Cosmological Bell Inequalities

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AndyFest 2015

Warping the Universe:

A celebration of the Science of Andrew Strominger
COURSE 7

LECTURES ON BLACK HOLES

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Two dimensional black holes

Wormholes, black hole pair creation,…

Black branes

Black hole condensation & conifold

Black three brane entropy

Greybody factors and 2d CFTs

Doubling of supersymmetry in some near horizon regions,…

dS/CFT

Soft gravitons
Non trivial classical solutions

New Quantum effects

Gravity
Cosmological Bell Inequalities

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Warping the Universe:

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• According to the theory of inflation, primordial fluctuations were produced by quantum mechanical effects in the early universe.
• The fluctuations we see now are classical
• Each Fourier mode is a time dependent harmonic oscillator.

\[ ds^2 = \frac{-d\eta^2 + dx^2}{\eta^2} \]

\[ S = \int \frac{d\eta}{\eta^2} \left( |\dot{\phi}|^2 - k^2 |\phi|^2 \right) \]
\[ k^3 [\phi(\eta), \eta \partial_\eta \phi] = \eta^3 k^3 \quad \rightarrow 0 \quad \text{as} \quad \eta k \rightarrow 0 \]

Fluctuations become classical as they exit the horizon
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Fluctuations become classical as they exit the horizon

At reheating we have a classical measure, or probability distribution.
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Fluctuations become classical as they exit the horizon

At reheating we have a classical measure, or probability distribution.

We do not measure the conjugate momentum! Or time derivatives!.

\[ |\Psi(\phi(x))|^2 = \mu[\phi(x)] \]

\[ \uparrow \]

Classical probability distribution.
• Can we distinguish this probability distribution from a purely classical one?
Testing ordinary quantum mechanics

• Many successful predictions!
Testing ordinary quantum mechanics

• Many successful predictions!.

• Fundamental deviation from local classical physics $\rightarrow$ Bell inequalities.
All operators, $A, A', B, B'$ have eigenvalues +1 or -1.

**e.g.** $A = \vec{n}.\vec{\sigma}$, $A' = \vec{n}'.\vec{\sigma}$
\[ C = AB - AB' + A'B + A'B' \]

Clauser, Horne, Simony, Holt, 1969

\[
|C|_{\text{QM, max}} = 2\sqrt{2} > 2 = |C|_{\text{classical, max}}
\]

\[ C = A(B - B') + A'(B + B') \]

\[ C^2 = 4 + [A, A'][B, B'] \]
• In cosmology, we only have commuting observables $\rightarrow$ cannot do the same.
Bell case

Cosmology?

QUANTUM TO CLASSICAL MEASUREMENT

Late Universe
Reheating
Early Universe

Time

A
B'
Bell case

No interesting inequality!

Cosmology

Late early Universe

medium early Universe

Very early Universe

Reheating

QUANTUM TO CLASSICAL MEASUREMENT
Classical information
Proof of principle

• Choose a universe that will make this easy!
Proof of principle

• Choose a universe that will make this easy!

• No claim that this toy model agrees with our universe.
Proof of principle

• Choose a universe that will make this easy!

• No claim that this toy model agrees with our universe.

• Simply a universe where Bell inequalities can be tested with primordial fluctuations.
Designer Universe

- **Entangled state**: Massive particles with an internal "isospin" quantum number
- **Decider variables or detector settings**: Axion field with fluctuations at the locations of the particles.
- **Measurement**: Introduce growing masses which are isospin dependent, according to the isospin projection along an axis determined by the axion.
- **Communication of results**: Growing mass produces a classical perturbation on the inflaton hot spots in the curvature fluctuations. Axion should also be visible today.
Each step in detail...
Massive particle pairs

Particles whose mass depends on time.
  Heavy at early and late times.
  Become light at some specific time during inflation.
  Create well separated pairs of particles.

Particles carry ``isospin''. Create them into isospin singlets.
Distribution of massive particles
Detector Settings (Decider variables)

- Axion field with a time dependent $f_a$.

- $f_a$ becomes small during some time, a few efoldings after the massive particles were created. Then it becomes large again.

- Creates an axion field with fluctuations at a characteristic scale.
Axion fluctuations

\[ \theta \sim \theta + 2\pi \]
Measurement

- Mass is inflaton dependent and it increases to large values of order $M_{pl}$.
- Coupling to inflaton generates a classical perturbation in the inflaton.

\[ \zeta_{\text{part}}(x) = \frac{m(\eta = - |x|)}{2\sqrt{2\epsilon M_{pl}}} \times \left( \frac{1}{2\pi \sqrt{2\epsilon}} \frac{H}{M_{pl}} \right) \]

Size of quantum fluctuations
Hot spots

We see the effects of individual particles
Measurement of the isospin

Isospin dependent mass term.
Dependent on the axion field $\theta$.

\[
m_1^2(\phi) h^\dagger h + \lambda_2(\phi) h^\dagger(\sigma_x \cos n\theta + \sigma_y \sin n\theta) h =
= m_1^2(\phi) \left[ |h_1|^2 + |h_2|^2 \right] + \left[ \lambda_2(\phi) e^{in\theta} h_1^\dagger h_2 + c.c. \right]
\]

Leads to mass eigenvalues:

\[
m_\pm = \sqrt{m_1^2(\phi) \pm |\lambda_2(\phi)|}
\]
Time dependence

No breaking of isospin at early times

Large and distinct values

Distinguishable hot spots
Post inflationary observations

First map the hot and very hot spots, corresponding to $m_+$ and $m_-$. 
Post inflationary observations

Group them into pairs. View this as a measurement with values +1 or -1.
Measuring axion values

Axion $\rightarrow$ could give rise to isocurvature perturbations. Amplitude of perturbation $\rightarrow$ related to axion value
Constructing the observable

Outcome:

\[ \left( \pm 1 \theta_A ; \pm 1 \theta_B \right) \]

Settings of detectors

We can now form the C observable and check whether Bell’s inequalities are violated.

Quantum mechanics allows a violation of up to a factor of \( \sqrt{2} \).

In this model we indeed get such a violation.

This proves that the variable determining the type of hotspot we have is quantum.
Conclusions

• We have discussed a toy cosmological model which contains Bell inequality violating observables.

• Can we make them in more realistic models?

• There are other signatures of quantum mechanics: e.g. Looking at phase oscillations in the 3 or 4 point function produced by massive particles, with constant masses. This is an interference effect.

• Can we find more evidence in favor of the quantum nature of fluctuations?
Happy Birthday Andy