Supercurrents

Nathan Seiberg

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Tom Banks and NS arXiv:1011.5120
Thomas T. Dumitrescu and NS arXiv:1106.0031
Summary

- The supersymmetry algebra can have brane charges.
- Depending on the brane charges, there are different supercurrent multiplets.
- The nature of the multiplet is determined in the UV but is valid also in the IR. This leads to exact results about the renormalization group flow.
- Different supermultiplets are associated with different off-shell supergravities. (They might be equivalent on shell.)
- Understanding the multiplets leads to constraints on supergravity and string constructions.
The $4d \mathcal{N} = 1$ SUSY Algebra (imprecise)

[Ferrara, Porrati; Gorsky, Shifman]:

$$\{ \bar{Q}_{\dot{\alpha}}, Q_\alpha \} = 2\sigma^\mu_{\alpha\dot{\alpha}} (P_\mu + Z_\mu) ,$$
$$\{ Q_\alpha, Q_\beta \} = \sigma^\mu_{\alpha\beta} Z_{\mu\nu} .$$

- $Z_\mu$ is a string charge.
- $Z_{[\mu\nu]}$ is a complex domain wall charge.
- They are infinite – proportional to the volume.
- They are not central.
- They control the tension of BPS branes.
- Algebraically $P_\mu$ and $Z_\mu$ seem identical. But they are distinct...
The $4d \mathcal{N} = 1$ SUSY Current Algebra

\[ \begin{align*}
\{ \overline{Q}_\dot{\alpha}, S_{\alpha\mu} \} & = 2\sigma^\nu_{\dot{\alpha}\dot{\alpha}} (T_{\nu\mu} + C_{\nu\mu}) + \cdots , \\
\{ Q_\beta, S_{\alpha\mu} \} & = \sigma^\nu_{\dot{\alpha}\dot{\beta}} C_{\nu\rho\mu} .
\end{align*} \]

- $T_{\mu\nu}$ and $S_{\alpha\mu}$ are the energy momentum tensor and the supersymmetry current.
- $C_{[\mu\nu]}$, $C_{[\mu\nu\rho]}$ are conserved currents associated with strings and domain-walls. Their corresponding charges are $Z_\mu$, $Z_{[\mu\nu]}$ above.
- Note that $T_{\{\mu\nu\}}$ and $C_{[\mu\nu]}$ are distinct.
Properties of the Supercurrent Multiplet

- $T_{\mu\nu}$ is conserved and symmetric. It is subject to improvement (actually more general)

$$T_{\mu\nu} \to T_{\mu\nu} + \left( \partial_\mu \partial_\nu - \eta_{\mu\nu} \partial^2 \right) u .$$

- $S_{\alpha\mu}$ is conserved. It is subject to improvement

$$S_{\alpha\mu} \to S_{\alpha\mu} + \left( \sigma_{\mu\nu} \right)_\alpha^{\beta} \partial^\nu \eta_\beta .$$

- We impose that $T_{\mu\nu}$ is the highest spin operator in the multiplet.
- We consider only well-defined (gauge invariant) local operators.
The S-Multiplet

The most general supercurrent satisfying our requirements is the S-multiplet \( S_{\alpha \dot{\alpha}} \) (real)

\[
\overline{D}^{\dot{\alpha}} S_{\alpha \dot{\alpha}} = \chi_\alpha + \mathcal{V}_\alpha ,
\]

\[
\overline{D}_{\dot{\alpha}} \chi_\alpha = 0 , \quad D^\alpha \chi_\alpha = \overline{D}_{\dot{\alpha}} \chi^{\dot{\alpha}} ,
\]

\[
D_\alpha \mathcal{V}_\alpha + D_\beta \mathcal{V}_\alpha = 0 , \quad \overline{D}^2 \mathcal{V}_\alpha = 0 .
\]

