SU(8) FAMILY UNIFICATION WITH

BOSON - FERMION BALANCE


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MOTIVATION:

SM HAS BOSONS AND FERMIONS \rightarrow SUPERSYMMETRY?

BUT:

• NO DEFINITIVE MECHANISM FOR SUSY BREAKING
• SO FAR, SUSY NOT FOUND

TRY WEAKE PRINCIPLE THAN EXACT SUSY:

BOSON - FERMION BALANCE

THAT IS, EQUAL NUMBERS OF MASSLESS BOSON
AND FERMION HELICITY STATES
Starting point: $SO(8)$, $N=8$ Supergravity

<table>
<thead>
<tr>
<th>Particle</th>
<th>Helicities</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Graviton</td>
<td>2</td>
</tr>
<tr>
<td>8 Gravitinos</td>
<td>16</td>
</tr>
<tr>
<td>28 Vectors</td>
<td>128</td>
</tr>
<tr>
<td>56 Majorana Fermions</td>
<td>70</td>
</tr>
<tr>
<td>70 Scalars</td>
<td></td>
</tr>
</tbody>
</table>

$28 = \text{adjoint of } SO(8)$ too small to contain SM

Redistribute 70 scalar helicities to vectors

$\frac{70}{2} = 35$

$35 + 28 = 63$ can be adjoint of $SO(8)$ big enough!

<table>
<thead>
<tr>
<th>Spin</th>
<th>Weyl $\frac{3}{2}$</th>
<th>Weyl $1/2$</th>
<th>Complex</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3/2</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

"$SO(8)$ Graviton Multiplet"

1 $8_L$ 63 $56_L$ $\leftarrow SU(8) \text{ reps}$

"$SU(8)$ Matter Multiplet"

$SU(8) \text{ reps} \rightarrow \frac{18_L}{18_L}$ 56
SU(18) ANOMALY CANCELLATION

CHIRAL ANOMALY OF SPIN 3/2

= 3 TIMES CHIRAL ANOMALY OF SPIN 1/2

3 x ANOMALY (8L) = 3 x 1 = 3

2 x ANOMALY (\bar{8}L) = 2 x (-4) = -8

ANOMALY (56L) = 5

TOTAL ANOMALY = 3 - 8 + 5 = 0

[THIS COUNTING FIRST NOTED BY MARCUS (1985)]

[SPIN 3/2 ANOMALY DUFF (1982)
NIELSEN + RÖMER (1985)]

SO THE MODEL IS VIABLE
Will now show:

- Model contains fields of SM
- Model has a symmetry breaking path to SM

\[ \text{SU}(8) \rightarrow \text{SU}(3)_{\text{family}} \times \text{SU}(5) \times U(1)/Z_5 \]

"Flipped" SU(5) \[\rightarrow\] SM \[\rightarrow\] Electromagnetic

From 56 scalars

From dynamical symmetry breaking

Under \[\text{SU}(8) \rightarrow \text{SU}(3) \times \text{SU}(5) \times U(1)\]

\[56 \rightarrow (1,1) \ (-15) + (1,10) \ (9) + (3,5) \ (-7) + (3,10) \ (1)\]

\[U(1) \text{ generator } \neq 0\]
5

**MODULAR GROUND STATE**

\[ |\Omega\rangle = \text{GROUND STATE \underline{BEFORE DYNAMICAL SYMMETRY BREAKING}} \]

NEED \[ \langle \Omega | (1,1)(-15) | \Omega \rangle \neq 0 \]

\[ \Rightarrow |\Omega\rangle \text{ MUST BE A SUPERPOSITION OF U(1) EIGENSTATES DIFFERING BY IS UNITS} \]

**MODULAR GROUND STATE MODULO DIVISOR OF IS**

**ASSUME MOD 5 IS NATURE'S CHOICE**

\[ |\Omega\rangle = \sum_{h=-\infty}^{\infty} e^{i h \omega} |15n\rangle = |10\rangle + e^{i \omega} |15\rangle + e^{2i \omega} |10\rangle \]

\[ + e^{3i \omega} |15\rangle + e^{4i \omega} |10\rangle + \ldots \]

\[ \{3\} = \text{MODULO 5 EQUIVALENT TO ( )} \]

