Three roads not (yet) taken

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Thank You
• Question all assumptions, be rigorous wherever possible, do not settle for incomplete understanding
• The more references you have, more people will complain
• Publish what you understand, not what you do not understand

• Direct but not intimidating

• Contributions to QFT and string theory
What to talk about ???

Projects with Nati ?
(Top 10, 2 days – 3+ years)
Never underestimate the joy people derive from hearing something they already know.

— Enrico Fermi —
Three topics that Nati has not (yet) worked on – possibilities for the next 60 years
Topic 1: Theories with a number of supersymmetries that is not a power of 2

Nothing since 1987 (Schwimmer+Seiberg)? Nothing in $d > 2$? ($d = 3 \quad \mathcal{N} = 3, 5, 6; \quad d = 4 \quad \mathcal{N} = 3$)
d=4 $\mathcal{N}=3$ superconformal theories

- d=4 $\mathcal{N}=4$ theories are essentially classified.
- For $\mathcal{N}=2$ theories – much progress but still seem far from full classification.
- What about $\mathcal{N}=3$ ?
- Only free multiplet of $\mathcal{N}=3$ is $\mathcal{N}=4$ vector multiplet. So no free or weakly coupled pure $\mathcal{N}=3$ theories.
- Can have $6r$-dimensional Coulomb branch with $r$ free vector multiplets in IR.
d=4 $\mathcal{N}=3$ superconformal theories

- General properties of pure $\mathcal{N}=3$ SCFTs (OA+Evtikhiev, Cordova+Dumitrescu+Intriligator):
  1. No $\mathcal{N}=3$-preserving marginal or relevant deformations ($\mathcal{N}=2$ : relevant, not marginal)
  2. No global symmetries (just $\text{SU}(3)_R \times \text{U}(1)_R$)
  3. $a=c$
(Garcia-Etxebarrio+Regalado)

- N M2-branes in M theory on $C^4/Z_k$ preserve $d=3$ $\mathcal{N}=6$ SUSY, gives $d=3$ $\mathcal{N}=6$ SCFTs ($\mathcal{N}=8$ for $k=2$)
- For $k=2,3,4,6$ can consider instead $(C^3\times T^2)/Z_k$ (for specific $\tau$ when $k>2$)
- But now can lift to N D3-branes in F-theory on $(C^3\times T^2)/Z_k$ (orientifolds of type IIB for $k=2$, S-folds for $k=3,4,6$)
- A family of $d=4$ $\mathcal{N}=3$ SCFTs
Recent examples

- Naively labeled by $N,k$
- Extra "discrete torsion" parameters (OA+Tachikawa):
  - $k=2$: 4 well-known orientifolds
  - $k=3$: $L=1,3$
  - $k=4$: $L=1,4$
  - $k=6$: $L=1$
- Dimensions of $N$ "Coulomb branch generators": $(k, 2k, 3k, \ldots, (N-1)k, NL)$
- Dual to F-theory on $\text{AdS}_5 \times (S^5 \times T^2)/\mathbb{Z}_k$
Special examples

• Minimal pure $\mathcal{N}=3$ theory is $N=1$, $k=L=3$ with a single generator of dimension 3
• Theories with $N=2$ and $L=1$ happen to give $\mathcal{N}=4$ SYM theories:
  • $k=3 : \text{SU}(3)$
  • $k=4 : \text{SO}(5)$
  • $k=6 : \text{G}_2$ (brane construction !)
Some questions

• Can we find more pure $\mathcal{N}=3$ theories?
  o Deformations of $\mathcal{N}=1$ theories?
  o Bootstrap? (More constrained than $\mathcal{N}=2$)

• Can we classify all pure $\mathcal{N}=3$ theories?
Topic 2: Conformal bootstrap

Nothing (at least since $d=2$ RCFTs) ?

(Based on work very much in progress with Alday, Bissi, Perlmutter)
Conformal bootstrap

- **CFT**: primary operators $O_i(x)$ and OPE coefficients $c_{ijk}$. Consistency of the OPE in $\langle O_1(x_1)O_2(x_2)O_3(x_3)O_4(x_4)\rangle$ requires that

$$\sum_n c_{12n}c_{34n} G_n(u, v) \approx \sum_n c_{14n}c_{23n} G_n(v, u)$$

for conformal cross-ratios $u, v$ and “(super)conformal blocks” $G_n$ depending on operator dimensions.