Equivalently,

\[
\overline{D}^{\dot{\alpha}} S_{\alpha \dot{\alpha}} = \overline{D}^2 D_\alpha V + D_\alpha X ,
\]

\[
\overline{D}_{\dot{\alpha}} X = 0 , \quad V = V^\dagger
\]

but \( V \) and \( X \) do not have to be well defined.
Components the S-Multiplet

\[
\overline{D}^{\dot{\alpha}} S_{\alpha \dot{\alpha}} = \chi_\alpha + \mathcal{Y}_\alpha
\]

\[
S_\mu = j_\mu + \theta^\alpha S_{\alpha \mu} + \bar{\theta}^{\dot{\alpha}} \bar{S}_{\dot{\mu}}^{\dot{\alpha}} + (\theta \sigma^\nu \bar{\theta}) T_{\mu \nu} + \ldots
\]

It includes 16+16 operators:

- Energy momentum tensor \( T_{\mu \nu} \) (10 - 4 = 6)
- String current \( \epsilon_{\mu \nu \rho \sigma} F^{\rho \sigma} \) in \( \chi_\alpha = \overline{D}^2 D_\alpha V \) (3)
- A complex domain wall current \( \epsilon_{\mu \nu \rho \sigma} \partial^\sigma x \) in \( \mathcal{Y}_\alpha = D_\alpha X \) (2)
- A non-conserved R-current \( j_\mu \) (4)
- A real scalar (1)
- 16 fermionic operators
Improvements of the S-Multiplet

The S-multiplet is not unique. The defining equation

$$\overline{D}^{\dot{\alpha}} S_{\alpha \dot{\alpha}} = \chi_{\alpha} + Y_{\alpha}$$

is invariant under the transformation

$$S_{\alpha \dot{\alpha}} \rightarrow S_{\alpha \dot{\alpha}} + [D_{\alpha}, \overline{D}_{\dot{\alpha}}]U,$$

$$\chi_{\alpha} \rightarrow \chi_{\alpha} + \frac{3}{2} \overline{D}^2 D_{\alpha} U,$$

$$Y_{\alpha} \rightarrow Y_{\alpha} + \frac{1}{2} D_{\alpha} \overline{D}^2 U,$$

with real $U$ (well-defined up to an additive constant).

This changes $S_{\alpha \mu}, T_{\mu \nu}$, the string and domain wall currents by improvement terms.
In special cases the S-multiplet is decomposable:
Special Cases

\[ \overline{D}^\alpha S_{\alpha \dot{\alpha}} = \chi_\alpha + \mathcal{Y}_\alpha \]

In special cases the S-multiplet is decomposable:

- If \( \chi_\alpha = \frac{3}{2} \overline{D}^2 D_\alpha U \) with a well defined \( U \), we can set it to zero. This happens when there is no string charge (the string current can be improved to zero). Then, the S-multiplet is decomposed to a real (vector superfield) \( U \) and the Ferrara-Zumino multiplet (below).

- If \( \mathcal{Y}_\alpha = \frac{1}{2} \overline{D}^2 U \) with a well defined \( U \), we can set it to zero. This happens when there is no domain wall charge. Here the S-multiplet is decomposed to a real (vector superfield) \( U \) and the R-multiplet (below).

- If both are true with the same \( U \), we can set \( \chi_\alpha = \mathcal{Y}_\alpha = 0 \). This happens when the theory is superconformal.
Special Cases

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- If \( \mathcal{V}_\alpha = D_\alpha X = \frac{1}{2} D_\alpha \overline{D}^2 U \) with a well defined \( U \), we can set it to zero. This happens when there is no domain wall charge. Here the S-multiplet is decomposed to a real (vector superfield) \( U \) and the R-multiplet (below).
Special Cases

\[ \overline{D}^{\dot{\alpha}} S_{\alpha \dot{\alpha}} = \chi_{\alpha} + \mathcal{Y}_{\alpha} \]

In special cases the S-multiplet is decomposable:

- If \( \chi_{\alpha} = \frac{3}{2} \overline{D}^{2} D_{\alpha} U \) with a well defined \( U \), we can set it to zero. This happens when there is no string charge (the string current can be improved to zero). Then, the S-multiplet is decomposed to a real (vector superfield) \( U \) and the Ferrara-Zumino multiplet (below).

