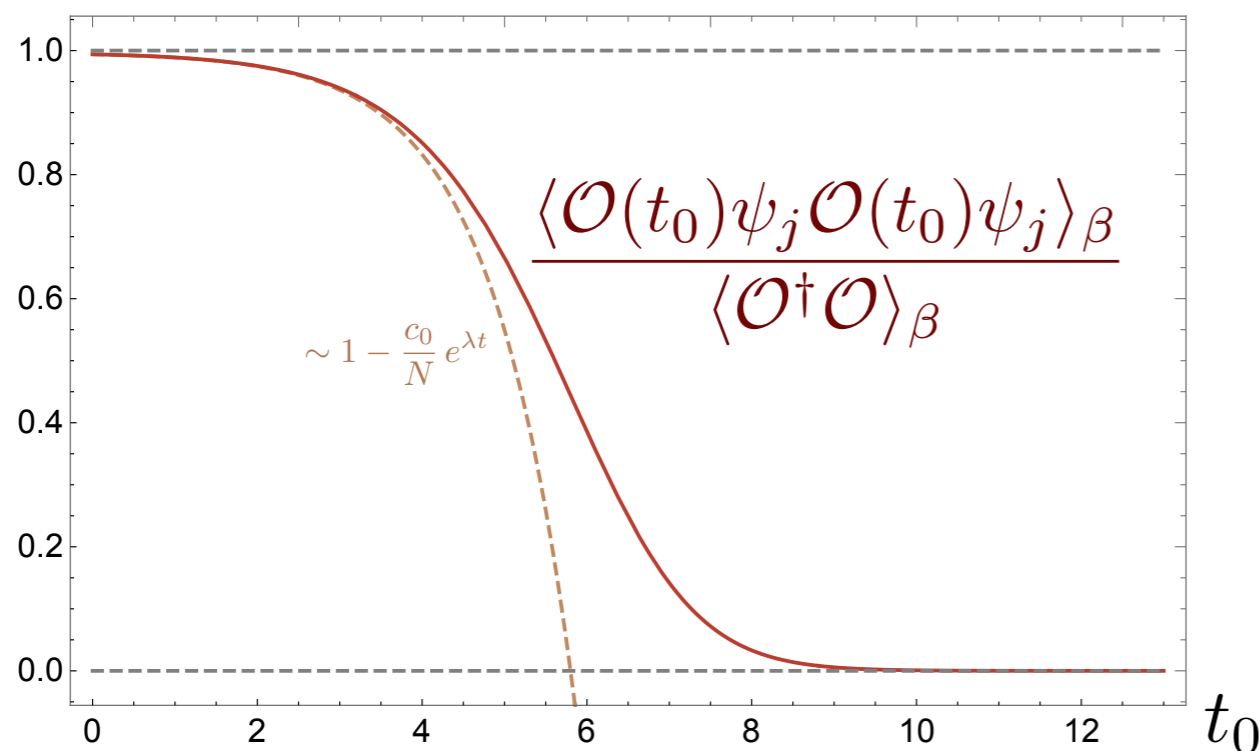
▶ Generalize to thermal state: $\langle \mathcal{O} | \hat{n}_\beta | \mathcal{O} \rangle = \langle \text{TFD} | \mathcal{O}^\dagger \hat{n}_\infty \mathcal{O} | \text{TFD} \rangle$ (up to regularisation)

▶ Explicit evaluation shows: $\frac{\langle \mathcal{O} | \hat{n}_\beta | \mathcal{O} \rangle}{\langle \mathcal{O}^\dagger \mathcal{O} \rangle_\beta} = n_{\max} \left[1 - \frac{1}{N} \sum_{j=1}^N \frac{\langle \mathcal{O}(t_0) \psi_j \mathcal{O}(t_0) \psi_j \rangle_\beta}{\langle \mathcal{O}^\dagger \mathcal{O} \rangle_\beta} \right]$

