Reparametrization modes in CFTs & applications

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(IAS)

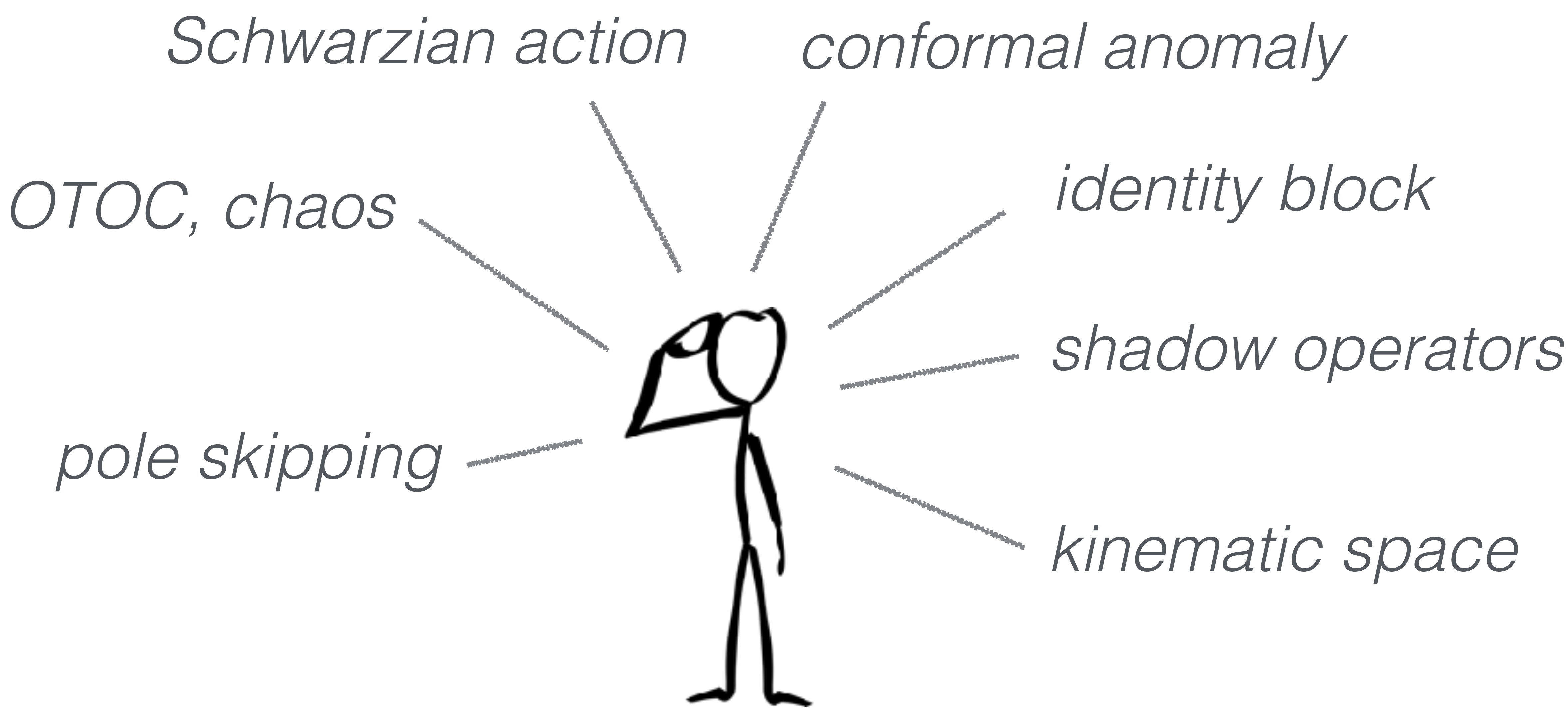
- 1909.05847 with **W. Reeves** & **M. Rozali**
- 2005.xxxxx with **T. Anous**
- (also 1808.02898 with M. Rozali)

Summary of talk

- Topic: correlation functions in CFTs with large c and large gap
- A theory of *reparametrization fields* that describes universal aspects of energy-momentum tensor/graviton exchanges
- Applications:
  - *Effective field theory* for quantum chaos
  - Do certain *conformal block* calculations very efficiently

Summary of talk

- Will also provide a language to piece together various ideas from SYK, CFT, bootstrap, holography



Outline

- Reparametrization fields & quantum chaos
  - SYK
  - CFT
  - Comments on thermal correlators
- Conformal blocks
  - Connection with shadow operators
  - Virasoro blocks (d=2)

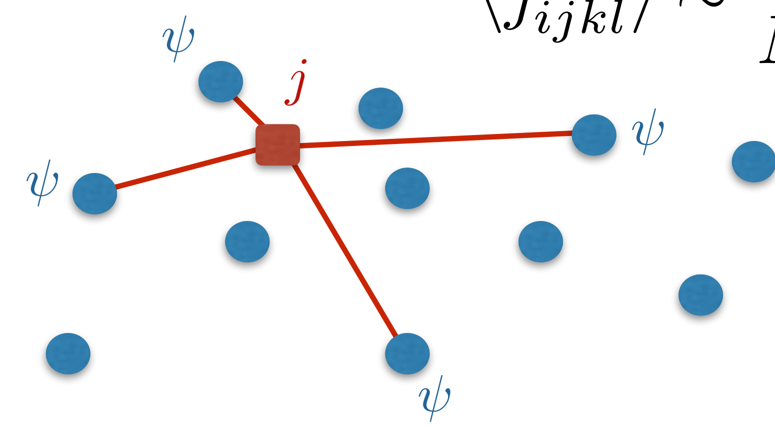
Reparametrization modes in SYK

Reminder: SYK model

- N Majorana fermions with random, Gaussian couplings

$$H = -\sum_{ijkl}^{N} j_{ijkl}\, \psi_i \psi_j \psi_k \psi_l$$

$$\langle j_{ijkl}^2 \rangle \sim \frac{\mathcal{J}^2}{N^3}$$



- IR: emergent conformal symmetry, $\tau \to f(\tau)$
  - Spontaneously and explicitly broken to *SL(2,R)*
  - Large N effective field theory of reparametrization Goldstone: *Schwarzian action*

$$S \propto -\frac{N}{\mathcal{J}} \int d\tau \ \{f(\tau), \tau\}$$

[Kitaev '15]
[Maldacena-Stanford '16] ...

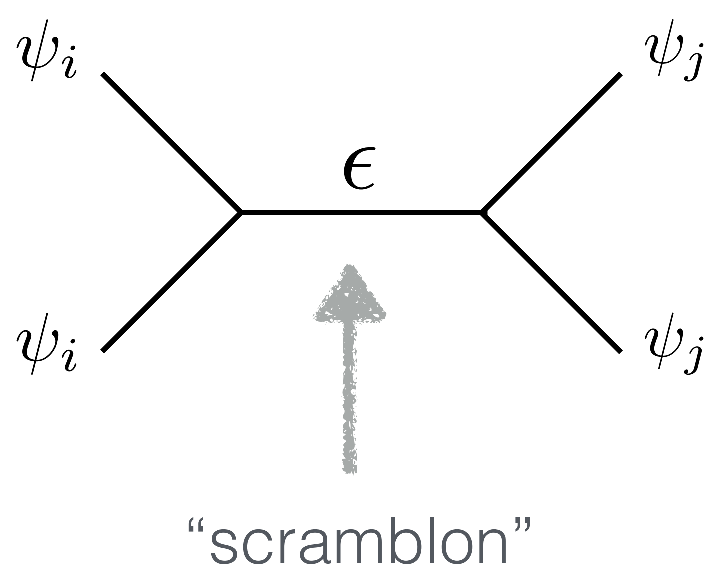
Schwarzian action

- Schwarzian describes dominant "gravity" effects

$$S \propto -\frac{N}{\mathcal{J}} \int d\tau \ \{f(\tau), \tau\}$$

- E.g.: enhanced contribution to *4-pt. OTOC*:

$$\frac{\langle \psi_i(t)\psi_j(0)\psi_i(t)\psi_j(0)\rangle^{\text{reg.}}_{\text{conn.}}}{\langle \psi_i\psi_i\rangle\langle\psi_j\psi_j\rangle} \sim \frac{\beta\mathcal{J}}{N}\ e^{\frac{2\pi}{\beta}t}$$



$$f(\tau) = \tau + \epsilon(\tau)$$

Schwarzian action

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- Schwarzian is *"hydrodynamic"* in the sense that it captures energy conservation [Jensen '16] ...

$$\partial_\tau E \equiv \partial_\tau \left[-\frac{N}{\mathcal{J}} \;\{f(\tau), \tau\}\right] = 0$$

- Is Lyapunov growth more generally described by *effective field theory of energy mode?* [Blake-Lee-Liu '18] ...

Schwarzian action

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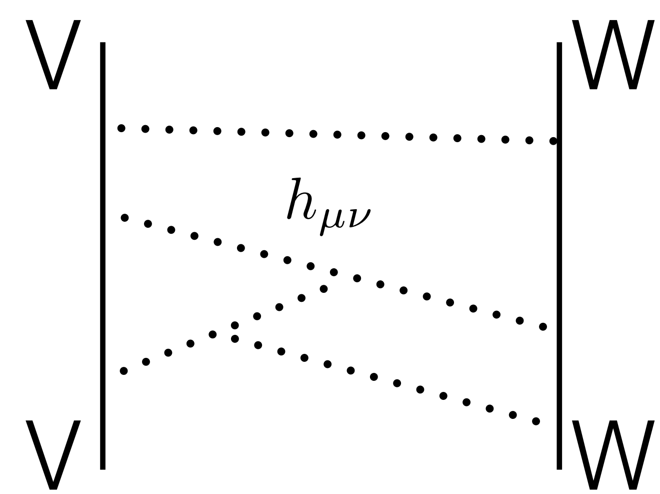
- E.g., in 2d CFTs: stress-energy tensor exchanges also maximally chaotic [Roberts-Stanford '14] ...