- If \( \mathcal{Y}_{\alpha} = D_{\alpha} X = \frac{1}{2} D_{\alpha} \overline{D}^{2} U \) with a well defined \( U \), we can set it to zero. This happens when there is no domain wall charge. Here the S-multiplet is decomposed to a real (vector superfield) \( U \) and the R-multiplet (below).

- If both are true with the same \( U \), we can set \( \chi_{\alpha} = \mathcal{Y}_{\alpha} = 0 \). This happens when the theory is superconformal.
The Ferrara-Zumino (FZ) Multiplet

When $\chi_\alpha = \frac{3}{2} D^2 D_\alpha U$ we find the most familiar supercurrent – the FZ-multiplet

$$D^\dot{\alpha} J_{\alpha \dot{\alpha}} = D_\alpha X,$$
$$D_{\dot{\alpha}} X = 0.$$

- It contains 12+12 independent real operators: $j_\mu$ (4), $T_{\mu \nu}$ (6), $x$ (2) and $S_{\alpha \mu}$ (12).
- It exists only if there are no string currents – it does not exist if there are FI-terms $\zeta$ or if the Kähler form is not exact. Nontrivial

$$C_{\mu \nu} \sim \zeta \epsilon_{\mu \nu \rho \sigma} F^{\rho \sigma},$$
$$C_{\mu \nu} \sim i \epsilon_{\mu \nu \rho \sigma} K_{i \bar{i}} \partial^\rho \phi^i \partial^\sigma \phi^{\bar{i}}$$

are obstructions to its existence.
The R-Multiplet

When \( \mathcal{Y}_\alpha = D_\alpha X = \frac{1}{2} D_\alpha \overline{D}^2 U \) we find the R-multiplet

\[
\overline{D}^{\dot{\alpha}} R_{\alpha \dot{\alpha}} = \chi_\alpha, \\
\overline{D}_{\dot{\alpha}} \chi_\alpha = 0, \quad D^\alpha \chi_\alpha = \overline{D}_{\dot{\alpha}} \chi^{\dot{\alpha}}.
\]

- This multiplet includes 12+12 operators. Among them is a string current. But there is no domain wall current.
- \( j_\mu = R_\mu \) is a conserved R-current – the theory has a \( U(1)_R \) symmetry.
- This multiplet exists even when the Kähler form is not exact or the theory has FI-terms.
- \( S_{\alpha \mu}, T_{\mu \nu} \) differ from those in the FZ-multiplet by improvement terms.
Example: Wess-Zumino Models

Every theory has an S-multiplet. Example: a WZ theory

\[ S_{\alpha\dot{\alpha}} = 2K_{ij} D_\alpha \Phi^i \bar{D}_{\dot{\alpha}} \bar{\Phi}^j , \]
\[ \chi_\alpha = \bar{D}^2 D_\alpha K , \]
\[ X = 4W . \]

- All the operators are globally well defined.
- Since \( X \) has to be well defined up to adding a constant, we can allow multi-valued \( W \).
Example: Wess-Zumino Models

- If the Kähler form is exact, there are no strings and the S-multiplet can be improved to the FZ-multiplet

\[ \mathcal{J}_{\alpha\dot{\alpha}} = 2K_{ij} D_\alpha \Phi^i \bar{D}_{\dot{\alpha}} \bar{\Phi}^j - \frac{2}{3} [D_\alpha, \bar{D}_{\dot{\alpha}}] K \]

\[ X = 4W - \frac{1}{3} \bar{D}^2 K \]
Example: Wess-Zumino Models

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\[ \mathcal{J}_{\alpha\dot{\alpha}} = 2K_{i\bar{j}} D_\alpha \Phi^i \bar{D}_{\dot{\alpha}} \bar{\Phi}^\bar{j} - \frac{2}{3} [D_\alpha, \bar{D}_{\dot{\alpha}}] K , \]
\[ X = 4W - \frac{1}{3} \bar{D}^2 K . \]