- “Crossing equation” necessary but not sufficient for consistent CFT.
Holographic bootstrap

- QG on $\text{AdS}_{d+1}$ is a CFT$_d$ → automatically obeys crossing. When weakly coupled can expand correlators in “Witten diagrams”:

\[
\langle OOOOO \rangle = \text{ [Diagram]} + \text{ [Diagram]} + \text{ [Diagram]} + \text{ [Diagram]}
\]

- Dual to a “large N CFT” with “single-trace ops” $O_i$, $\Delta_i \sim m_i$, “double-trace” $[O_i O_j]$, etc.

$c_{O_i O_j O_k} = 3$-point coupling $= O(1/N)$,
$c_{O_i O_j [O_i O_j]} = O(1)$, $\Delta_{[O_i O_j]} = \Delta_i + \Delta_j + O(1/N^2)$, ...
Holographic bootstrap

• A field theory like $\Phi^4$ or $\Phi^3$ on AdS$_{d+1}$ can also give a solution to crossing, though not full CFT$_d$ (can be decoupled sector).
• Any bulk theory gives a solution to crossing perturbatively in $1/N$. In $<\text{OOOOO}>$:
  O(1) : Just disconnected diagram, [OO]
  O(1/N^2) : only O’ and correction to [OO]
• At order $1/N^2$ reverse also seems true! (Heemskerk, Penedones, Polchinski, Sully; Alday, Bissi, Lukowski;…) (when gap)
One-loop or $O(1/N^4)$

• In a QFT (or effective field theory like SUGRA), tree-level action determines also loop amplitudes, up to a finite number of coupling constants (“renormalization conditions”). Should be true also in AdS.

• One-loop bulk diagrams not yet computed, except in $\phi^4$ (Penedones, Fitzpatrick+Kaplan). Contribute to $\langle OOOO \rangle$ at $O(1/N^4)$:
One-loop or $O(1/N^4)$

- So can we take a solution to crossing at $O(1/N^2) = \text{a tree-level bulk theory}$, and use only crossing to compute a solution at $O(1/N^4) = \text{one-loop diagrams}$?
- Preliminary results: yes! (Up to inevitable freedom in changing bulk couplings)
- More precisely, can do it if only $[OO]$ appears at $O(1/N^4)$. If also $[O'O']$ appear, may need input from additional 4-point functions at order $O(1/N^2)$. 
One-loop or $O(1/N^4)$

- So can use crossing to compute one-loop diagrams in AdS in theories like $\Phi^4$ or $\Phi^3$ or 5d $\mathcal{N}=8$ SUGRA on AdS$_5$, just from tree-level $<\text{OOOOO}>$. Position/Mellin space.
- In theories like $\mathcal{N}=4$ SYM, even at very strong coupling ($=10d$ SUGRA), need more input from $<O_m O_m O_n O_n>$, but then should be able to compute $<O_2 O_2 O_2 O_2>$ at $O(1/N^4)$ from leading large $N$ answers. (Up to undeterminable local bulk couplings.)
Higher-loops

- In principle can extend to higher loops = higher orders in $1/N$, but new bulk couplings appear, and, related to this, also higher-trace operators appear in OPE, so need extra tree-level information (like 5-point functions) to get full answer.
Topic 3 : Disordered field theories

Nothing ?

(Based on OA, Komargodski, Yankielowicz + work in progress with Narovlansky)
Disorder

• Random inhomogeneities common in condensed matter
• Model as QFTs with random coupling constants (“quenched disorder”). Analyze typical behavior in ensemble
• My motivation: How does renormalization group work in the presence of disorder? Are there new types of fixed points (phases)?
Simplifying assumptions

• Disorder couples to a single scalar operator,

\[ S[h] = S_0 + \int d^d x \ h(x)O(x) \]

• Coupling \( h \) varies independently and randomly (Gaussian) at every point,

\[ h(x) = 0, \quad h(x)h(y) = c^2 \ \delta(x - y) \]