5C \[ \rightarrow (1,1)(-15) \{0\} + (1,10)(9) \{-4\} + (3,5)(-7) \{-2\} + (3,10)(1) \{1\} \]

**WILL SHOW MODULO 5 REPRESENTATIVES THAT CORRESPONDS TO PATH TO SM:** **ASSUME UNIQUE MOD 5 REPRESENTATIVE IS PICKED BY DYNAMICAL SYMMETRY BREAKING**
<table>
<thead>
<tr>
<th>Field</th>
<th>Spin</th>
<th>SU(8) Rep</th>
<th>Helicities</th>
<th>Branching → SU(3) × SU(5) × U(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{μν}$</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>$^{1}_{1}$ (3,1)(-5) {0} + (1,5)(3) {-23}</td>
</tr>
</tbody>
</table>
| $\lambda_{μ}^α$ | WEYL \(\frac{3}{2}\) | $8_\iota$ | 16         | $(3,1)(0) \{0\} + (8,1)(0) \{0\} + (1,24)(0) \{0\}$  
|           |      |           |            | $ + (3,\overline{5})(-6) \{2\} + (\overline{3},5)(0) \{-23\}$ |
| $A_{μ}^A$ | 1    | 63        | 126        | $(3,1)(-15) \{0\} + (1,\overline{10})(9) \{13\} + (\overline{3},5)(-7) \{23\} + (3,10)(1) \{1\}$ |
| $V^{[A,B]}_\nu$ | WEYL \(\frac{1}{2}\) | $56_\iota$ | 112        | $(3,1)(-15) \{0\} + (1,\overline{10})(9) \{13\} + (\overline{3},5)(-7) \{23\} + (3,10)(1) \{1\}$ |
| $\lambda_1[αβ]$ | WEYL \(\frac{1}{2}\) | $\overline{56}_\iota$ | 56        | $(3,1)(10) \{5\} + (1,\overline{10})(-6) \{-13\} + (\overline{3},\overline{5})(2) \{-3\}$ |
| $\lambda_2[αβ]$ | WEYL \(\frac{1}{2}\) | $\overline{56}_\iota$ | 56        | $(3,1)(10) \{5\} + (1,\overline{10})(-6) \{-13\} + (\overline{3},\overline{5})(2) \{-3\}$ |
| $ϕ^{[αβ]}_β$ | COMPLEX 0 | 56        | 112        | $(3,1)(-15) \{0\} + (1,\overline{10})(9) \{-13\} + (3,10)(1) \{1\}$  
|           |      |           |            | $ + (\overline{3},\overline{5})(-7) \{-23\}$ |

**States with flipped SU(5) representations**
Symmetry breaking: $SU(6) \rightarrow SU(3) \times SU(5) \times U(1)$

**Benhar using scalar $\phi^{(x, y, z)}$**

\[ A_\mu^{(1, 1)}(0) \{0\} + (8, 1) (0) \{0\} + (1, 24) (0) \{0\} \]

\[ U(1) \uparrow \quad SU(3) \quad SU(5) \]

\[ + (3, 5) (-8) \{2\} + (\bar{3}, 5) (0) \{\bar{2}\} \]

Absorb 30 real components of $\phi^{(x, y, z)}$ to become massive.

\[ \phi^{(x, y, z)} \]

\[ (1, 1)(-15) \{0\} + (1, 10)(9) \{-1\} + (3, 10)(1) \{1\} \]

6 gets vacuum expectation. Residual scalars.

\[ + (\bar{3}, 5)(-7) \{-2\} \]

30 real components that are absorbed by vector $(\bar{3}, 5)(8) \{-2\}$ and conjugate.

