IMPlications of Planet-Bound Dark Matter

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- Gravitationally Bound Dark Matter
- Placing Direct Limits on the Mass of Earth-Bound Dark Matter
- Can the Flyby Anomaly Be Attributed to Earth-Bound Dark Matter?
- Planet-Bound Dark Matter and the Internal Heat of Uranus, Neptune, and Hot-Jupiter Exoplanets
Gravitationally Bound Dark Matter

Cosmology suggests that only \( ~4\% \)
of the mass-energy density of the universe is ordinary baryonic matter.

\( ~23\% \) is gravitationally attractive

"dark matter"

\( ~73\% \) is gravitationally repulsive

"dark energy"

Little is known about dark matter:

- Is it bosonic or fermionic?
- Is it self-annihilating (either its own antiparticle, or equal abundances of particle and antiparticle) or non-self-annihilating?
- What are mass (masses) and non-gravitational interactions?
- Hints of direct detection DAMA/LIBRA/PAMELA
DARK MATTER CAN BE GRAVITATIONALLY BOUND ON DIFFERENT SCALES

- GALACTIC HALO DARK MATTER

\[ \rho \sim 0.3 \text{ GeV/cm}^3 \]

- SOLAR SYSTEM-BOUND DARK MATTER?

\[ \rho < 10^5 \text{ GeV/cm}^3 \]

FROM STUDY OF PLANETARY ORBITS

(Frére, Luing, Vertongen
Severo, Jelenk
Toric
Khriplovich, Litteva)

- EARTH AND PLANET-BOUND DARK MATTER?

SUBJECT OF THIS TALK:

BOUNDS

IMPLICATIONS
PLACING DIRECT LIMITS ON THE MASS OF EARTH-BOUND DARK MATTER

Can set a direct limit on the total Earth-bound dark matter mass lying between the radius ∼ 389,000 km of Moon's orbit, and the radius ∼ 12,300 km of LAGEOS geostatic satellite orbit.

For a satellite of negligible mass in circular orbit around body of mass $M$, measurement of orbit radius $R$ and orbit period $T$ gives $GM$:

$$GM = \frac{4\pi^2 R^3}{T^2}$$

- LAGEOS tracking gives $GM_\oplus$ Earth mass $M_\oplus$ includes dark matter within the LAGEOS orbit.

- Lunar orbiters give $GM_m$ (assume moon-bound dark matter mass to be negligible).
MORE ACCURATE DETERMINATION OF $M_m$ COMES FROM TRACKING EROS ASTEROID, WHICH IS INFLUENCED BY EARTH'S AND MOONY GRAVITY - GIVES

$$R / m = \frac{g M_\oplus + 6 \Delta M_\oplus}{GM_m}$$

POSSIBLE EARTH-BOUND DARK MATTER CONTRIBUTION

$$= \frac{GM_\oplus}{GM_m} (1 + \delta) \quad \delta = \frac{\Delta M_\oplus}{M_\oplus}$$

COMBINED EARTH-MOON SYSTEM

LUNAR LASER RANGING DETERMINES THE COMBINED MASS (TIMES 6) OF THE EARTH-MOON SYSTEMS:

$$GM_{\text{combined}} = GM_\oplus + GM_m + GM_{\Delta m}$$

$M_{\Delta m}$ = DARK MATTER MASS LYING BETWEEN THE RADIUS OF MOON AND LAGGOS ORBITS

SO

$$GM_{\Delta m} = GM_{\text{combined}} - GM_\oplus - GM_m$$

LUNAR ORBITER DETERMINATION
OR USING EROS

\[ G_{\text{Combined}} - G_M = \frac{G M}{R_{\oplus}/m} \]

\[ = G M_{dm} + G M_{ms} = G M_{dm} + \frac{M_{ms}}{M_{\oplus}} G M_{\oplus} \]