  - Virasoro *identity block*

  - Is there some "effective description"?

  - Do we learn something universal about scrambling and nonlinear gravity?



Schwarzian action

- Is Lyapunov growth more generally described by *effective field theory of energy mode?* [Blake-Lee-Liu '18] ...
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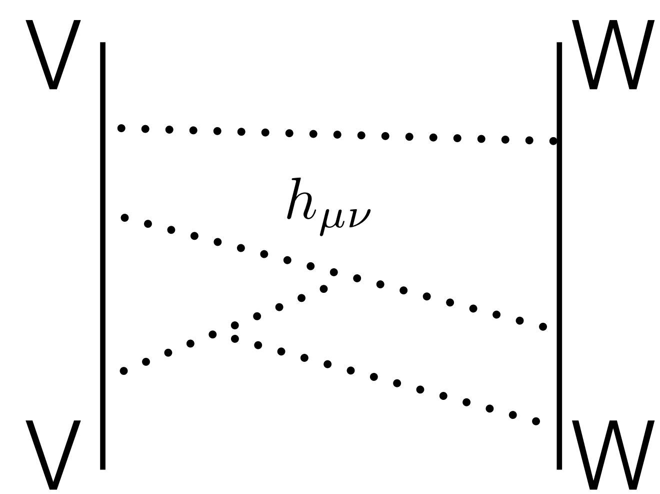

Effective theory of the “scramblon” in CFT_2

- Consider large c, large gap CFT and a reparametrization

$$(z, \bar{z}) \;\rightarrow\; (f(z,\bar{z}), \bar{f}(z,\bar{z}))$$

- $(f, \bar{f})$ have an *effective action* determined by $\langle T_{\mu\nu} \cdots T_{\rho\sigma} \rangle$

$$e^{-W[f,\bar{f}]} = \frac{1}{Z_0} \int [d\Phi]\, e^{-S_{CFT} - \int d^2 z \left\{ \frac{\bar{\partial} f}{\partial f}\, T \,+\, \mathrm{c.c.} \right\}}$$

- Understood in general (for finite reparametrizations) by [Alekseev-Shatashvili '89] (c.f. [Polyakov '87] [Witten '88]...)

- For simplicity, consider $(z, \bar{z}) \;\rightarrow\; (z + \epsilon(z,\bar{z}), \bar{z} + \bar{\epsilon}(z,\bar{z}))$

- $(\epsilon, \bar{\epsilon})$ have an *effective action* determined by $\langle T_{\mu\nu} \cdots T_{\rho\sigma} \rangle$
- Quadratic action:

$$\iint d^2 z_1\, d^2 z_2\; \bar{\partial}\epsilon_1\, \bar{\partial}\epsilon_2\, \langle T(z_1) T(z_2) \rangle \quad + \quad (\text{anti-holo.})$$

fixed by conformal symmetry

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- The quadratic action becomes local

[… because: $\bar{\partial}_1 \langle T(z_1) T(z_2) \rangle \sim \partial^3 \, \delta^{(2)}(z_1 - z_2)$]

$$\frac{c}{24\pi} \int d^2 z \; \bar{\partial}\epsilon \, \partial^3 \epsilon \quad + \quad (\text{anti-holo})$$

- $(\epsilon, \bar{\epsilon})$ have an *effective action* determined by $\langle T_{\mu\nu} \cdots T_{\rho\sigma} \rangle$

$$\frac{c}{24\pi} \int d^2 z \, \bar{\partial}\epsilon \, \partial^3 \epsilon \quad + \quad (\text{anti-holo})$$

- Reminiscent of quadratic Schwarzian
    - E.g., similar symmetries & charges (however, conformal symmetry not explicitly broken)
- Euclidean propagator:

$$\langle \epsilon(k, \bar{k}) \, \epsilon(-k, -\bar{k}) \rangle \propto \frac{1}{c} \frac{1}{(k^0 + ik^1)(k^0 - ik^1)^3}$$

$$\boxed{\langle \epsilon(z, \bar{z}) \, \epsilon(0,0) \rangle \propto \frac{1}{c} \, z^2 \, \log(z\bar{z})}$$

[FH-Reeves-Rozali '19]

Vertex rules

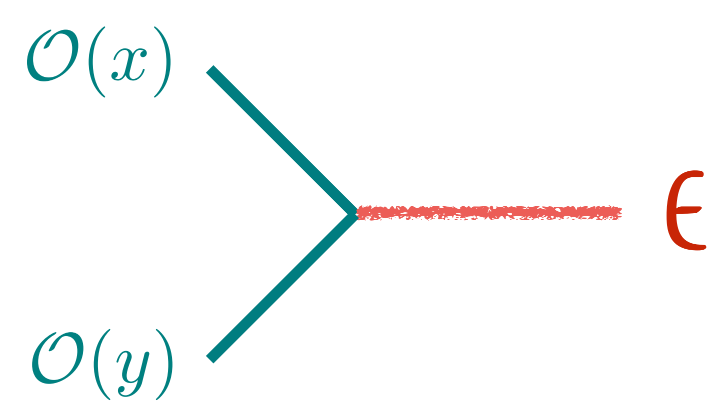
$$\langle \epsilon(z,\bar{z})\, \epsilon(0,0) \rangle \propto \frac{1}{c}\, z^2 \, \log(z\bar{z})$$

- Reparametrizations also couple to external fields:

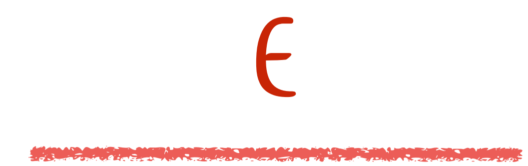
$$\langle \mathcal{O}(z_1)\mathcal{O}(z_2)\rangle \longrightarrow [\partial f(z_1)\,\partial f(z_2)]^{\Delta}\ \langle \mathcal{O}(f(z_1))\,\mathcal{O}(f(z_2))\rangle \qquad f(z) = z + \epsilon$$

$$= \langle \mathcal{O}(z_1)\mathcal{O}(z_2)\rangle \left\{ 1 + \ \Delta \left[\partial\epsilon(z_1) + \partial\epsilon(z_2) - 2\,\frac{\epsilon(z_1) - \epsilon(z_2)}{z_1 - z_2} \right] \right\}$$

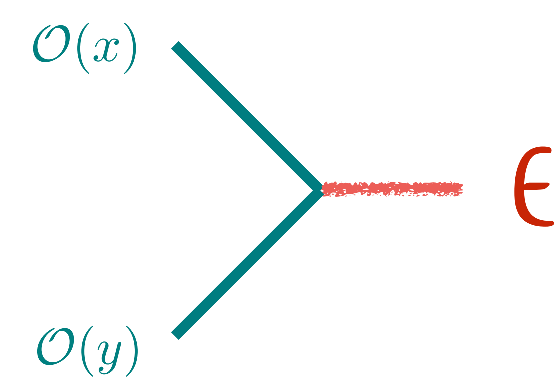
$$= \langle \mathcal{O}(z_1)\mathcal{O}(z_2)\rangle \left\{ 1 + \ \mathcal{B}^{(1)}_{\Delta}(z_1, z_2) \right\}$$



$$\langle \epsilon(z,\bar{z})\,\epsilon(0,0)\rangle \propto \frac{1}{c}\; z^2\;\log(z\bar{z})$$



$$\mathcal{B}^{(1)}_{\Delta}(z_1,z_2) \quad = \quad \Delta\left[\partial\epsilon(z_1)+\partial\epsilon(z_2)-2\,\frac{\epsilon(z_1)-\epsilon(z_2)}{z_1-z_2}\right]$$



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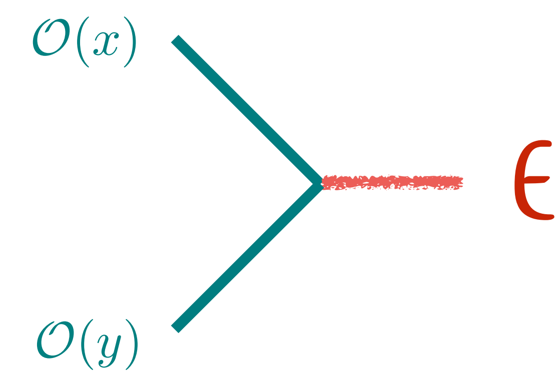
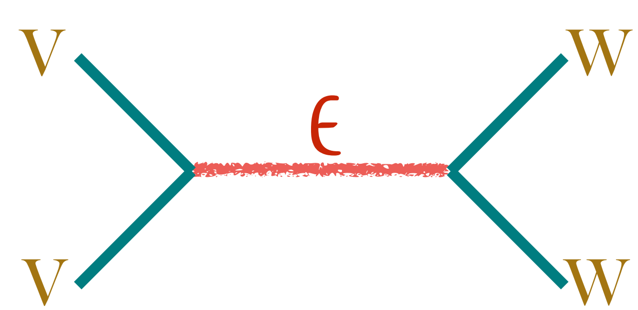