**Note:** $8 \equiv -7 \mod(15)$
"STANDARD" SU(5)

GEORGI + GLASNOW
PRL 32, 438 (1974)

SU(5) → SM

Q = T^3_L + Y < (LANGacker Definition)

Y = \frac{1}{6} SLANSKY U(1)_SM GENERATOR

PHYS. REP. 79, 1 (1981)

SU(5) REPS:

- 10
- \bar{5}

"FLIPPED" SU(5)

BARR
PLB 112, 219 (1982)

DERENDINGER, KIM, NANOPOULOS
PLB 139, 170 (1984)

ANTONIOU, ELLIS, HAGELIN, NANOPOULOS
PLB 194, 231 (1987)

SU(5) x U(1)_X → SM

Q = T^3_L + Y

Y = \frac{1}{5} (X - \bar{7})

X = \frac{1}{6} SLANSKY U(1)_SM GENERATOR

SU(5) REPS:

- 10
- \bar{5}
- 1
- 1
- 5

X VALUES:

- 1
- -3
- 5

DESCENT FROM SO(10) 16 REPS.

⇒ ALL SU(5) x U(1)_X ANOMALIES CANCEL

ALSO, ALL SU(2)_{FAMILY} x SU(5) x U(1)_X " " " 
<table>
<thead>
<tr>
<th>((SU(2)<em>L, SU(3)</em>{color})_\gamma)</th>
<th>NAME</th>
<th>STD. SUL5</th>
<th>FLIPPED SUL5</th>
</tr>
</thead>
<tbody>
<tr>
<td>((1, \bar{3})_{-2/3})</td>
<td>(u_L^c)</td>
<td>10</td>
<td>5</td>
</tr>
<tr>
<td>((2, 1)_{-1/3})</td>
<td>((e_L, \nu_L) = \ell_L)</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>((1, 1)_{0})</td>
<td>(N_L)</td>
<td>ABSENT</td>
<td>10</td>
</tr>
<tr>
<td>((1, \bar{3})_{1/3})</td>
<td>(d_L^c)</td>
<td>5</td>
<td>10</td>
</tr>
<tr>
<td>((2, 3)_{1/6})</td>
<td>((u_L, \nu_L) = q_L)</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>((1, 1)_{1})</td>
<td>(e_L^c)</td>
<td>10</td>
<td>1</td>
</tr>
</tbody>
</table>
SYMMETRY BREAKING TO SM

$SU(5) \rightarrow SU(2) \times SU(3) \times U(1)$

$\overline{5} = (2,1) (-3) + (1, \overline{3}) (2)$

$10 = (1,1) (6) + (1, \overline{3}) (-4) + (2,3) (1)$

$24 = (1,1) (0) + (3,1) (0) + (1, \overline{3}) (0) + (2,3) (-5) + (2, \overline{3}) (5)$

**STANDARD S U (5): USES (1,1)(0) OF REAL SCALAR 24**

**FLIPPED S U (5): USES (1,1)(6) OF COMPLEX SCALAR 10**

$U(1)$ GENERATOR TO MODULAR GROUND STATE MOD (6)

IN 24 VECTOR, $(2,3)(-5) + (2, \overline{3})(5)$ ABSORB 12 REAL COMPONENTS OF COMPLEX 10 TO BECOME MASSIVE

THESE COME FROM $(2,3)(1)$ AND CONJUGATE

$(2,3)(-5) \equiv -5 \ mod \ (6)$
Symmetry Breaking to SM - Continued

\[ SU(5) \rightarrow SU(3)^{\text{family}} \times U(1)^{\text{family}} \times SU(5) \]

\[ \uparrow \quad U(1) \times \]

Use scalar \( S_6 \)

\[ \text{Flipped } SU(5) \]

Residual \( (1,10) \{-1\} + (3,10) \{1\} \)

Correct assignments to break flipped \( SU(5) \rightarrow SM \)

\[ 10 \supset (1,1)(6) \quad \begin{cases} \tau = \frac{1}{6} (6) = 1 \\ x = 1 \\ y = \frac{1}{5} (x-\tau) = 0 \\ T^3_L = 0 \ (\text{singlet}) \Rightarrow Q = 0 \end{cases} \]