SINCE \( \frac{M_{ms}}{M_{\oplus}} = 0.0123 \), THIS GIVES

\[ G_{\text{Combined}} - G_M = \frac{G M_{dm}}{R_{\oplus}/m} = G M_{dm} + 0.0123 G M_{\oplus} \]

\[ > G M_{dm} \]

\[ = G M_{dm} (1 + 0.01) \] IF \( M_{\oplus} \sim M_{dm} \)

\[ \text{NUMERICAL} \quad \text{(ALL CONVERTED TO GARYCENTRIC)} \]
\[ \text{(SIAA Tsyganov, JPL) DYNAMICAL TIME} \]

LAGEOS \( \Rightarrow G M = 398,600.4356 \pm 0.0008 \) \( \text{km}^3 \text{s}^{-2} \)

LUNAR RANGING \( \Rightarrow G_{\text{Combined}} = 903,503.2357 \pm 0.0014 \) \( \text{km}^3 \text{s}^{-2} \)

EROS \( \Rightarrow R_{\oplus}/m = 82.300570 \pm 0.000606 \)

+LAGEOS \( G M \) \( \Rightarrow G M_m = 992.8000 \pm 0.0003 \) \( \text{km}^3 \text{s}^{-2} \)
Combining these, 

\[ G M_{dm} = (0.0001 \pm 0.0016) \text{ km}^3 \text{ s}^{-2} \]

\[ = (0.3 \pm 4) \times 10^{-9} \, G M_\odot \]

\[ \rho \]

Dominant error comes from 

\[ G M_{\text{combined}} \text{ from lunar laser ranging} \]

\[ \therefore M_{dm} \leq 4 \times 10^{-9} M_\odot \]

If this bound were attained, and the mass were uniformly distributed below the moon's orbit, the density would be

\[ \rho \sim 6 \times 10^{10} \, 6eV/c^2 \text{ cm}^{-3} \]

Much larger than the limit on sun-bound dark matter \((\sim 10^5 \, 6eV/c^2 \text{ cm}^{-3})\) and even still larger than the galactic halo density — so there is only a weak constraint on earth-bound dark matter.
CAN THE FLYBY ANOMALY BE ATTRIBUTED TO EARTH-BOUND DARK MATTER?

Ando, D. et al.,
PRD 100, 091102 (2008)

**Incoming Asymptote**

**EARTH**

**Outgoing Asymptote**

**Outgoing Velocity Extrapolated from Incoming Velocity**

**Does not agree with measured value**

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<th>Parameter</th>
<th>Galileo I</th>
<th>Galileo IV</th>
<th>Near Earth</th>
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\[
\frac{\Delta V_{oo}}{V_{oo}} = \frac{1}{2} \frac{\Delta E}{E} = K (\cos \delta - \cos \delta_0)
\]
\( \delta_{\lambda 0} = \text{INCOMING, OUTGOING DECLINATION} \)
\( \lambda \quad \text{LATITUDE ANALOG IN} \)
\( \text{CELESTIAL COORDINATE} \)
\( \text{SYSTEM} \)

\[ K = \frac{2\omega R_E}{c} = 3.099 \times 10^{-6} \]

\( \omega = \text{EARTH ANGULAR ROTATION VELOCITY} \)
\( R_E = \text{EARTH RADIUS} = 6,371 \text{ km} \)

(AS WE SHALL SEE) ANALYSIS OF DRAG-FREE
CLOSER ORBITS DOES NOT SUGGEST THIS FORMULA,
WHICH SHOULD BE TREATED AS PURELY EMPIRICAL)

FOUR POSSIBILITIES:

\( \circ \text{ EFFECT IS AN ARTIFACT} - \text{SOME ESSENTIAL} \)
\( \text{PHYSICS HAS BEEN OMITTED FROM THE} \)
\( \text{ORBITAL CALCULATION} \)

\( \circ \text{ NEW ELECTROMAGNETIC PHYSICS} \)

\( \circ \text{ NEW GRAVITATIONAL PHYSICS} \) \( (\text{NON-MOND}) \)

\( \circ \text{ EFFECT COMES FROM COLLISIONS WITH} \)
\( \text{EARTH-BOUND DARK MATTER} \cdots \text{ANALYSE THIS} \)
ELASTIC AND INELASTIC DARK MATTER SCATTERING

Assume:

- Both initial particles nonrelativistic
  \[ |\vec{u}_1| \ll c \quad |\vec{u}_2| \ll c \]