- “Feynman rules” for reparametrization field
- At *large c*, this gives a *systematic perturbation theory*

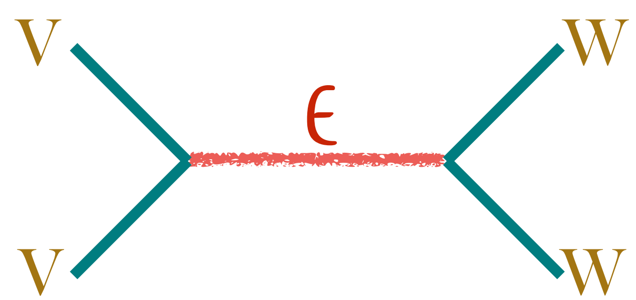
  equivalent to energy-momentum exchanges

$$\langle \mathcal{B}^{(1)}_{\Delta_V} \mathcal{B}^{(1)}_{\Delta_W} \rangle =$$



$$\langle \epsilon(z)\epsilon(0)\rangle = \frac{6}{c}\, z^2 \;\left[\log z + \log \bar{z}\right]$$

$$\langle \mathcal{B}^{(1)}_{\Delta_V} \mathcal{B}^{(1)}_{\Delta_W} \rangle =$$



$$= \frac{2h_V h_W}{c} \left[z^2 \; {}_2F_1(2,2,4,z) + 12\, \frac{\bar{z}}{z} \; {}_2F_1(-1,-1,-2,z) \; {}_2F_1(1,1,2,\bar{z}) \right]$$

"global stress tensor block"

"global stress tensor shadow block"

Thermal states:

$$\left(z = e^{i(\tau + i\sigma)}\right)$$

$$\langle \epsilon(\tau, \sigma) \epsilon(0,0) \rangle \sim \frac{1}{c} \sin^2\left(\frac{\tau + i\sigma}{2}\right) \log\left(1 - e^{i\,\mathrm{sgn}(\sigma)\,(\tau + i\sigma)}\right)$$

Thermal states:

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$$\langle \epsilon(\tau,\sigma)\epsilon(0,0)\rangle \sim \frac{1}{c}\,\sin^2\left(\frac{\tau+i\sigma}{2}\right)\,\log\left(1-e^{i\,\mathrm{sgn}(\sigma)\,(\tau+i\sigma)}\right)$$

[FH-Rozali '18][Cotler-Jensen '18]

- Starting point for *EFT of quantum chaos* in 2d CFT
- E.g., captures Lyapunov growth of OTOC:

$$\frac{\langle W_1 W_2 V_3 V_4\rangle_{\beta,\mathrm{Eucl.}}}{\langle VV\rangle_{\beta,\mathrm{Eucl.}}\langle WW\rangle_{\beta,\mathrm{Eucl.}}} = 1 + \left\langle \mathcal{B}^{(1)}_{\Delta_W}(1,2)\,\mathcal{B}^{(1)}_{\Delta_V}(3,4)\right\rangle_\beta + \ldots$$

OTOC: analytically continue to real time, with Euclidean separations held fixed
—> Get exponentially growing term if crossing branch cut of log

$$\frac{\langle W(t,\sigma)[V(0,0),W(t,\sigma)]V(0,0)\rangle_\beta}{\langle VV\rangle_\beta\langle WW\rangle_\beta} \sim \frac{1}{c}\,e^{\frac{2\pi}{\beta}(t-|\sigma|)}$$

[Shenker-Stanford '13][Roberts-Stanford '14][Maldacena-Shenker-Stanford '15]

Reparametrization modes in $d=2h$

Quadratic action

- Similar to d=2, stress tensor correlators are singular in d=2h. In particular:

$$\partial_\mu \langle T^{\mu\nu}(x) T^{\rho\sigma}(y)\rangle_{\text{conn.}} \propto C_T \left\{ \partial^\nu \partial^\rho \partial^\sigma - \frac{d-1}{d}\, \eta^{\nu(\rho}\partial^{\sigma)}\, \Box - \frac{1}{d^2}\, \eta^{\rho\sigma}\partial^\nu \Box \right\} \Box^{\frac{d-2}{2}} \delta^{(d)}(x-y)$$

=> quadratic action for reparametrization $x^\mu \to x^\mu + \epsilon^\mu$

$$W_2[\epsilon] = -\frac{1}{2}\int d^dx\, d^dy\; \partial_\mu \epsilon_\nu(x)\; \partial_\rho \epsilon_\sigma(y)\; \langle T^{\mu\nu}(x) T^{\rho\sigma}(y)\rangle_{\text{conn.}}$$

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=> quadratic action for reparametrization $x^\mu \to x^\mu + \epsilon^\mu$

$$W_2[\epsilon] = -\frac{1}{2}\int d^d x\, d^d y\, \partial_\mu \epsilon_\nu(x)\, \partial_\rho \epsilon_\sigma(y)\, \langle T^{\mu\nu}(x) T^{\rho\sigma}(y)\rangle_{\text{conn.}}$$

$$\langle \epsilon^\mu(x) \epsilon^\nu(0)\rangle = \frac{1}{C_T}\left(\eta^{\mu\nu} - 2\,\frac{x^\mu x^\nu}{x^2}\right) x^2\, \log\left(\mu^2 x^2\right)$$

[FH-Reeves-Rozali '19]

$$\langle \epsilon^\mu(x)\epsilon^\nu(0)\rangle = \frac{1}{C_T}\left(\eta^{\mu\nu} - 2\,\frac{x^\mu x^\nu}{x^2}\right) x^2\ \log\left(\mu^2 x^2\right)$$

- μ^2 is a scale needed in order to write sensible expression. Related to *conformal anomaly*.

- For example, recall: $\langle T_{\mu\nu}(x)T_{\rho\sigma}(0)\rangle \sim C_T\,\mathcal{D}^{(4)}_{\mu\nu\rho\sigma}\frac{1}{(x^2)^{d-2}}$

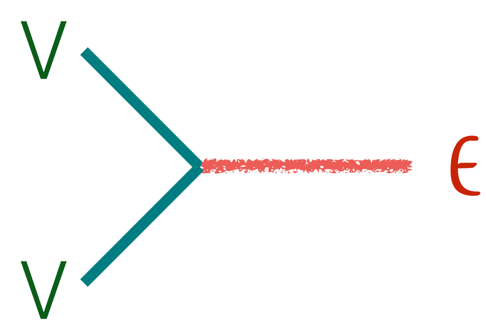
  - $(x^2)^{-d+2}$ is too singular

  - Regularization introduces a scale

$$\langle \epsilon^\mu(x)\epsilon^\nu(0)\rangle = \frac{1}{C_T}\left(\eta^{\mu\nu} - 2\,\frac{x^\mu x^\nu}{x^2}\right) x^2\ \log\left(\mu^2 x^2\right)$$

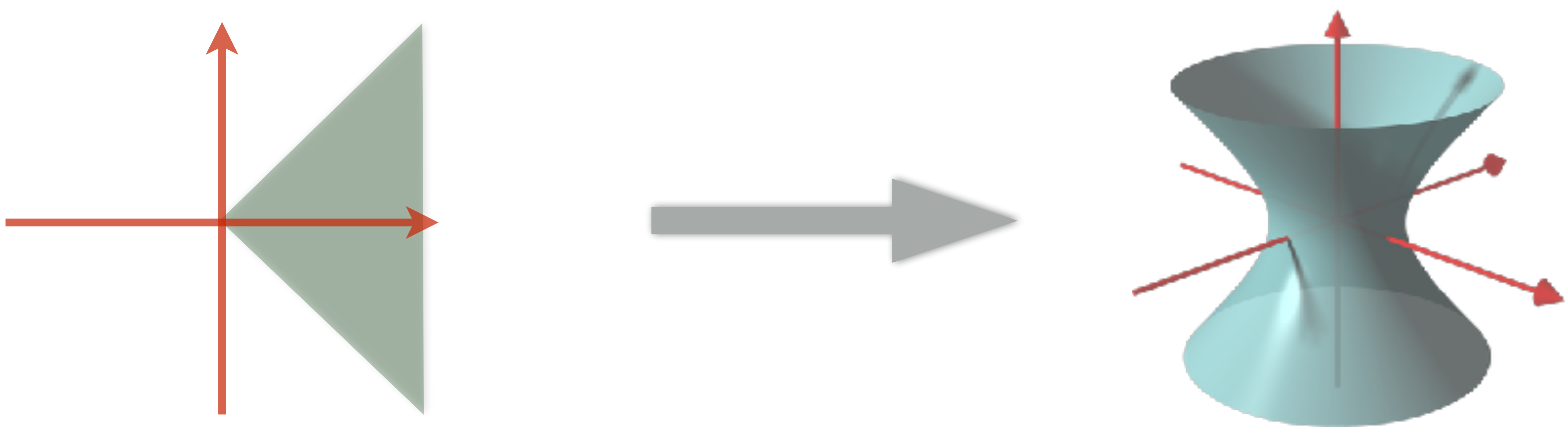
- Similarly, *bilinear couplings* are just reparametrized two-point functions:

$$\mathcal{B}^{(1)}_{\epsilon,V}(x,y) = \Delta_V\ \left\{\frac{1}{d}\left(\partial_\mu\epsilon^\mu(x) + \partial_\mu\epsilon^\mu(y)\right) - 2\,\frac{\left(\epsilon(x)-\epsilon(y)\right)^\mu (x-y)_\mu}{(x-y)^2}\right\}$$