\( (3,10) \) can also break family symmetry \( SU(3) \rightarrow SU(2) \times U(1) \)

After \( SU(5) \) breaking \( 10 \) decomposes into three components \( 10_A, 10_B, 10_C \)

\( (3,10_{A,B,C}) \) can completely break family symmetry \( SU(3) \)
**Fermion State Summary**

(Su3 family, Su(5))

\[ \chi \quad (3,1) \{5\} + (1, \bar{10}) \{-1\} + (\overline{3},5) \{3 \equiv -2\} + (3,10) \{1\} \]

\[ \lambda_1 \quad (3,1) \{5\} + (1,10) \{-1\} + (\overline{3},5) \{-3 \equiv 2\} \]

\[ \lambda_2 \quad (3,1) \{5\} + (1,10) \{-1\} + (\overline{3},5) \{-3\} \]

---

3 sets with flipped Su(5) assignments

3 vector-like sets

Ax + Ay + Ax + X: 1307.5770

\[ \text{rotate away proton delay?} \]

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DARK MATTER CANDIDATE?

**Possible Condensates:**

- **After family breaking**, three \((1,10) \{-1\}\) with \((3,10) \{1\}\)

- \(3 \times 3 \Rightarrow 1\) pairings

\[ \chi (\overline{3},5) \{3\} \quad \lambda (3,1) \{5\} \Rightarrow (1,5) \{3 \equiv -2\} \]

\[ \chi (3,10) \{1\} \quad \lambda (\overline{3},5) \{-3\} \Rightarrow (1,5) \{-3 \equiv 2\} \]

\[ \chi_1 (3,1) \{5\} \quad \lambda_2 (\overline{3},5) \{-3\} \Rightarrow (1,5) \{2\} \]

\[ \{ \text{Mechanism to get SM Higgs (more to say on this)} \} \]
ASYMPTOTIC FREEDOM AND GLOBAL SYMMETRIES

\( \text{SU}(8) \) is asymptotically free: \( C(s) = \text{index of } \mathfrak{so}(8) \text{ rep. of spin } s \)

\[
\frac{1}{3} \left[ 11C(1) - 26C(\text{Weyl } \frac{3}{2}) - 2C(\text{Weyl } \frac{1}{2}) - C(\text{complex} 0) \right]
\]

\[= \frac{1}{3} \left[ 11 \times 16 - 26 \times 1 - 2 \times (15 + 2 \times 6) - 15 \right] = 27 > 0
\]

GLOBAL CHIRAL SYMMETRIES

- **Overall** U(1) rephasing of all fermions - broken by usual instanton + anomaly mechanism

- **Relative** U(1) rephasing of \( \Sigma_{6L} \chi \)

- **Relative** U(2) rephasing and mixing of two \( \overline{28}_L \chi_{12} \)
Dynamical Symmetry Breaking: General Formalism

- From left chiral spinors, can form two types of scalars

\[ \bar{\Psi}_{L_1} \Psi_{L_2} = 0 \]
\[ \bar{\Psi}_{L_1} \Psi_{L_2} = \bar{\Psi}_{L_1}^T \gamma^0 \Psi_{L_2} \]
\[ = \bar{\Psi}_{L_2}^T \gamma^0 \Psi_{L_1} \] \( \neq 0 \)

Allowed form of condensate

Group rep. content of condensate is direct product of rep. content of \( \Psi_1 \) and \( \Psi_2 \)

- Vector gluon potential for \( A+B \rightarrow A+B \)

\[ V = \frac{g^2}{2\pi} K(A+B; A, B) \]

\[ K(A+B; A, B) = C_2(A+B) - C_2(A) - C_2(B) \]

\( C_2 = \) Casimir \( \geq 0 \)

\( C_2(1) = 0 \)

Special cases:

\[ K(A; A, A) = C_2(A) - C_2(A) - C_2(A) = -2 C_2(A) < 0 \]

\[ K(A; A, A) = C_2(A) - C_2(A) - C_2(B) = - C_2(B) < 0 \]

\[ K(A; A, 1) = C_2(A) - C_2(A) - C_2(1) = 0 \]