- Center of mass scattering amplitude \( f(\theta) \) depends only on polar scattering angle \( \theta \)

Outgoing nucleon velocity change, averaged over scattering angles, is

\[
\langle \vec{v}_1 \rangle = \frac{m_2 \vec{u}_2 - m_1 \vec{u}_1}{m_1 + m_2} + \alpha \langle \cos \theta \rangle \frac{\vec{u}_1 - \vec{u}_2}{1 \vec{u}_1 - \vec{u}_2}
\]

With \( \alpha \) the positive square root of

\[
\alpha^2 = \frac{(m_2^2 - m_1^2)}{(m_1 + m_2)(m_1 + m_2)} \left( \vec{u}_1 - \vec{u}_2 \right)^2 + \Delta m \frac{m_2}{m_1(m_1 + m_2)} \left[ 2 \vec{c} - \frac{(m_1 \vec{u}_1 + m_2 \vec{u}_2)^2}{(m_1 + m_2)(m_1 + m_2)} \right]
\]

And with

\[
\langle \cos \theta \rangle = \frac{\int_0^{\pi} \sin \theta \cos \theta |f(\theta)|^2}{\int_0^{\pi} \sin \theta |f(\theta)|^2}
\]
ELASTIC SCATTERING CASE:
\[ \Delta m = 0 \quad m_2' = m_2 \quad \Rightarrow \]
\[ \langle \Delta \vec{v}_1 \rangle = -2 \frac{m_2}{m_1 + m_2} (\vec{u}_1 - \vec{u}_2) \left( \frac{\sin^2 \theta}{2} \right) \]

INELASTIC CASE: Assume \( \frac{\Delta m}{m_2} \geq \frac{m_1}{m_2} \) are 0

\( \Rightarrow \vec{v}_1 \) dominated by second term

\[ \langle \Delta \vec{v}_1 \rangle \approx \frac{\vec{u}_1 - \vec{u}_2}{1 \vec{u}_1 - \vec{u}_2} \left( \frac{2 \Delta m m_2'}{m_1 (m_1 + m_2)} \right)^{1/2} \cos \theta \]

For \( \vec{u}_1 \approx 10^4 \text{ km/s} = 10^6 \text{ cm/s} \), the velocity change in the inelastic case is larger than that in the elastic case by \( \frac{\vec{v}_1}{\vec{u}_1} \approx 10^5 \), and opposite in sign.

\[ \text{FORCE per UNIT MASS} = \delta F = \int d^3 u_2 \langle \Delta \vec{v}_1 \rangle |\vec{u}_1 - \vec{u}_2| \sigma (x_0, \vec{u}_2) \]

Velocity change in single scatter flux cross section dark matter spatial + velocity distribution
INTEGRATING \( \frac{W}{\text{UNIT MASS}} \) ALONG SPACECRAFT

TRAJECTORY \( \Rightarrow \)

\[
\delta \frac{1}{2} (\ddot{\mathbf{r}}_1 - \ddot{\mathbf{r}}_2) = \dot{\mathbf{v}}_1 \cdot \delta \dot{\mathbf{v}}_1 = \int_{\mathbf{x}_1}^{\mathbf{x}_2} \frac{d^2}{dt^2} \cdot \delta \mathbf{F}.
\]

\[
= \int_{\mathbf{x}_1}^{\mathbf{x}_2} \int d^3 u \frac{d^2}{dt^2} \cdot (\ddot{\mathbf{u}}_1 - \ddot{\mathbf{u}}_2) = \rho (\mathbf{x}, \ddot{\mathbf{u}}_2)
\]

IF \( \rho (\mathbf{x}, \ddot{\mathbf{u}}_2) = \rho (\mathbf{x}, -\ddot{\mathbf{u}}_2) \) THEN:

- **ELASTIC**: AVERAGED VELOCITY CHANGE OPPOSITE TO \( \frac{d^2 \mathbf{F}}{dt^2} \)
  \( \Rightarrow \) POSITIVE DRAG COEFFICIENT
  REDUCTION IN SPACECRAFT VELOCITY