Application: Rindler OTOCs

- Could again use this to study OTOCs
- In d>2: Rindler wedge is conformal to $\mathbb{H}^{d-1} \times \mathbb{R}$



- Can study *thermal physics on hyperbolic space* via conformal transformation

- Stress tensor contribution to *thermal 4pt function* on $\mathbb{H}^{d-1}$ from a single ϵ^{μ} exchange: $\langle \mathcal{B}_V^{(1)}(x_1,x_2)\mathcal{B}_W^{(1)}(x_3,x_4)\rangle$
- Analytic continuation to second sheet gives *OTOC*

- Stress tensor contribution to *thermal 4pt function* on $\mathbb{H}^{d-1}$ from a single ϵ^μ exchange: $\langle \mathcal{B}_V^{(1)}(x_1, x_2) \mathcal{B}_W^{(1)}(x_3, x_4) \rangle$
- Analytic continuation to second sheet gives *OTOC*
- <u>Result:</u>

$$\langle V(t, \mathbf{d}) W(0,0) V(t, \mathbf{d}) W(0,0) \rangle \sim \langle VV \rangle \langle WW \rangle \times \left[1 + \# \, e^{\lambda_L t + \mathbf{d}/v_B} \right]$$

$$\lambda_L = \frac{2\pi}{\beta} \,, \quad v_B = \frac{1}{d-1}$$

(agrees with [Perlmutter '16])

Remarks on

EFT of chaos

Pole skipping

Pole skipping

- Due to *relation between chaos and stress tensor*, one can already see some chaos characteristics in $\langle TT \rangle$
  - $\langle T^{00}(\omega, k) T^{00}(-\omega, -k) \rangle$ has poles and zeros
  - Poles are continuation of hydro diffusion pole $\omega + iD(\omega, k)k^2 = 0$
  - Zeros occur out of hydro regime, when $\omega \sim T$
  - Observation: in complex frequency space poles and zeros collide — *the pole is "skipped"* — at the Lyapunov values:

$$(\omega, k)_{\text{skip}} = \big(i\lambda_L,\ iv_B\big)$$

[Grozdanov-Schalm-Scopelliti '17]
[Blake-Lee-Liu '18]

$$(\omega, k)_{\text{skip}} = \big(i\lambda_L,\ iv_B\big)$$

In CFT:

- E.g. d=2 CFT: $\langle T(\omega,k)T(-\omega,-k)\rangle \propto c\,\dfrac{\omega(\omega^2+1)}{\omega-k}$

$$(\omega, k)_{\text{skip}} = \big(i\lambda_L,\ iv_B\big)$$

In CFT:

- E.g. d=2 CFT: $\langle T(\omega,k)T(-\omega,-k)\rangle \propto c\,\dfrac{\omega(\omega^2+1)}{\omega-k}$

More generally:

- Quadratic action: $W_2[\epsilon] = \displaystyle\iint d^d\xi\, d^d\xi'\ \partial_\mu\epsilon_\nu\,\partial_\rho\epsilon_\sigma\,\langle T^{\mu\nu}(\xi)T^{\rho\sigma}(\xi')\rangle$

  => roughly: $\langle\epsilon\epsilon\rangle \sim \dfrac{1}{\partial\partial\langle TT\rangle_{\text{sing.}}}$

- $\langle\epsilon\epsilon\rangle$ grows exponentially <-> $\partial\partial\langle TT\rangle_{\text{sing.}} \sim (\omega - i\lambda)\times f(\omega,k)$

- E.g. d=2 CFT: $\langle T(\omega,k)T(-\omega,-k)\rangle \propto c\,\frac{\omega(\omega^2+1)}{\omega-k}$

- In d>2 CFT:

$$\langle T^{00}(\omega_E,k)T^{00}(-\omega_E,-k)\rangle$$
$$\propto C_T\left(k^2+\left(\frac{d}{2}\right)^2\right)\left(k^2+\left(\frac{d-2}{2}\right)^2\right)\frac{\Gamma\left[\frac{1}{2}\left(\omega_E\pm ik+\frac{d-2}{2}\right)\right]}{\Gamma\left[\frac{1}{2}\left(\omega_E\pm ik-\frac{d-6}{2}\right)\right]}\Bigg|_{\text{reg.}}$$

[FH-Reeves-Rozali '19]

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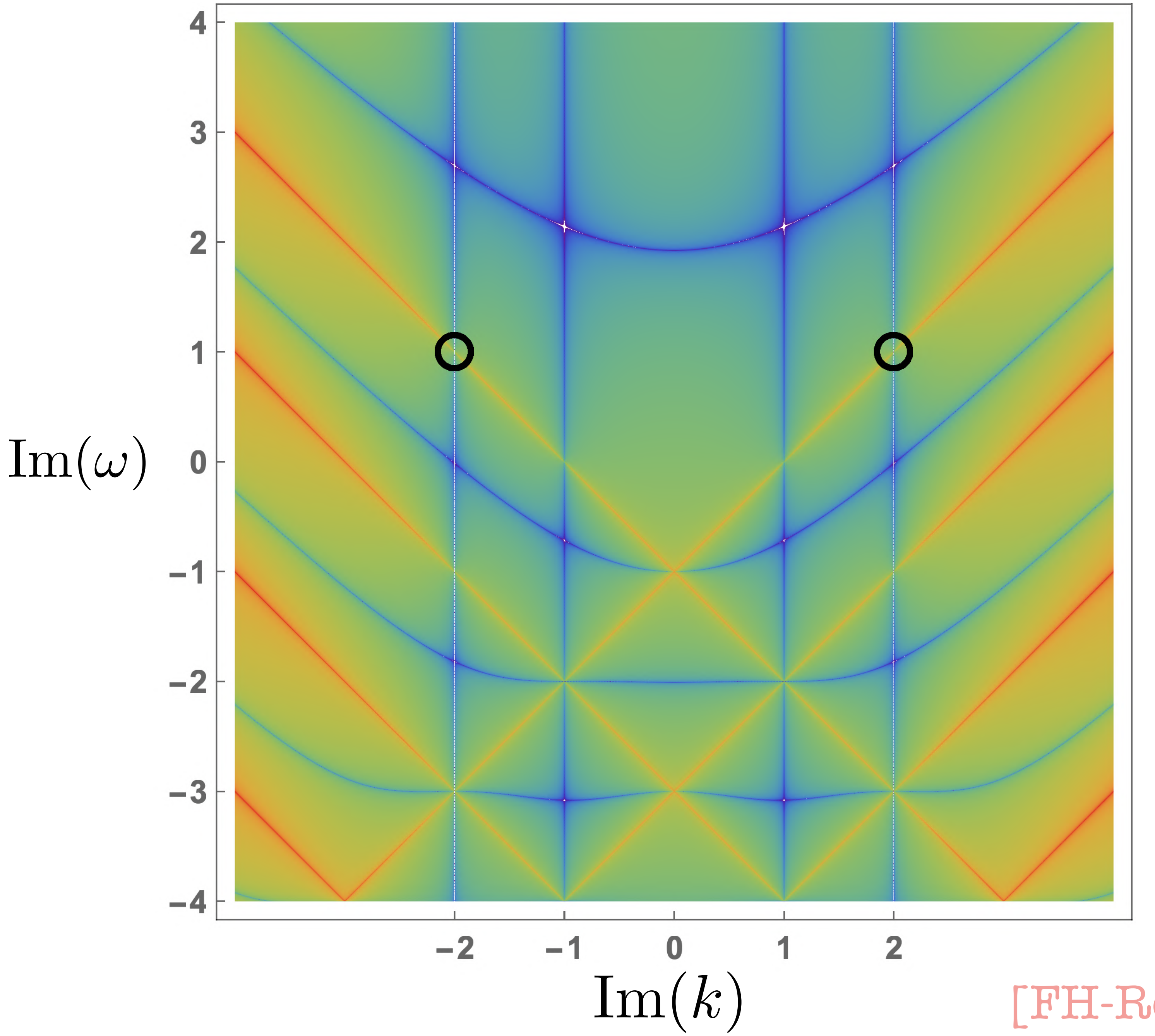
zeros
(special to stress tensor)

poles (and more zeros)

[FH-Reeves-Rozali '19]

$$\langle T^{00}(\omega_E, k) T^{00}(-\omega_E, -k)\rangle$$
$$\propto C_T \left(k^2 + \left(\frac{d}{2}\right)^2\right)\left(k^2 + \left(\frac{d-2}{2}\right)^2\right) \frac{\Gamma\left[\frac{1}{2}\left(\omega_E \pm ik + \frac{d-2}{2}\right)\right]}{\Gamma\left[\frac{1}{2}\left(\omega_E \pm ik - \frac{d-6}{2}\right)\right]}\Bigg|_{\text{reg.}}$$

e.g. d=4:



[FH-Reeves-Rozali '19]

Sub-maximal chaos (w.i.p.)