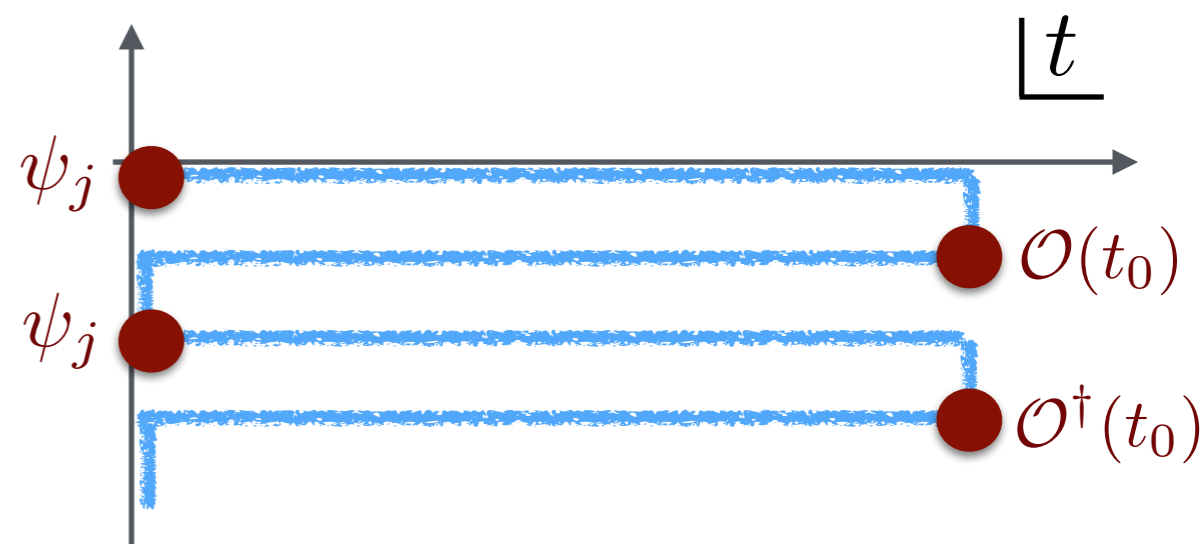
▶ OTOCs received a lot of attention as measures of quantum chaos

[Larkin/Ovchinnikov '68] [Kitaev '14]
[Shenker/Stanford '14] ... [FH/Rozali '18] ...

▶ They exhibit **exponential growth** when $t_0 \sim \beta \log(N)$, then saturate



“out-of-time-order”
correlation function

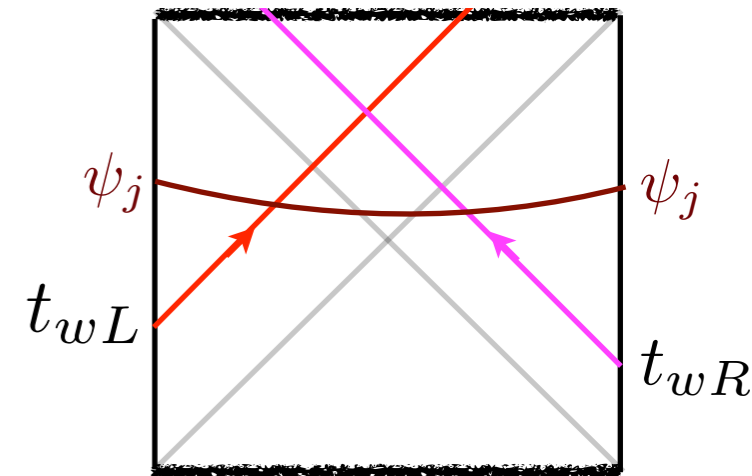


▶ “Epidemic” operator growth in quantum circuit matches more detailed OTOC calculations in SYK, Schwarzian, CFTs, ...