- **INELASTIC**: AVERAGED VELOCITY CHANGE PARALLEL TO \( \frac{d^2 \mathbf{F}}{dt^2} \)
  \( \Rightarrow \) NEGATIVE DRAG COEFFICIENT
  INCREASE IN SPACECRAFT VELOCITY

TO GET NEGATIVE DRAG ON SOME TRAJECTORIES, POSITIVE ON OTHERS, NEED EITHER

- TWO-COMPONENT DARK MATTER, WITH DIFFERENT SPATIAL DENSITIES \( \rho (\mathbf{x}, \ddot{\mathbf{u}}_2) \) GOVERNING THE INELASTIC AND ELASTIC CASES

OR

- SINGLE COMPONENT WITH \( \rho (\mathbf{x}, \ddot{\mathbf{u}}_2) = \rho (\mathbf{x}, -\ddot{\mathbf{u}}_2) \)
• Quantitative Estimates: $10^6$ fractional velocity change over time interval $T$ needs

$$10^6 \sim T \frac{\Delta v}{v} \frac{c}{c} < \frac{c}{v} / \frac{c}{v}$$

$$\Rightarrow \Delta v \sim 10^6 \frac{v}{c} \frac{c}{c} \frac{c}{c} \frac{c}{c}$$

"near" flyby $T = 3.7 \times 10^5$

$$\frac{v}{c} \sim 10^6 \text{ cm s}^{-1}$$

Define $\overline{\rho}_m = \overline{\rho}_d = \text{dark matter mass density}$

\begin{align*}
\text{Elastics:} & \quad \overline{\rho}_m \sim 10^{-16} \text{ cm}^{-1} \left( m_1 + m_2 \right) \geq 10^{-12} \text{ GeV cm}^{-3} \\
\text{Inelastics:} & \quad \overline{\rho}_m \sim 10^{-20} \text{ cm}^{-1} \left[ m_1 \left( m_1 + m_2 \right) \right]^{1/2} \geq 10^{-20} \text{ GeV cm}^{-3}
\end{align*}

For $\sigma = 1 \text{ picobarn} = 10^{-24} \text{ cm}^2$, \text{elastic} $\overline{\rho}_m \sim 10^{16} \text{ GeV/cm}^3$

\text{inelastic} $\overline{\rho}_m \sim 10^{16} \text{ GeV/cm}^3$

For $\sigma = 1 \text{ millibarn} = 10^{-27} \text{ cm}^2$, \text{elastic} $\overline{\rho}_m \sim 10^{11} \text{ GeV/cm}^3$

\text{inelastic} $\overline{\rho}_m \sim 10^7 \text{ GeV/cm}^3$

All much greater than \text{galactic halo} $\overline{\rho}_m = 0.3 \text{ GeV/cm}^3$

Flyby velocity changes occur within radius 70,000 km for dark matter mass within this radius not to exceed $4 \times 10^4 \text{ M}_\odot$ need $\overline{\rho}_m \leq 10^{13} \text{ (GeV/cm}^3\text{)} \text{ cm}^{-3}$ requires $\sigma_{\text{inel}} > 10^{-33} \text{ cm}^2$

$\sigma_{\text{el}} > 10^{-29} \text{ cm}^2$
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DARK MATTER ACCUMULATION CASCADE?

SOLAR SYSTEM MOVES THROUGH GALAXY
WITH $v_{ss} = 220 \text{ km s}^{-1}$

LET $f_{ss} = \text{PROBABILITY OF CAPTURE OF A}
\text{DARK MATTER PARTICLE IN EARTH ORBIT}
\text{RADIUS } R \approx 1.5 \times 10^8 \text{ km}$

CAPTURE PARTICLES IN ANNULUS OF AREA $2\pi R dR$
OVER $T_{ss} \approx 1.5 \times 10^{17} \text{s}$, REDISTRIBUTION INTO
VOLUME $4\pi R^2 dR \Rightarrow$ AT RADIUS $R$

$$\frac{\rho_{\text{miss.}}}{\rho_{\text{ms. halo}}} \sim \frac{f_{ss} v_{ss} T_{ss}}{2\pi} \sim 10^{11} f_{ss}$$