If the theory has an R-symmetry, the S-multiplet can be improved to the R-multiplet (even when the Kähler form is not exact)

\[ \mathcal{R}_{\alpha\dot{\alpha}} = 2K_{i\bar{j}} D_\alpha \Phi^i \bar{D}_{\dot{\alpha}} \bar{\Phi}^\bar{j} - [D_\alpha, \bar{D}_{\dot{\alpha}}] \sum_i R_i \Phi^i \partial_i K , \]
\[ \chi_\alpha = \bar{D}^2 D_\alpha \left( K - \frac{3}{2} \sum_i R_i \Phi^i \partial_i K \right) , \]
Constraints on RG-Flow

Consider a SUSY field theory, which has an FZ-multiplet in the UV (e.g. a gauge theory without FI-terms). Hence, the FZ-multiplet exists at every energy scale. This constrains the low-energy theory:

- No string charge in the SUSY algebra
- No FI-terms, even for emergent gauge fields (previous arguments by [Shifman, Vainshtein; Dine; Weinberg]).
- The Kähler form of the target space is exact (previous argument by [Witten]).
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  - The Kähler form of the target space is exact (previous argument by [Witten]).

- Consider a SUSY field theory with a $U(1)_R$ symmetry. It has an R-multiplet at every energy scale. Hence, there are no domain wall charges in the supersymmetry algebra and in particular, no BPS domain walls.
The S-Multiplet in $3d$

- The S-multiplet for $\mathcal{N} = 2$ in $3d$ is given by

$$\overline{D}^\beta S_{\alpha\beta} = \chi_\alpha + \mathcal{Y}_\alpha,$$

$$\overline{D_\alpha} \chi_\beta = \frac{1}{2} C \varepsilon_{\alpha\beta}, \quad D^\alpha \chi_\alpha = -\overline{D}^\alpha \chi_\alpha,$$

$$D_\alpha \mathcal{Y}_\beta + D_\beta \mathcal{Y}_\alpha = 0, \quad \overline{D}^\alpha \mathcal{Y}_\alpha = -C,$$

where $C$ is a complex constant.

- It leads to a new term in the SUSY current algebra:

$$\{Q_\alpha, S_{\beta\mu}\} = \frac{1}{4} \overline{C} \gamma_{\mu\alpha\beta} + \cdots.$$

We interpret it as a space-filling brane current (not affected by improvements). This is consistent with dimensional reduction from $4d$. 

Application: Partial SUSY-Breaking

- If $C \neq 0$, the vacuum preserves at most two of the four supercharges. SUSY can be partially broken from $\mathcal{N} = 2$ to $\mathcal{N} = 1$.

- It happens because of a deformation of the current algebra [Hughes, Polchinski].

- This is fundamentally different from spontaneous breaking, where the current algebra is not modified.

- The nature of the multiplet and the value of $C$ are determined in the UV. Hence, if $C = 0$ in the UV (e.g. in conventional SUSY gauge theories), there cannot be partial SUSY breaking.
Partial SUSY-Breaking

Other places with the same phenomenon:

- In the $2d \mathcal{N} = (0, 2) \mathbb{CP}^1$ model instantons generate nonzero $C$ (earlier work by [Witten; Tan, Yagi]).
- $\mathcal{N} = (2, 2)$ in $2d$
  - Three different space-filling brane currents can break $(2, 2) \rightarrow (1, 1), (2, 0), \text{ or } (0, 2)$.
  - Simple models realize these possibilities [Hughes, Polchinski; Losev, Shifman].
- $\mathcal{N} = 2 \rightarrow \mathcal{N} = 1$ breaking in $4d$ [Hughes, Liu, Polchinski; Antoniadis, Partouche, Taylor; Ferrara, Girardello, Porrati].
Constraints on Linearized SUGRA

Linearized SUGRA is obtained by adding to the flat space Lagrangian

$$\mathcal{L}_{\text{flat space}} + \int d^4 \theta H^\mu S_\mu + \mathcal{O}(H^2)$$

where $S_\mu$ is the supercurrent and $H^\mu$ is a superfield containing the deformation of the metric.
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Standard SUGRA ("old-minimal SUGRA") uses the FZ-multiplet

\[ \mathcal{L} = \mathcal{L}_{\text{flat space}} + h^{\mu\nu} T_{\mu\nu} + \psi^{\mu\alpha} S_{\mu\alpha} + \bar{\psi}^{\mu\dot{\alpha}} \bar{S}_{\mu\dot{\alpha}} + b^\mu j_\mu + M x + \bar{M} \bar{x} + \ldots \]

It exists only when the FZ-multiplet exists; i.e. when the Kähler form is exact [Witten and Bagger] and when there is no FI-term.
Constraints on Linearized SUGRA

If the theory has a global $U(1)_R$ symmetry we can use “new-minimal SUGRA”, which is based on the R-multiplet.