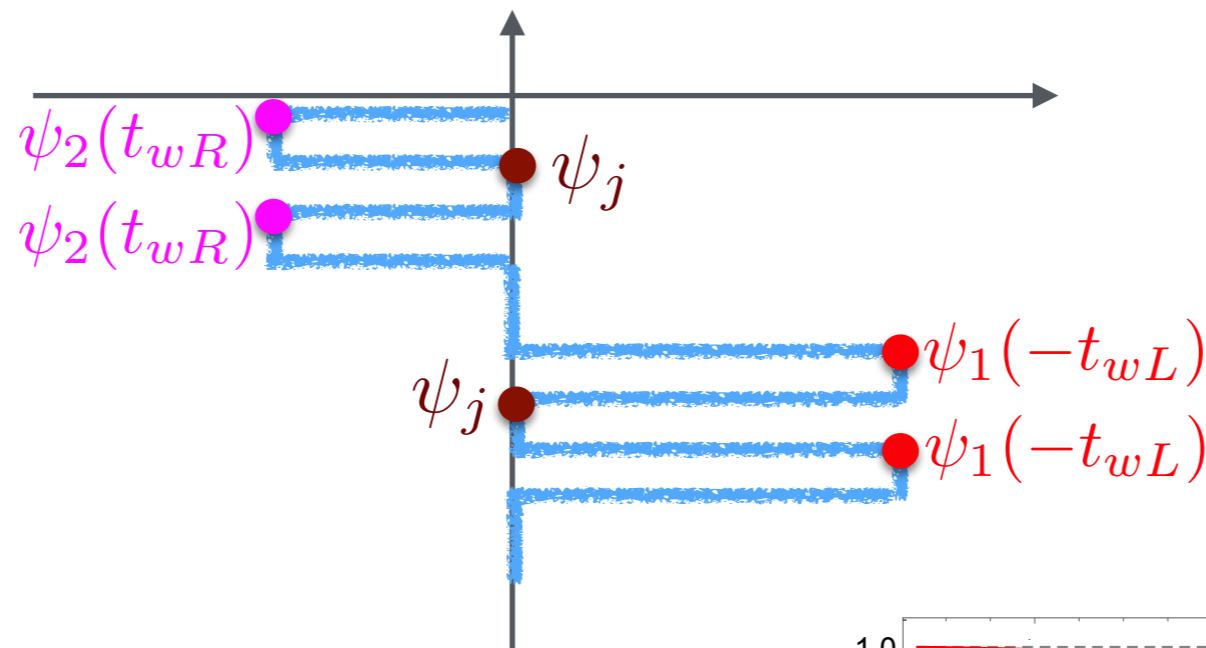
Interior collisions and OTOCs

- Apply this formalism to our collision setup:

$$|\Psi\rangle \equiv \psi_1^L(t_{wL})\psi_2^R(t_{wR})|\text{TFD}\rangle$$

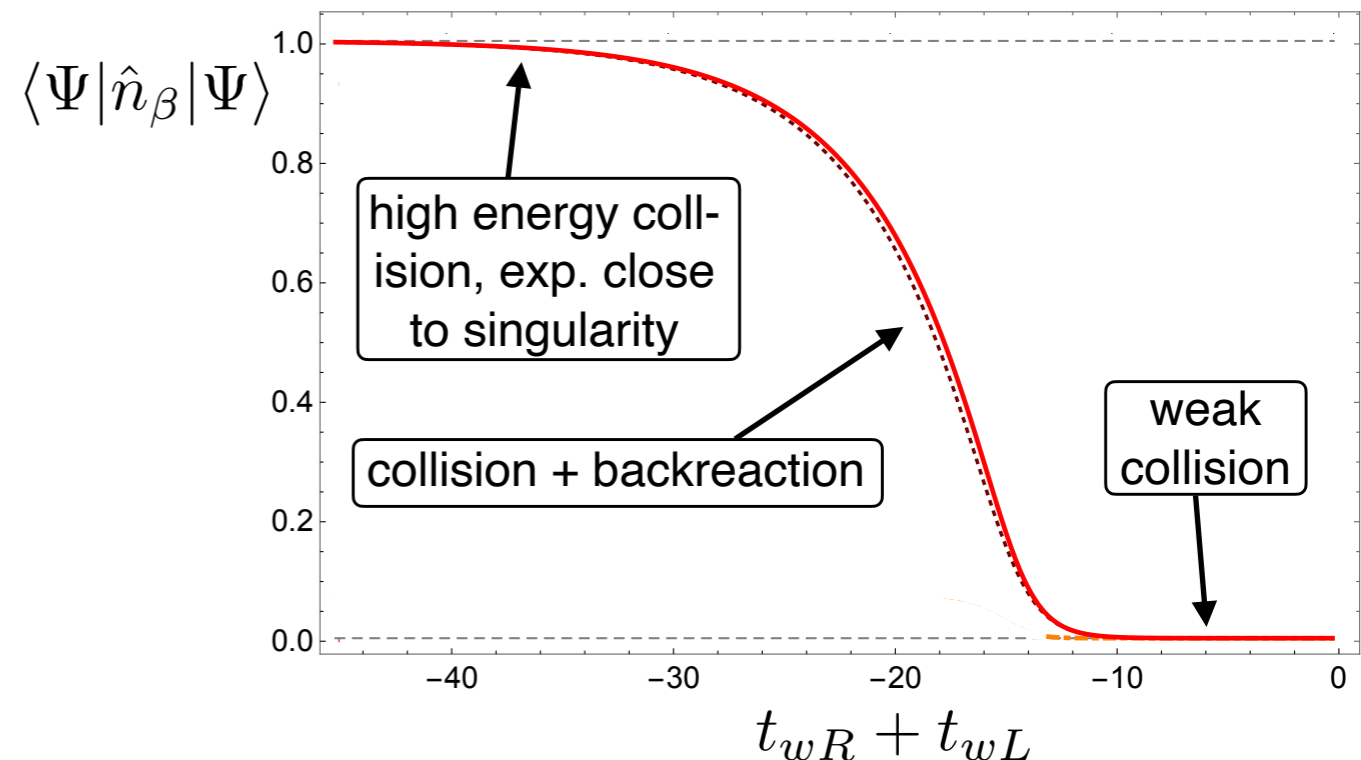


$$\langle\Psi|\hat{n}_\beta|\Psi\rangle :$$



- Proposal: this 6-point OTOC computes the “size” of the double perturbation. It matches the fraction of healthy gates in the corresponding circuit.

[FH/Zhao '21] [FH/Streicher/Zhao '21]



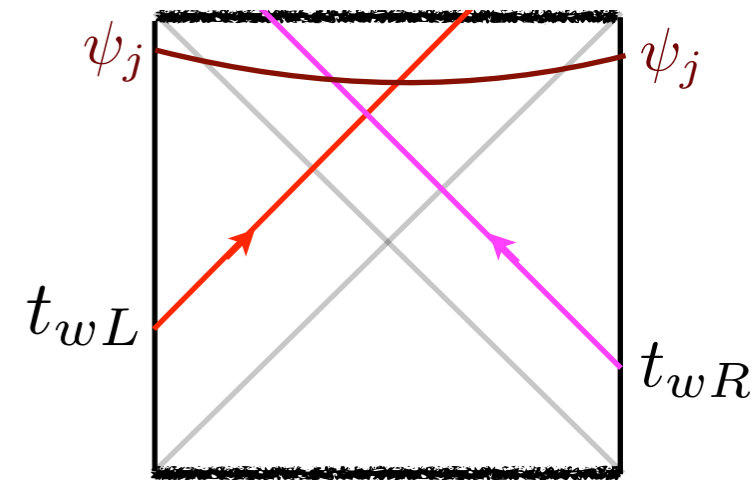
Summary & Outlook

Summary

- ▶ Entanglement & other information theoretic concepts: emergence of local spacetime dynamics in AdS/CFT

- ▶ Probe connection between interior geometry and L/R entanglement:

strong collision \Rightarrow $\left\{ \begin{array}{l} \bullet \text{ disrupt L/R entanglement} \\ \bullet \text{ decorrelate } \langle \text{TFD} | \psi_j^L \psi_j^R | \text{TFD} \rangle \\ \bullet \text{ "remove" interior geometry} \end{array} \right.$



- ▶ Quantum circuit model captures non-linear gravity effects, two-sided correlation functions, operator growth

Questions

- ▶ Quantum circuit is coarse-grained representation of geometry (scale ℓ_{AdS}). Can we refine it?
 - ▶ Derive local Einstein equations?
- ▶ Spacetime volume is an unusual way to characterize a scattering process. Make connection with S-matrix?
- ▶ Two-sided setup was crucial (TFD, operator size, ...). How to deal with one-sided black holes, or pure states?
- ▶ How to define operator size in CFTs, QFTs, ...?