KNOWN LIMIT ON $\rho_{\text{ms.}}$ IS $3 \times 10^5 \rho_{\text{ms. halo}}$

$\Rightarrow f_{ss} \lesssim 3 \times 10^{-6}$

ANALOGOUS CALCULATION FOR EARTH MOVING IN
SOLAR SYSTEM $\Rightarrow$

$$\frac{\rho_{\text{ms.}}}{\rho_{\text{ms. halo}}} \sim \frac{v_e v_e T_{ss}}{9R} \sim 2 \times 10^{13}$$

$70,000 \text{ km}$

$\Rightarrow$ EVEN WITH SMALL $v_e$, COULD GET DARK MATTER
DENSITIES LARGE ENOUGH TO EXPLAIN THE FLYBY ANOMALY,
IF $v_e$ IS LARGE ENOUGH
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**Constraints**

**Closed Orbit Constraints**

Most general form of drag force that gives zero cumulative drag for all closed satellite orbits

\[ \delta W = \int_{0}^{2\pi} \mathbf{D}(\mathbf{r}, \mathbf{v}) \cdot d\mathbf{r}^2 \]

\[ \int_{0}^{2\pi} \mathbf{D}(\mathbf{r}, \mathbf{v})(\mathbf{r}(0), \mathbf{v}(0)) = 0 \Rightarrow \]

\[ \mathbf{D}(\mathbf{r}, \mathbf{v}) = \sum_{k=1}^{\infty} \left( a_k \sin k\theta + b_k \cos k\theta \right) \]

\[ b_0 = 0 \]

For a hyperbolic flyby orbit with deflection angle \(2\theta_0\),

\[ \int_{-\pi}^{\pi} (\mathbf{v}_f - \mathbf{v}_i) = 2 b_0 \theta_0 + 2 \sum_{k=1}^{\infty} \frac{b_k}{k} \sin k\theta_0 \]

Details of near-earth environment appear through the \(b_k\).

Kinematics of vanishing drag anomaly for closed orbits does not give the Anderson et al. fitting formula. There may be drag anomaly in satellite orbits.
QUESTION: IF ONE FITS ALL SATELLITES TO
DRAG = D_1 \times AREA + D_2 \times MASS,

○ IS THERE EVIDENCE FOR D_1?
○ IF NOT, WHAT BOUNDS CAN ONE PLACE ON D_2?

ASSUMING NOW NO FINE-TUNING TO CANCEL b_0,
THE RATE AT WHICH THE RADIUS OF AN ORBITING
BODY INCREASES OR DECREASES CAN BE USED TO
BOUND A DRAG FORCE ACTING ON IT. A BOUND
ON \vec{F}_m ACTING ON ORBIT. WHEN OPTICAL
DEPTH IS \ll RADIUS OF ORBITING BODY, \vec{F}_m
DROPS OUT AND WE GET A BOUND ON \vec{F}_m

\text{Earth} \quad a \approx 1.5 \times 10^6 \text{ km ORBIT RADIUS}
\quad d_q \leq 1.5 \text{ cm / ORBIT}
\Rightarrow \vec{F}_m \quad \text{S.R.} < 2 \times 10^5 \text{ (GeV/}c^2\text{)} \text{ cm}^{-2}
\text{BASED ON INELASTIC DARK MATTER SCATTERING}
\Rightarrow \text{EARTH CAPTURE FRACTION IN CASCADE}
\text{SCENARIO MUST OBEY}
\quad \frac{f_e}{\sigma} \geq \frac{0.2 \times 10^{-35}}{6} \text{ cm}^2
\quad \sigma = 10^{-33} \text{ cm}^2 \Rightarrow f_e \geq 0.2 \times 10^{-2} \quad \sigma = 10^{-27} \text{ cm}^2 \Rightarrow f_e \geq 0.2 \times 10^{-9}
MOON
\[ A_m \sim 389,000 \ \text{km} \]
\[ \Delta A_m \lesssim 0.28 \ \text{cm} / \text{orbit} \]

\[ \Rightarrow \ \rho_{\text{m.c.}} \lesssim 10^4 \ \text{(GeV/c}^2\text{)} \ \text{cm}^{-3} \]

\[ \Rightarrow \ \text{DARK MATTER DENSITY AT MOON'S ORBIT} \]
\[ \text{MUST BE } \ll \text{DENSITY WITHIN } 70,000 \ \text{km} \]