- Challenge: effective field theory for chaos with $\lambda_L < \frac{2\pi}{\beta}$
- Stress tensor physics (identity block) => $\lambda_L = \frac{2\pi}{\beta}$
- *Sub-maximal chaos* due to other exchanges
  - Lowest twist Regge trajectory gives collective correction: $\delta\lambda_L \sim j(0) - 2$ ("pomeron")

    [Costa-Goncalves-Penedones '12]

    [Kravchuk-Simmons—Duffin '18]
  - In bulk: stringy corrections $\delta\lambda_L \sim \frac{\ell^2_{string}}{\ell^2_{AdS}}$

    [Shenker-Stanford '15]

Connection with conformal blocks

Conformal blocks

- Consider OPE in (Euclidean) CFT:

$$V(x)V(y) = \frac{1}{(x-y)^{2\Delta_V}} \sum_{\mathcal{O}} C_{VV\mathcal{O}} \, |x-y|^{\Delta} \left[\mathcal{O}(y) + \text{conformal descendants} \right]$$

Conformal blocks

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"OPE block"

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"OPE block"

- Conformal block decomposition of 4-point function:

$$\langle V(x_1)V(x_2)W(x_3)W(x_4)\rangle = \frac{1}{x_{12}^{\Delta_V} x_{34}^{\Delta_W}} \sum_{\mathcal{O}} C_{VV\mathcal{O}} \, C_{WW\mathcal{O}} \, G_{\Delta}^{(\ell)}(u,v)$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \; v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

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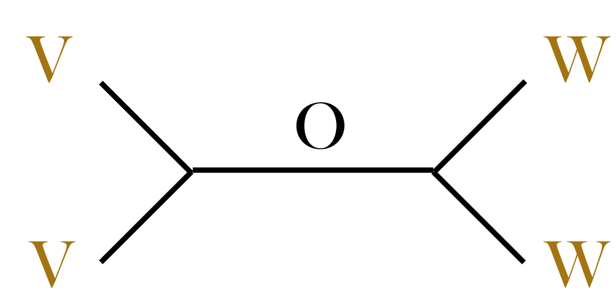
$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \quad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

“conformal block”



- Can find (global) conformal blocks by solving *Casimir eq.:*

$$\mathcal{C}[SO(2,d)]\, G_{\Delta}^{(\ell)} = [\ell(\ell+d-2) - \Delta(d-\Delta)]\, G_{\Delta}^{(\ell)}$$

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- Casimir has two solutions with same eigenvalue:

$$G_{\Delta}^{(\ell)}(u,v) \qquad\qquad G_{\tilde{\Delta}=d-\Delta}^{(\ell)}(u,v)$$

"physical" block — *"shadow" block*

- Distinguish them by short distance behavior:

$$G_{\Delta}^{(\ell)} \overset{u\to 0,\, v\to 1}{\sim} u^{\frac{\Delta-\ell}{2}}(1-v)^{\ell} + \dots \qquad G_{d-\Delta}^{(\ell)} \overset{u\to 0,\, v\to 1}{\sim} u^{\frac{d-\Delta-\ell}{2}}(1-v)^{\ell} + \dots$$

[Dolan-Osborn ’11] [Simmons-Duffin ’12] ...

Shadow operators

- Shadow block can be thought of as the conformal block associated with a *shadow operator* $\widetilde{\mathcal{O}}$

$$\widetilde{\mathcal{O}}(x) = \int d^d\xi \, \langle \widetilde{\mathcal{O}}(x) \widetilde{\mathcal{O}}(\xi) \rangle \, \mathcal{O}(\xi) \qquad (\widetilde{\Delta} = d - \Delta)$$

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- Define projector onto conformal multiplet of $\mathcal{O}$:

$$|\mathcal{O}| = \int d^d\xi \, \mathcal{O}(\xi)|0\rangle\langle 0|\widetilde{\mathcal{O}}(\xi)$$

Compute conformal block by projection onto $\mathcal{O}$:

$$G_\Delta^{(\ell)} \stackrel{?}{=} \langle V(x_1)V(x_2)|\mathcal{O}_{\Delta,\ell}|W(x_3)W(x_4)\rangle$$

[Simmons-Duffin '12] ...

- Define projector onto conformal multiplet of $\mathcal{O}$:

$$|\mathcal{O}| = \int d^d\xi \; \mathcal{O}(\xi)|0\rangle\langle 0|\widetilde{\mathcal{O}}(\xi)$$

- Can understand *OPE block* from this:

$$V(x)V(y)|0\rangle\Big|_{\mathcal{O}+\text{global desc.}} = |\mathcal{O}|V(x)V(y)|0\rangle$$

$$= \int d^d\xi \; \langle V(x)V(y)\widetilde{\mathcal{O}}(\xi)\rangle \, \mathcal{O}(\xi)|0\rangle$$

$$\equiv \mathfrak{B}_{\mathcal{O}}(x,y)|0\rangle$$

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“conformal partial wave”

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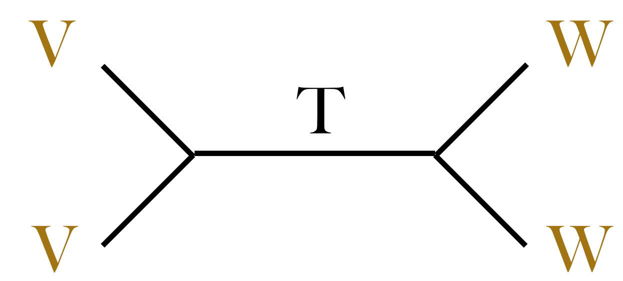
"conformal partial wave"

- The shadow block can be projected out by imposing correct short distance behavior

"monodromy projection"

[Dolan-Osborn '11] [Simmons-Duffin '12] ...

Stress tensor block



- Interesting and universal physics is encoded by the *stress tensor block*

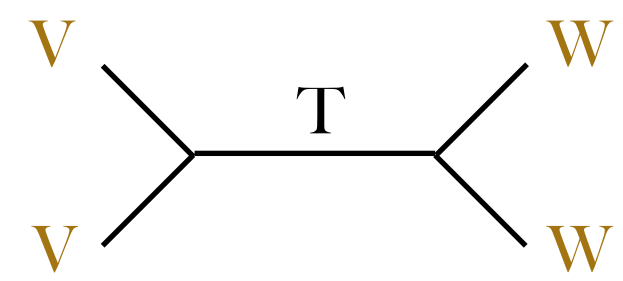
- Encodes graviton exchanges in AdS/CFT

  - Probe pure gravity sector (gravitational scattering, bulk locality,…)

  - Important example: V, W = twist operators

Stress tensor block

- Interesting and universal physics is encoded by the *stress tensor block*



- Special feature of stress tensor shadow:

$$\widetilde{T}_{\mu\nu}(x) \equiv \mathbb{P}_{\mu\nu}^{\ \ \rho\sigma}\, \partial_\rho \mathcal{E}_\sigma(x)\,, \qquad \mathcal{E}_\sigma(x) = \int d^d\xi\; K^{\alpha\beta}{}_\sigma(x-\xi)\, T_{\alpha\beta}(\xi)$$

symmetric-traceless projector

Stress tensor block

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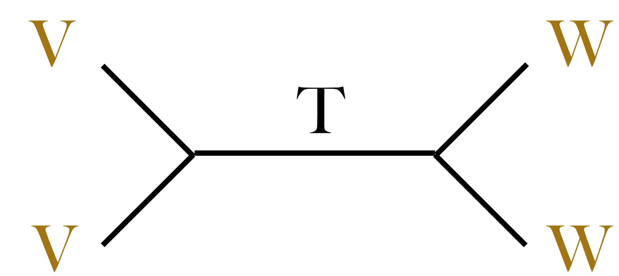

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symmetric-traceless projector

- Operator $\mathcal{E}_\mu$ inherits 2-point function from <TT>:

$$\langle \mathcal{E}^\mu(x)\mathcal{E}^\nu(0)\rangle \propto \left(\eta^{\mu\nu} - 2\,\frac{x^\mu x^\nu}{x^2}\right) x^2 \, \log\left(\mu^2 x^2\right)$$

- Two-point function of $\mathcal{E}_\mu$ follows from conf. symm.:

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- This is exactly the same as the two-point function of reparametrization fields, $\langle \epsilon^\mu \epsilon^\nu \rangle$

Calculations of stress tensor global blocks can be recast as reparametrization mode diagrams.

  - EFT of the global stress tensor block (in thermal state: becomes EFT of maximal chaos)

[FH-Reeves-Rozali '19]

Calculations of stress tensor global blocks can be recast 1-to-1 as reparametrization mode exchanges.

- Bilocal operators:

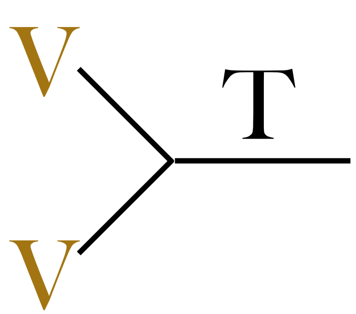
$$\mathfrak{B}_T(x,y) = \mathcal{B}^{(1)}_{\Delta_V}(x,y)$$

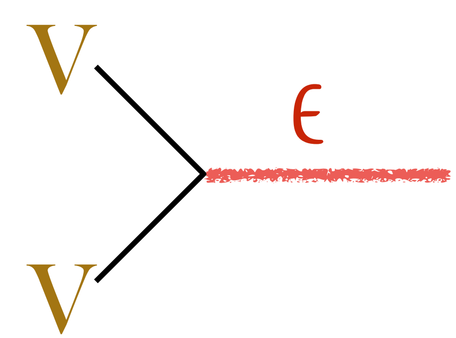
$$= \int d^d\xi \, \frac{\langle V(x)V(y)\widetilde{T}(\xi)\rangle}{\langle V(x)V(y)\rangle} T(\xi)$$

$$\Delta_V \left[\frac{1}{d}\left(\partial\epsilon(x)+\partial\epsilon(y)\right) - 2\,\frac{(\epsilon(x)-\epsilon(y))^\mu (x-y)_\mu}{(x-y)^2}\right]$$

"stress tensor OPE block"

"reparam. mode vertex"





- Writing $\widetilde{T}_{\mu\nu} = \mathbb{P}^{\rho\sigma}_{\mu\nu}\,\partial_\rho \epsilon_\sigma$ and using conformal Ward identity, can show equivalence explicitly

Calculations of stress tensor global blocks can be recast 1-to-1 as reparametrization mode exchanges.

