Kozai–Lidov oscillations

- Kozai (1962 - asteroids); Lidov (1962 - artificial satellites)
- Arise most simply in restricted three-body problem (two massive bodies on a Kepler orbit + a test particle)
- E.g., wide binary star + planet orbiting one member of the binary
- In Kepler potential $\Phi = -\frac{GM}{r}$, eccentric orbits have a fixed orientation
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*generic axisymmetric potential*
Kozai-Lidov oscillations

• now subject the Kepler orbit to a weak, time-independent external force $F$ from the companion star

• because the orbit orientation is fixed even weak external forces act for a long time in a fixed direction relative to the orbit and therefore change the angular momentum or eccentricity

• if $F \sim \varepsilon$ then \textit{timescale} for evolution $\sim 1/\varepsilon$ but \textit{nature} of evolution is independent of $\varepsilon$
Kozai-Lidov oscillations

Consider a planet orbiting one member of a binary star system:

- because the force from the companion star is weak we can average over both planetary and binary star orbits
- keep only the quadrupole term from the companion
- because of averaging the gravitational potential from the companion is fixed, so energy $E$ is conserved ($E = -GM_*/2a$ so semi-major axis $a$ is conserved)
- for circular companion orbit the potential is axisymmetric so $J_z$ is conserved
- accidentally, it turns out that $J_z$ is conserved even if companion orbit is eccentric
Averaged Hamiltonian is

\[ H = \epsilon [5e^2 \sin^2 i \sin^2 \omega - (1 - e^2) \cos^2 i - 2e^2] \]

where

\[ \epsilon \equiv \frac{3GM_e a^2}{8(1 - e^2)^{3/2} a_c^3}. \]

Action-angle variables are

- Angular momentum: \( J_1 = [GM_*(1 - e^2)]^{1/2} \)
- Argument of pericenter: \( J_2 = J_1 \cos i \)
- Longitude of node: \( \theta_1 = \omega \)
- Argument of pericenter: \( \theta_2 = \Omega \)

Hamiltonian is independent of \( \Omega \) so \( J_2 \) is conserved. Remaining motion has one degree of freedom and follows \( H = \text{constant contours} \).

\[ \frac{dJ_1}{dt} = -\frac{\partial H}{\partial \omega}, \quad \frac{d\theta_1}{dt} = \frac{\partial H}{\partial J_1}. \]
Let $\mathbf{j}$ point in the direction of the angular momentum vector with magnitude $|\mathbf{j}| = (1 - e^2)^{1/2}$. Let $\mathbf{e}$ point towards pericenter with magnitude $e$. Then

$$H = \epsilon [5(\mathbf{e} \cdot \mathbf{n})^2 - (\mathbf{j} \cdot \mathbf{n})^2 - 2e^2]$$

where $\mathbf{n}$ is the normal to the companion orbit. The equations of motion are

$$\frac{d\mathbf{j}}{d\tau} = \mathbf{e} \times \nabla_e H + \mathbf{j} \times \nabla_j H$$

$$\frac{d\mathbf{e}}{d\tau} = \mathbf{j} \times \nabla_e H + \mathbf{e} \times \nabla_j H$$

where $\tau = t/(GM_\star a)^{1/2}$. 
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Action-angle variables are:
- z-angular momentum
- longitude of node
- angular momentum
- argument of pericenter

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\[ \frac{dJ_1}{dt} = -\frac{\partial H}{\partial \omega}, \quad \frac{d\theta_1}{dt} = \frac{\partial H}{\partial J_1}. \]
Kozai-Lidov oscillations

- Initially circular orbits remain circular if and only if the initial inclination is $< 39^\circ = \cos^{-1}(3/5)^{1/2}$
- For larger initial inclinations, the phase plane contains a separatrix
- Circular orbits cannot remain circular, and are excited to high inclination and eccentricity — not a rigid hoop (surprise #1)
- Circular orbits are chaotic (surprise #2)
Kozai-Lidov oscillations

• circular orbits cannot remain circular, and are excited to high inclination and eccentricity (surprise # 1)

• circular orbits are chaotic (surprise # 2)