LAGEOS

BOUNDS ON RESIDUAL ACCELERATIONS

\[ \Rightarrow \ \rho_{\text{m.c.}} \lesssim 3 \times 10^{-26} \ \text{(GeV/c}^2\text{)} \ \text{cm}^{-1} \ (\text{inclined}) \]

\[ \Rightarrow \ \text{DARK MATTER DENSITY AT LAGEOS ORBIT} \]
\[ \text{RADIUS MUST BE } \ll \text{DENSITY AT RADIUS RELEVANT FOR FLYBY ANOMALY} \]

** STELLAR (AND SOLAR) DYNAMICS CONSTRAINTS**

EFFECT OF DARK MATTER CAPTURE ON STELLAR DYNAMICS DISCUSSED BY FAIRBAIRN, SCOTT + EOSJØ

\[ \Rightarrow \ \rho_{\text{m.s.s.}} \lesssim 10^{-33} \ \text{cm}^{-3} \ (5 \text{ to } 50) \ \text{GeV/c}^2 \ \text{cm}^{-3} \]

FOR SELF-ANNIHILATING DARK MATTER

FOR NON-SELF-ANNIHILATING DARK MATTER, THIS RESTRICTION CAN BE WEAKENED BY FACTOR \( \sim 10^5 \), WHEN DARK MATTER
SECONDARY ESCAPES FROM SUN

EARTH AND SATELLITE HEATING CONSTRAINTS

EARTH HEATING - IF DARK MATTER SECONDARY ESCAPES FROM EARTH, ONLY KINETIC ENERGY OF RECOILING NUCLEON IS DEPOSITED

\[ \Delta T \sim m_1 \left( \frac{v}{v_1} \right)^2 \]

\[ \frac{\Delta T}{\Delta m c^2} \sim \frac{m_2}{2m_1} \quad \text{DEPENDS ON DARK MATTER MASS } m_2 \]

FOR \( m_2 \sim 10 \text{ keV} \), GET BOUND

\[ \rho_{m_2} R @ \lesssim 10^9 \text{ (GeV/cm}^3 \text{) cm}^{-3} \]

AGAIN, IMPLIES THAT DARK MATTER DENSITY MUST BE DEPLETED NEAR EARTH (SIMILAR TO CONCLUSION FROM LAGGEUS CONSTRAINT)
FLYBY TEMPERATURE GAIN

\[ \text{Temp Gain} \sim \frac{T_i}{1 (\text{GeV})} \times 10^{-6} (\tilde{u}, 1 \sim \frac{1}{2} m_i \tilde{u}, 1) \times 10^{-6} (\tilde{u}, 1) \]

INELASTIC: \[ \text{Temp Gain} \sim \frac{1}{2} \times 10^{-6} m_2 (\tilde{u}, 1) c \]
\[ \sim 0.2 \text{ K} \left( \frac{m_2 c^2}{\text{MeV}} \right) \]

ELASTIC: \[ \text{Temp Gain} \sim \frac{1}{2} \times 10^{-6} m_2 (\tilde{u}, 1) (\tilde{u}, 1 - \tilde{u}, 1) \]
\[ \sim 10^{-5} \text{ K} \left( \frac{m_2 c^2}{\text{MeV}} \right) \]

\( \Rightarrow \) DARK MATTER MASS \( \leq 6 \text{ GeV} \)

COULD CALORIMETRY IN HIGH ORBITING SPACECRAFT BE USED FOR DARK MATTER DETECTION?