- Bilocal operators:

$$\mathfrak{B}_T(x,y) \quad = \quad \mathcal{B}^{(1)}_{\Delta_V}(x,y)$$

$$= \int d^d\xi \, \frac{\langle V(x)V(y)\widetilde{T}(\xi)\rangle}{\langle V(x)V(y)\rangle} T(\xi)$$

$$\Delta_V \left[\frac{1}{d} \left(\partial\epsilon(x) + \partial\epsilon(y) \right) - 2 \, \frac{(\epsilon(x) - \epsilon(y))^\mu (x-y)_\mu}{(x-y)^2} \right]$$

"stress tensor OPE block" "reparam. mode vertex"

- lhs: have to compute *conformal integrals*
- rhs: EFT-type exchanges of ϵ^μ

- For example, *stress tensor 4-point (global) block:*

$$F^{(\ell=2)}_{\Delta=d} = \langle \mathcal{B}^{(1)}_{\Delta_V}(x_1,x_2)\mathcal{B}^{(1)}_{\Delta_W}(x_3,x_4)\rangle \propto \left(\frac{4}{d}+\frac{1-v}{u}\log v\right)$$

—> Matches the known CPW associated with stress tensor exchanges in d=2,4,6.

[Dolan-Osborn '11] [FH-Reeves-Rozali '19]

- Note: calculation independent of *d*

- In particular, consider *d=2:*

$$\langle \epsilon(z)\epsilon(0)\rangle = \frac{6}{c}\, z^2 \;\, [\log z + \log \bar{z}]$$

$$\langle \mathcal{B}^{(1)}_{\Delta_V}\, \mathcal{B}^{(1)}_{\Delta_W}\rangle = \frac{2h_V h_W}{c}\, \left[z^2 \; {}_2F_1(2,2,4,z) + 12\, \frac{\bar{z}}{z}\; {}_2F_1(-1,-1,-2,z)\; {}_2F_1(1,1,2,\bar{z}) \right]$$

"global block" "shadow block"

- Can do *monodromy projection* at the level of $\langle \epsilon\epsilon \rangle$:
  - Simply drop the term $\sim z^2 \log \bar{z}$
  - Useful in calculations
- Q: can one do this in d>2 ?

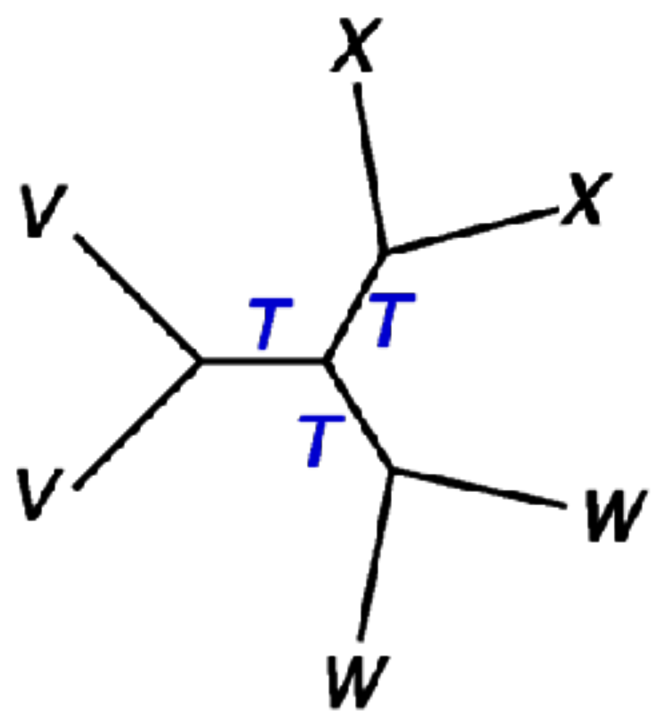
Some explicit calculations

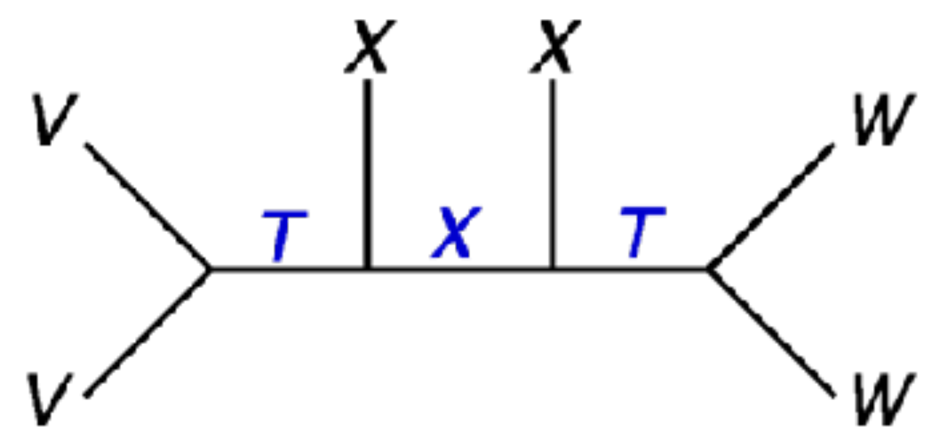
Six-point identity block in d=2

- *Higher-point conformal blocks* relevant for:
  - Entanglement entropy for multiple intervals (mutual information, …)
  - Eigenstate thermalization
  - Chaos (higher-point OTOCs)
  - Generalized bootstrap?
  - …

Six-point identity block in d=2

- Two types of *OPE channels*:



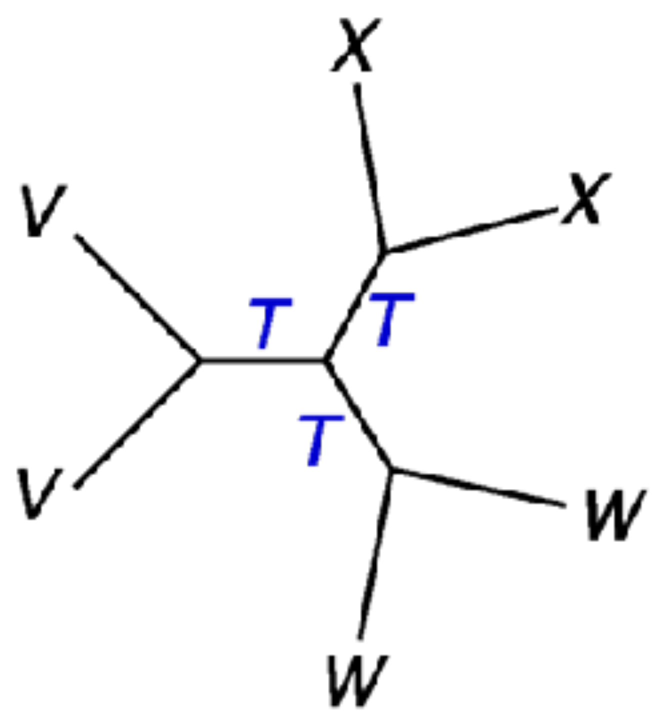


- "star channel"
- Pure gravity sector
- Universal (in d=2)

- "comb channel"
- Understood quite generally in terms of hypergeometric series/integrals [Rosenhaus '18]

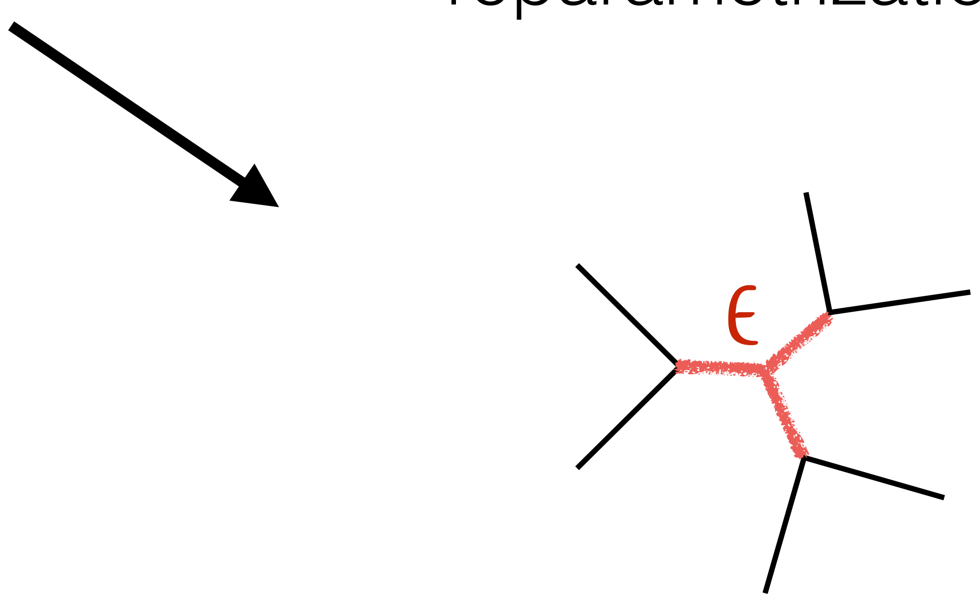
Six-point identity block in d=2

- Two types of *OPE channels*:

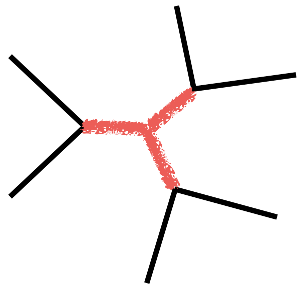


  - “star channel”
  - Pure gravity sector
  - Universal (in d=2)

- Corresponds 1-to-1 to a cubic self-interaction of reparametrization:



$$\langle \mathcal{B}_V^{(1)}(1,2)\mathcal{B}_X^{(1)}(3,4)\mathcal{B}_W^{(1)}(5,6)\rangle \sim \langle \epsilon\epsilon\epsilon\rangle$$



Six-point “star channel” block

$\langle \mathcal{B}_V^{(1)}(1,2)\mathcal{B}_X^{(1)}(3,4)\mathcal{B}_W^{(1)}(5,6)\rangle \sim \langle \epsilon\epsilon\epsilon\rangle$

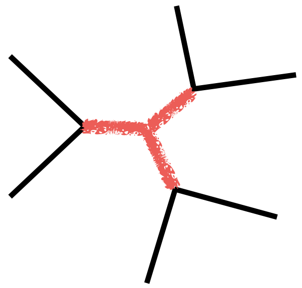
- Easy! No need to do any conformal integrals, nested hypergeometric sums etc.

- Just need $\langle \epsilon\epsilon\epsilon\rangle$

  - Follows by simple integration of identities like:

$$\langle \bar\partial\epsilon\,\bar\partial\epsilon\,\bar\partial\epsilon\rangle \propto \langle \widetilde{T}\widetilde{T}\widetilde{T}\rangle \qquad \langle \partial^3\epsilon\,\partial^3\epsilon\,\partial^3\epsilon\rangle \propto \langle TTT\rangle$$

- *Monodromy projection:* only keep purely holomorphic part of $\langle \epsilon\epsilon\epsilon\rangle$



Six-point "star channel" block

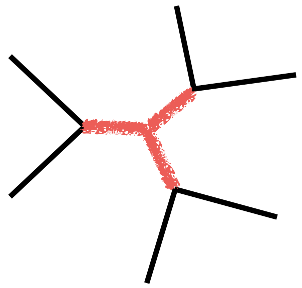
$\langle \mathcal{B}^{(1)}_V(1,2)\mathcal{B}^{(1)}_X(3,4)\mathcal{B}^{(1)}_W(5,6)\rangle \sim \langle \epsilon\epsilon\epsilon\rangle$

$$\langle \bar{\partial}\epsilon\, \bar{\partial}\epsilon\, \bar{\partial}\epsilon\rangle \propto \langle \widetilde{T}\widetilde{T}\widetilde{T}\rangle \qquad \langle \partial^3\epsilon\, \partial^3\epsilon\, \partial^3\epsilon\rangle \propto \langle TTT\rangle$$

- *Monodromy projection:* only keep purely holomorphic part of $\langle \epsilon\epsilon\epsilon\rangle$

$$\langle \epsilon_1\epsilon_2\epsilon_3\rangle_{phys.} \sim \frac{1}{c^2}\, z_{12}z_{23}z_{31}\left[\mathrm{Li}_2\left(\frac{z_{12}}{z_{13}}\right) - \mathrm{Li}_2\left(\frac{z_{23}}{z_{13}}\right)\right]$$

Six-point "star channel" block



$\langle \mathcal{B}_V^{(1)}(1,2)\mathcal{B}_X^{(1)}(3,4)\mathcal{B}_W^{(1)}(5,6)\rangle \sim \langle \epsilon\epsilon\epsilon\rangle$

- Result:

$$G^{(T,\,\mathrm{star})}_{V,V;X,X;W,W} = \left\langle \mathcal{B}_V^{(1)}(1,2)\,\mathcal{B}_X^{(1)}(3,4)\,\mathcal{B}_W^{(1)}(5,6)\right\rangle$$

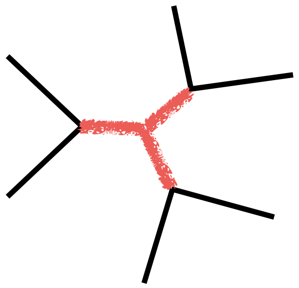
$$\sim \frac{h_V h_X h_W}{c^2}\left[\mathcal{I}(z,u,v)+\mathcal{I}(z,v,u)+\mathcal{I}\left(\frac{1}{z},\frac{u}{z},\frac{v}{z}\right)+\mathcal{I}\left(\frac{1}{z},\frac{v}{z},\frac{u}{z}\right)\right]$$

$$\begin{aligned}\mathcal{I}(z,u,v) \equiv 1+\frac{1}{(u-v)(1-z)}\Bigg[&u\left(\frac{u(v-vz+z-u)}{1-u}+2(z-v)\right)\log u\\ &-(1-u)\left(\frac{(1-u)(zv+u)}{u}+2(z-v)\right)\log(1-u)\\ &-2(uv-z)\big(\mathrm{Li}_2(u)-\mathrm{Li}_2(1-u)\big)\Bigg]\end{aligned}$$

$$z=\frac{z_{13}z_{24}}{z_{23}z_{14}}\,,\qquad u=\frac{z_{13}z_{25}}{z_{23}z_{15}}\,,\qquad v=\frac{z_{13}z_{26}}{z_{23}z_{16}}$$

[Anous-FH w.i.p.]

Six-point "star channel" block



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$$\sim \frac{h_V h_X h_W}{c^2}\left[\mathcal{I}\left(z,u,v\right)+\mathcal{I}\left(z,v,u\right)+\mathcal{I}\left(\frac{1}{z},\frac{u}{z},\frac{v}{z}\right)+\mathcal{I}\left(\frac{1}{z},\frac{v}{z},\frac{u}{z}\right)\right]$$

- Solves 6-point Casimir equations & boundary conditions
- Interesting transcendentality/branch cut structure
- Questions:
  - *6-point OTOCs?* —> self interactions in the chaos EFT
  - *HHLLLL block?* —> ETH, entanglement/mutual information in excited states, …

[Anous-FH w.i.p.]

Summary

- Single reparametrization mode exchanges are in 1-to-1 correspondence with *global stress tensor blocks*

  - *Easy calculations:* never need to deal with conformal integrals, hypergeometric sums, Casimir PDEs etc.

  - *Monodromy projection* can be done from the outset (at least in d=2)

Q: What about Virasoro blocks?

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Q: What about Virasoro blocks?

Virasoro blocks in d=2

Virasoro blocks in d=2

$$V(z_1)V(z_2)\big|_T \sim \frac{C_{VV1}}{|z_{12}|^{2(\Delta_V-1)}}\,[\mathbf{1} + \text{Virasoro descendants}]$$

- Virasoro identity block in AdS/CFT: gravitational scattering/graviton exchanges
- Proposal for generalization of shadow operator approach:

Virasoro identity blocks are computed by higher order exchanges of reparametrization fields.

$$z \to f(z) = z + \epsilon(z,\bar{z}) + \ldots : \qquad \mathcal{B}_h(z_1,z_2) = \left(\frac{\partial f(z_1)\,\partial f(z_2)}{(z_1-z_2)^2}\right)^h$$

Example: light-light block

Example: light-light block

- [Cotler-Jensen '18] already used ϵ^μ to reproduce the "eikonalization" of the *LLLL block*

$$\langle V(\infty)V(1)W(z)W(0)\rangle \qquad \alpha \equiv \frac{\Delta_V \Delta_W}{c} \sim \mathcal{O}(1)$$

- Ladder diagrams dominate in EFT

exponentiation!

$$+ \ldots = e^{2\alpha\, z^2\, {}_2F_1(2,2,4,z)} + \ldots$$

$\sim \alpha$ $\quad \sim \alpha^2$ $\quad \sim \alpha^3$

Example: heavy-light block

- HHLL Virasoro block in d=2: $\langle H(\infty)H(0)L(1)L(z,\bar{z})\rangle$

$$\mathcal{V}_0 = \alpha^{\Delta_L} z^{-\frac{1-\alpha}{2}\Delta_L}\left(\frac{1-z}{1-z^\alpha}\right)^{\Delta_L}$$

$$\alpha \equiv \sqrt{1-\mu}\,, \quad \mu \equiv \frac{12\Delta_H}{c}$$

$$(\Delta_H \sim \mathcal{O}(c),\ \Delta_L \sim \mathcal{O}(1))$$

[Fitzpatrick-Kaplan-Walters '14]