“Background field” \( h(x) \) becomes dynamical

• Euclidean QFT (2\(^{nd}\) order phase transitions); can generalize to couplings constant in time (more complicated)
Precise setup

- Disorder-averaged correlation functions

\[
\langle O_1(x_1) \ldots O_n(x_n) \rangle = \int [Dh] e^{-\frac{1}{2c^2} \int d^d x \, h^2(x)} \frac{\int [D\Phi] O_1(x_1) \ldots O_n(x_n) e^{-S[h]}}{\int [D\Phi] e^{-S[h]}}
\]

- Not

\[
\frac{1}{Z} \int [Dh] e^{-\frac{1}{2c^2} \int d^d x \, h^2(x)} \int [D\Phi] O_1(x_1) \ldots O_n(x_n) e^{-S[h]}
\]

- Connected disordered correlators such as

\[
\langle O_1(x_1) O_2(x_2) \rangle^{\text{conn}} = \langle O_1(x_1) O_2(x_2) \rangle - \langle O_1(x_1) \rangle \langle O_2(x_2) \rangle
\]

generated by disordered free energy

\[
W_D = \int [Dh] \log(Z[h]) e^{-\frac{1}{2c^2} \int d^d x \, h^2(x)}
\]

- Independent of general disordered correlators
Replica trick

- A general method is replica trick:
  \[ W_D = \int [Dh] \log(Z[h]) e^{-\frac{1}{2c^2} \int d^d x \, h^2(x)} = \]
  \[ = \frac{d}{dn} |_{n=0} \int [Dh] Z^n[h] \, e^{-\frac{1}{2c^2} \int d^d x \, h^2(x)} \]
  \[ Z^n[h] = \int \prod_{A=1}^{n} [D\Phi_A] \, e^{-\sum_{A=1}^{n} S_A[h(x)]]} \]

So a limit of standard QFTs.

- Same for correlators. Different origin for \( O(x) \) in general disordered correlators (\( O_1(x) \)) and in connected correlators (\( \frac{1}{n} \sum_{A=1}^{n} O_A(x) \)).
RG flow

- Limit of standard QFTs → couplings flow as usual + extra coupling $c^2$
- So can have standard ($c=0$) and disordered scale-invariant fixed points
- In connected correlators standard RG equation, renormalization of local operators
- In general correlators, have extra “anomalous dimension” and mixings, e.g.

$$
\left( M \frac{\partial}{\partial M} + \beta_i \frac{\partial}{\partial \lambda_i} + \beta_{c^2} \frac{\partial}{\partial c^2} + 2\gamma \right) \langle O_1(x_1)O_2(x_2) \rangle + \tilde{\gamma} \langle O_1(x_1)O_2(x_2) \rangle^{conn} = 0.
$$
RG flow

• At fixed point we get “logarithmic CFTs” (Cardy), e.g.:

\[
\langle O(0)O(x) \rangle^{\text{conn}} \propto 1/|x|^{2\Delta}
\]

\[
\langle O(0)O(x) \rangle \propto \log(x)/|x|^{2\Delta}
\]

Related to degeneracy in replica theory.

• New types of critical exponents.

• Can perform exact computations at large $N$ (field theory / holography). (Subtleties)

• Time-independent disorder – what can be said? Replica theory non-local in time; what does RG equation look like? Fixed points? Lifshitz scaling (Hartnoll+Santos)?
Summary

- These were some suggestions for the next 60 years (120 years)
- But Nati has often preferred to take the road less travelled by, and that has made all the difference...

The Road Not Taken
By Robert Frost

TWO roads diverged in a yellow wood,
And sorry I could not travel both
And be one traveler, long I stood
And looked down one as far as I could
To where it bent in the undergrowth;

Then took the other, as just as fair,
And having perhaps the better claim,
Because it was grassy and wanted wear;
Though as for that the passing there
Had worn them really about the same,

And both that morning equally lay
In leaves no step had trodden black.
Oh, I kept the first for another day!
Yet knowing how way leads on to way,
I doubted if I should ever come back.

I shall be telling this with a sigh
Somewhere ages and ages hence:
Two roads diverged in a wood, and I—
I took the one less traveled by,
And that has made all the difference.
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