FLYBY STRUCTURAL DISRUPTION

IF EACH INDIVIDUAL NUCLEON RECOIL SHOULD NOT PRODUCE STRUCTURAL CHANGE, NEED

\( \left\{ T_i \leq E_{\text{binding}} \right\} \)

INELASTIC: \( m_2 c^2 < \left( m_1 c^2 E_{\text{binding}} \right)^{1/2} \sim 1 \text{ GeV} \)

FOR \( E_{\text{binding}} \sim 10 \text{ eV} \)
SUMMARY ON FLYBY - ESTIMATES DO NOT RULE OUT DARK MATTER EXPLANATION (FOR EXAMPLE, DO NOT REQUIRE $f_0 > 1$)
- BUT CONSTRAINTS ARE SEVERE

- NEED EXOTHERMIC INELASTIC SCATTERING OF DARK MATTER ON ORDINARY MATTER
- DARK MATTER MUST BE WELL WITHIN MOON'S ORBIT AND DEPLETED NEAR EARTH'S SURFACE
- CASCADING ACCUMULATION MECHANISM REQUIRED TO REACH NEEDED DARK MATTER DENSITY
- DARK MATTER MASS MUST BE WELL BELOW A GeV
- interacting cross section with nucleons must be relatively high
  \( 10^{-33} \text{ cm}^2 < \sigma < 10^{-27} \text{ cm}^2 \)
- DARK MATTER MUST BE NON-SELF-ANNIHILATING
PLANET-BOUND DARK MATTER, AND
THE INTERNAL HEAT OF URANUS, NEPTUNE,
AND HOT-JUPITER EXOPLANETS

LET \( f = \text{fraction of dark matter annihilation energy that is deposited in a planet when a dark matter particle is accreted} \)

\( f \) CAN BE \( \ll 1 \) FOR EXAMPLE, IF THE SECONDARY \( m_2 \) IS VERY WEAKLY INTERACTING AND ESCAPES, \( f \approx \frac{1}{2} \frac{m_2}{m_1} \)

IF \( m_2 \ll m_1 = \text{nucleon mass} \), THEN \( f \ll 1 \)

CONSIDER A PLANET WITH OUTWARD ENERGY FLOW PER UNIT AREA OF SURFACE \( \equiv H \)

ASSUME IT IS IMMERSED IN A DARK MATTER CLOUD, WITH MASS DENSITY \( \rho_m \) AND MEAN VELOCITY \( v \sim \left( \frac{6 \, M_{\text{planet}}}{R_{\text{cloud}}} \right)^{1/2} \)
NEAR PLANET'S SURFACE
INCLUDING A SOLID ANGLE FACTOR OF $\frac{1}{2}$, THE CONDITION FOR ALL OF $N$ TO BE SUPPLIED BY DARK MATTER CAPTURE IS

$$\frac{1}{2} \rho m c^2 v f = N$$

$$\Rightarrow$$ DARK MATTER DENSITY AT ENERGY FLUX EQUILIBRIUM IS

$$\rho_m = \frac{K_{\text{planet}}}{f}$$

$$K_{\text{planet}} = \frac{2N}{c^3 V} \sim \frac{2N}{c^3} \left( \frac{R_{\text{planet}}}{GM_{\text{planet}}} \right)^{1/2}$$

FROM PLANETARY NEUTRINO FLOW DATA (de Lavaud & Lieuauve) GET

$$K_{\text{Earth}} = 0.12 \text{ GeV/}c^2 \text{ cm}^{-3}$$

$$K_{\text{Jupiter}} = 1.6 \text{ GeV/}c^2 \text{ cm}^{-3}$$

$$K_{\text{Saturn}} = 1.0 \text{ GeV/}c^2 \text{ cm}^{-3}$$

$$K_{\text{Uranus}} < 0.04 \text{ GeV/}c^2 \text{ cm}^{-3}$$

$$K_{\text{Neptune}} = 0.3 \text{ GeV/}c^2 \text{ cm}^{-3}$$
For Eq(1), \( M \) for equilibrium \( U \) in the possible range for sun-bound or planet-bound dark matter.

Thus, a substantial fraction of planetary internal heat generation could come from dark matter accretion; could account for unexplained residual heat production in Earth, jovian planets, and "not-Jupiter" exoplanets.

Uranus anomalies: rotation axis tilted 98° with respect to plane of solar system, and very low heat production \( H \) - much less than Neptune.

If heat production is largely associated with a planet-bound dark matter cloud, then the collision thought to have tilted the axis of Uranus could also have knocked it out of its associated dark matter cloud, leaving Uranus with a much reduced internal heat production.