- On shell it is equivalent to the “old-minimal” formalism.
- Even though the $U(1)_R$ symmetry of the matter theory is gauged, the resulting theory has a global $U(1)_R$ symmetry.
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- We conclude that theories without an FZ-multiplet can be coupled to SUGRA only if they have a global $U(1)_R$ symmetry. This is consistent with earlier work of [Freedman; Barbieri, Ferrara, Nanopoulos, Stelle; Kallosh, Kofman, Linde, Van Proeyen].
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- We conclude that theories without an FZ-multiplet can be coupled to SUGRA only if they have a global $U(1)_R$ symmetry. This is consistent with earlier work of [Freedman; Barbieri, Ferrara, Nanopoulos, Stelle; Kallosh, Kofman, Linde, Van Proeyen].
- Excluding gravity theories with global symmetries, such models are not acceptable.

This constrains many supergravity and string constructions.
Constraints on Linearized SUGRA

A rigid theory without an FZ-multiplet and without a $U(1)_R$ symmetry can be coupled to linearized SUGRA using the S-multiplet.
A rigid theory without an FZ-multiplet and without a $U(1)_R$ symmetry can be coupled to linearized SUGRA using the S-multiplet.

- The resulting SUGRA (16/16 SUGRA) has more degrees of freedom.
- One way to think about it is to add to the matter system another propagating chiral superfield such that it has an FZ-multiplet and then use the standard formalism [Siegel].
- This is familiar from heterotic compactifications, where the additional propagating degrees of freedom are the dilaton, the dilatino and the two-form $B$. 
Conclusions

- The supersymmetry current and the energy-momentum tensor are embedded in a supermultiplet.
- The S-multiplet is the most general supercurrent multiplet.
  - It has $16+16$ components.
  - It always exists.
  - In special situations it is decomposable – can be improved to a smaller multiplet.
- The most common supercurrent is the FZ-multiplet.
  - It has $12+12$ components.
  - It exists when there are no string charges in the SUSY algebra.
  - This happens when the Kähler form is exact and there are no FI-terms.
Conclusions

- If the theory has a $U(1)_R$ symmetry it has an R-multiplet
  - It has $12 + 12$ components.
  - It does not admit domain wall charges in the algebra.
Conclusions

- If the theory has a $U(1)_R$ symmetry it has an R-multiplet
  - It has $12+12$ components.
  - It does not admit domain wall charges in the algebra.
- This discussion constrains the dynamics:
  - If the UV theory has an FZ-multiplet, the low-energy theory has an exact Kähler form and no FI-terms – it does not have string charges in the SUSY algebra.
  - If the theory has a $U(1)_R$ symmetry, it has an R-multiplet and then it does not have charged domain walls.
  - Space-filling brane currents give rise to partial SUSY breaking (fundamentally different from spontaneous breaking).
  - If the corresponding current is not present in the UV, SUSY cannot be partially broken.
Conclusions

- This also constrains linearized supergravity
  - Only theories with an FZ-multiplet can be coupled to “old-minimal supergravity.”
  - Theories with nontrivial Kähler form or FI-terms can be coupled to “new-minimal supergravity”, but then they must have a continuous global $U(1)_R$ symmetry.
  - More general theories can be coupled through their S-multiplet. But the resulting theory has more degrees of freedom.
- Constraints on SUGRA/string constructions.
- There conclusions are limited to linearized supergravity. Additional consistent possibilities exist in intrinsic gravitational theories with Planck size coupling constants [Witten and Bagger; NS].