• as the initial inclination approaches 90°, the maximum eccentricity achieved in a Kozai oscillation approaches unity ⇒ tidal dissipation or collision (surprise # 3)

• mass and separation of companion affect period of Kozai oscillations, but not the amplitude (surprise # 4)
eccentricity oscillations of a planet in a binary star system

- $a_{\text{planet}} = 2.5 \text{ AU}$
- companion has inclination $75^\circ$, semi-major axis $750 \text{ AU}$, mass $0.08 \ M_\odot$ (solid) or $0.9 \ M_\odot$ (dotted)

(Takeda & Rasio 2005)
Kozai–Lidov oscillations

- circular orbits are excited to high inclination and eccentricity (surprise # 1)
- circular orbits are chaotic (surprise # 2)
- as the initial inclination approaches 90°, the maximum eccentricity approaches unity ⇒ tidal dissipation or collision (surprise # 3)
- mass and separation of companion affect period of Kozai oscillations, but not the amplitude (surprise # 4)
- small additional effects such as general relativity or octupole tidal potential can strongly affect the oscillations (surprise # 5)
1. Irregular satellites of the giant planets

Hill (or tidal, or Roche) radius

\[ r_H = a_p \left( \frac{m}{3M_\odot} \right)^{\frac{1}{3}} \]

represents approximately the maximum radius at which an orbit stays bound to the planet

- at \( r < 0.05r_H \), satellites of the giant planets tend to be on nearly circular, prograde orbits near the planetary equator ("regular" satellites). Probably formed from a protoplanetary disk

- at \( r > 0.05r_H \) the satellites have large eccentricities and inclinations, including retrograde orbits (irregular" satellites). Probably captured from heliocentric orbits

- irregular satellites are much smaller than regular ones but there are a lot more of them (97). Total satellite count:

there are no irregular satellites with inclinations between 40° and 140°

Nesvorny et al. (2003)
unstable due to K-L oscillations

unstable because too close to Hill radius

Nesvorny et al. (2003)
Kozai-Lidov oscillations explain the absence of irregular satellites at high inclinations.

unstable due to K-L oscillations
unstable because too close to Hill radius

Nesvorny et al. (2003)
2. Exoplanet eccentricities

Kozai-Lidov oscillations may excite eccentricities of planets in some binary star systems, but probably not all planet eccentricities:

- not all have stellar companion stars (so far as we know)
- suppressed by additional planets
- suppressed by general relativity (!)

red = binary
3. Formation of close binary stars

Binary stars are common: roughly 2/3 of nearby stars are in binaries, with a wide distribution of periods:

\[ dn \propto \exp \left[ -\frac{(\log P/P_0)^2}{2\sigma_P^2} \right] \]

\[ P_0 = 170 \text{ yr}, \ \sigma_P = 2.3 \]

(Duquennoy & Mayor 1991)
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(Duquennoy & Mayor 1991)

If formation of inner and outer binary in a hierarchical triple star is independent we expect (1) about \((2/3) \times (2/3) \approx 0.5\) of all systems to be triple and (2) characteristics of inner and outer binary to be independent.
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How can a tertiary companion that is 1000 X further away affect the formation of a binary star?

How do you form a binary with a separation of a few stellar radii when stars shrink by orders of magnitude during their formation?
Formation of close binary stars

follow orbit evolution of binary or triple star systems, including:

• secular evolution of orbit due to quadrupole tidal field from a tertiary
• apsidal precession due to rotational distortion of stars in the inner binary
• apsidal precession due to mutual tidal distortion of stars in the inner binary
• stellar spins
• tidal friction (Eggleton & Kiseleva-Eggleton 2001)
• relativistic precession

Fabrycky & Tremaine (2007)
Formation of close binary stars

• choose binary stars at random from the Duquennoy & Mayor (1991) distribution, then evolve under tidal friction

• choose triple stars by sampling twice from the binary-star distribution and discard if unstable, then evolve under tidal friction
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new population of binaries at periods of ~3 d
• combine the distributions assuming (a) 25% of systems are triple; (b) period distribution is cut off at 6 d (radius of dynamically stable protostars)

• Kozai-Lidov cycles may be responsible for almost all close binary stars
in this simple model, there is a strong peak near 40 and 140 degrees in the mutual inclinations of systems with $3 \, d < P_{\text{in}} < 10 \, d$

Muterspaugh et al (2007) list five triple systems in this period range.
4. Blue stragglers

Blue stragglers are stars in globular clusters that appear to be anomalously young.