Example: heavy-light block

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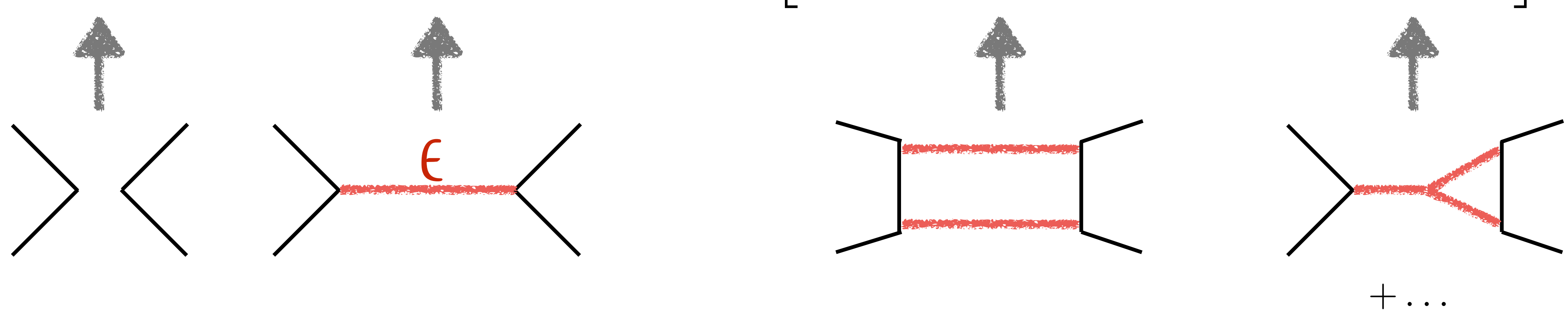
$$(\Delta_H \sim \mathcal{O}(c),\ \Delta_L \sim \mathcal{O}(1))$$

[Fitzpatrick-Kaplan-Walters '14]

$$\mathcal{V}_0 = 1 + \mu\,\Delta_L\left[\frac{z+1}{4(z-1)}\log z - \frac{1}{2}\right] + \frac{\mu^2}{2}\left[\Delta_L^2\left(\frac{z+1}{4(z-1)}\log z - \frac{1}{2}\right)^2 + \frac{\Delta_L}{8}(\cdots)\right] + \frac{\mu^3}{6}[\ldots] + \ldots$$



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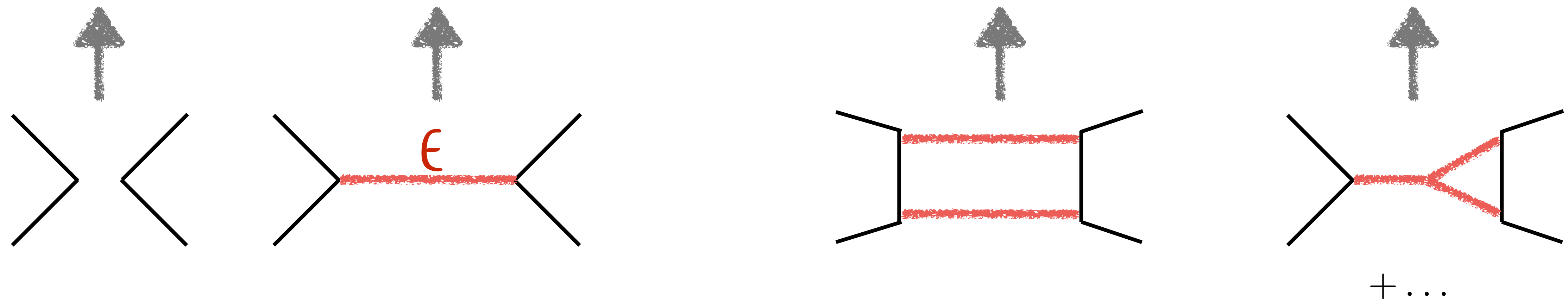

- Reparametrization mode diagrammatics: systematic way to construct HHLL block order by order

- 1/c corrections computed by loops [Cotler-Jensen '18]

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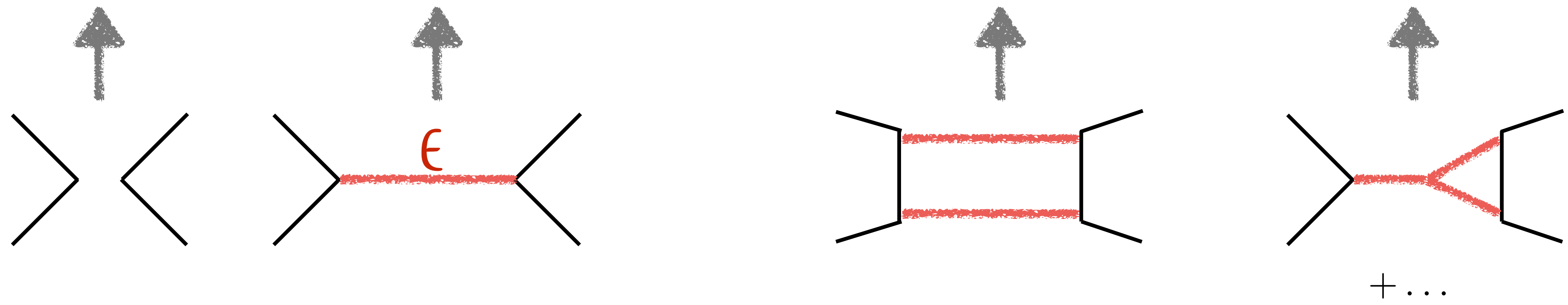

- Reparametrization mode diagrammatics: systematic way to construct HHLL block order by order

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- Q: Do multi-T contributions to HHLL exponentiate in d>2?

[Fitzpatrick-Huang(-Li) '19]

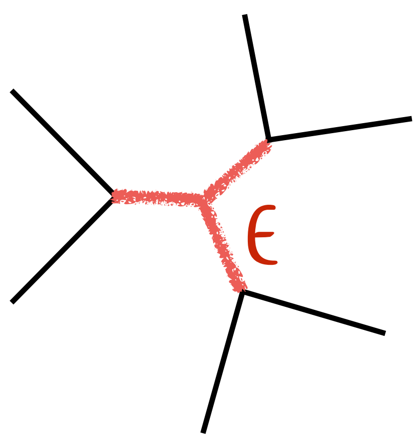
[Kulaxizi et al. '19] ...

  - Reparametrization mode approach might be useful

Six-point Virasoro identity block

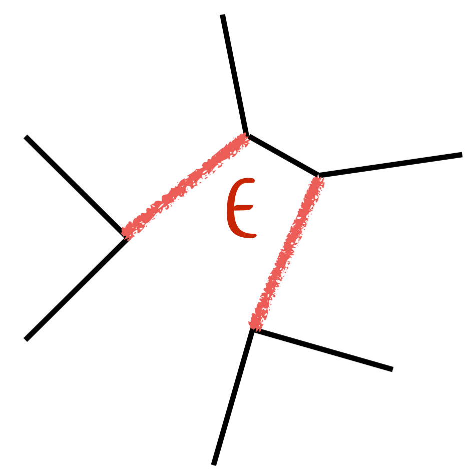
Six-point Virasoro identity block

- Computed by *all connected diagrams* in $\langle \mathcal{B}_V \, \mathcal{B}_X \mathcal{B}_W \rangle$
- Leading order at large c:



$\langle \mathcal{B}_V^{(1)}(1,2)\mathcal{B}_X^{(1)}(3,4)\mathcal{B}_W^{(1)}(5,6)\rangle \sim \langle \epsilon\epsilon\epsilon \rangle$

$+$



$\langle \mathcal{B}_V^{(1)}(1,2)\mathcal{B}_X^{(2)}(3,4)\mathcal{B}_W^{(1)}(5,6)\rangle \sim \langle \epsilon\epsilon \rangle \langle \epsilon\epsilon \rangle$

$$\mathcal{V}_{\text{id.}}^{(6)} = G^{(6,\,\text{global})} + G^{(6,\,\text{extra})} + \mathcal{O}(1/c)$$

[Anous-FH w.i.p.]

Six-point Virasoro identity block

$$\mathcal{V}_{\text{id.}}^{(6)} = G^{(6,\,\text{global})} + G^{(6,\,\text{extra})} + \mathcal{O}(1/c)$$

- Surprising: leading term $\neq$ global block
- $\mathcal{V}_{\text{id.}}^{(6)}$ exponentiates when $\Delta_V\,,\Delta_X\,,\Delta_Y \sim c^{2/3}$
- "Extra" term turns out to be the dominant one in Regge limit

$$\text{OTOC}_{\text{id.}}^{(6)} \approx \text{OTOC}^{(6,\,\text{extra})} \sim e^{\frac{2\pi}{\beta}(t-2\,t_*)}$$

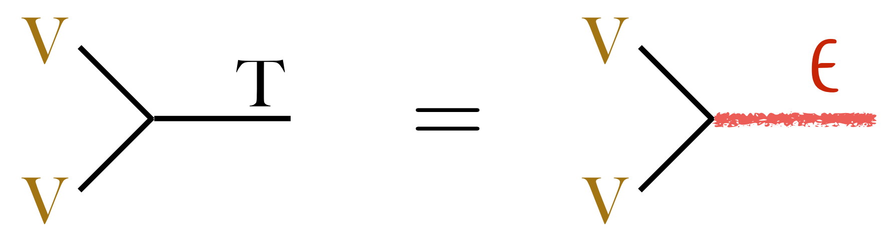
[Anous-FH w.i.p.]

Conclusion

Summary

- Can write down *systematic EFT for reparametrization modes* in CFT for d=2h perturbatively in 1/c

- Bilinear operators correspond to *"OPE blocks"*



- Reparametrization mode closely related to *shadow of T*

  - Explains why and how this theory computes conformal blocks, OTOCs etc.

  - 6-point block: exploit advantages to get new results

- Applications to thermal physics (OTOCs, ETH, …)

Outlook

- Compute more complicated blocks
- Hydrodynamic origin of quantum chaos?
  - Is there an EFT for *non-maximal chaos*?
- *Non-linear action* similar to Schwarzian (d=1) and Alekseev-Shatashvili (d=2) in d>2 ?
- Understand monodromy projection at the level of the reparametrization mode theory in d>2
- Odd dimensions?