Possible origins:
- stellar collision and merger
- mass transfer or coalescence in a primordial binary system
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Possible origins:

• stellar collision and merger

• mass transfer or coalescence in a primordial binary system

Problems:

• frequency is not correlated with expected collision rate (or any other cluster properties)

Leigh et al. (2007)
4. Blue stragglers

Possible origins:
- stellar collision and merger
- mass transfer or coalescence in a primordial binary system

Problems:
- frequency is not correlated with expected collision rate (or any other cluster properties)
- radial distribution is difficult to interpret (maybe both mechanisms operate?)

Ferraro (2005)
4. Blue stragglers

Possible origins:

• stellar collision and merger
• mass transfer or coalescence in a primordial binary system

Problems:

• frequency is not correlated with expected collision rate
• radial distribution is difficult to interpret
• binary fraction of blue stragglers in NGC 188 is three times that in solar neighborhood

Mathieu & Geller (2009)
4. Blue stragglers

Possible origin:

• stellar collision and merger

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• Kozai-Lidov oscillations in a triple system leading to merger (Perets & Fabrycky 2009)
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Possible origin:

• stellar collision and merger

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Problems:

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• binary fraction of blue stragglers in NGC 188 is three times that in solar neighborhood
These arise from white dwarfs that exceed the Chandrasekhar limit, either through:

- mass accretion from a main-sequence companion star
- mergers of white dwarf-white dwarf binaries

If most close binaries are in triples then most SN Ia progenitors are in triples so Kozai-Lidov oscillations will strongly affect rate (Thompson 2011)

- may explain “prompt” Ia supernovae
- predicts periodic gravitational pulses (Gould 2011)
- why have we not found nearby WD-WD binaries? Possible color contamination by main-sequence third body
6. Planetary migration

Planet-planet scattering + tidal friction may form hot Jupiters

• suppose scattering leads to an isotropic distribution of velocities

• tidal friction is only important for pericenter $q < 0.02$ AU, so must scatter onto nearly radial orbit. Probability $\sim q/a$

• if Kozai-Lidov oscillations are present angular momentum oscillates but $L_z$ is conserved. Probability of $q < 0.02$ AU at some point in the cycle is $\sim (q/a)^{1/2}$

• Kozai-Lidov oscillations due to outer planets are a critical part of all high-eccentricity migration scenarios
Kozai oscillations with tidal friction in a model of HD 80606b

initial conditions:

- $a = 5$ AU
- $i = 86 \pm$°
- $a_{\text{out}} = 1000$ AU

(Wu & Murray 2003)
distribution of projected obliquities

observed
Triaud et al. (2010)

KL oscillations
Fabrycky & Tremaine (2007)

planet-planet scattering
Nagasawa et al. (2008)
Kozai-Lidov oscillations may accelerate the merger of binary black holes (the “final parsec problem”) where external field may come from triaxial galaxy potential or a third black hole (Blaes et al. 2002, Yu 2002, Tanikawa & Umemura 2011)

8. Comets

Kozai-Lidov oscillations induced by the Galactic tidal field drive comets onto orbits that intersect the planetary system
Kozai-Lidov oscillations

- distant satellites of the giant planets have inclinations near 0 or 180° but not near 90°
- may excite eccentricities of planets in binary star systems, but probably not all planet eccentricities
- may enhance merger rate of binary black holes in the centers of galaxies
- source of long-period comets
- formation of close binary stars
- formation of blue stragglers
- formation of hot Jupiters
- obliquities of host stars of transiting exoplanets
- Type Ia supernovae, gamma-ray bursts, gravitational wave sources
- **homework**: why do Earth satellites stay up?