Both attractive
Dynamical Symmetry Breaking to Get SM Higgs

\[ SU(3): \quad 3 \times \overline{3} = 1 + 8 \quad \text{--- most attractive} \]
\[ SU(5): \quad 10 \times \overline{5} = 5 + 45 \]
\[ 1 \times 5 = 5, \quad 1 \times \overline{5} = \overline{5} \]

All family \( 3 \times \overline{3} \rightarrow 21 \) pairings:

\[
\begin{align*}
\nu \ (\overline{3}, 5) & \to (1, 5) \ (3 \equiv -2) \\
\lambda_1 \ (3, 10) & \to (1, 5) \ (-2) \\
\lambda_2 \ (\overline{3}, 5) & \to (1, 5) \ (2) \\
\end{align*}
\]

So get condensate \((1, 5) \ (-2)\) and its conjugate \((1, 5) \ (2)\).

**Global \( U(1/2) \) Chiral symmetry of \( \lambda_1 \) broken**

By condensate ⇒ get a Goldstone(s) with representation \((1, 5) \ (-2)\)

Thus is what is needed in flipped \( SU(5) \) to break SM electroweak

__Elimination of Higgs triplet (Antoniadis et al.)__

\[ 10 \{13\} \times 10 \{1\} \times 5 \ (-23) \subset (2,3)^{\frac{1}{6}} \times (2,3)^{\frac{1}{6}} \times (1,3)^{-\frac{1}{2}} \subset (1,1)_0 \]
Condensates to give spin $3/2$ a mass

\[ \nu_\mu^x + \psi \cdot (-5) \]

\[ X = (3,1) \quad (10) \]

\[ (3,5) \quad (-7) \rightarrow (2,5) \quad (-7) + (1,5) \quad (-7) \]

Need mod 10

Common divisor of 15 and 10 is 5

So again a special role for mod 5

These condensates (and others) break the global $U(2)$
ABSENCE OF SCALAR-FERMION YUKAWA COUPLINGS

CHIRALITY ⇒ YUKAWA COUPLINGS OF SPIN $1/2$ FERMIONS HAVE GENERAL FORM

\[ \Phi_{L1}^\dagger \chi_0 \Phi_{L2} \Phi \]
\[ \Phi_{L1,2} \text{ ANY OF } \chi, \lambda \]
\[ \Phi \text{ EITHER } \phi \text{ OR } \phi^* \]

NO WAY TO CONTRACT $SU(8)$ INDICES TO FORM A SINGLET:

$\chi \chi$ 6 UPPER INDICES

$\chi \lambda$ 4 LOWER INDICES

$\chi \lambda$ 3 UPPER, TWO LOWER INDICES

$\phi$ THREE UPPER INDICES

$\phi^*$ THREE LOWER INDICES

NO YUKAWA COUPLINGS INVOLVING SPIN $1/2$ FERMIONS MUST BE GENERATED BY RADIATIVE CORRECTIONS

SO AFTER $SU(8)$ BREAKING, SPIN $1/2$ FERMIONS REMAIN MASSLESS
Supersymmetries in limit of zero gauge coupling g

Two conserved rep. 8 supercurrents
\[ J_\mu^a = \gamma^\nu (\partial_\nu \phi^{[a \beta \gamma]} \gamma_\mu \chi_{[\beta \gamma]} ) \quad a = 1, 2 \]

One conserved singlet supercurrent
\[ J^\mu = \gamma^\nu (\partial_\nu \phi^{* \alpha \beta \gamma}] \gamma_\mu \chi_{[\alpha \beta \gamma]} ) \]

\[ \partial_\mu J_\mu^a = 0 \quad \partial_\mu J^\mu = 0 \]

Not invariances of scalar sector self-couplings
\[ S_{\text{self}} (\phi) = \phi^{x \alpha \beta} \phi^{x \alpha \beta} \left( g_1 \phi^{x \alpha \beta} \phi^{x \alpha \beta} + g_2 \phi^{x \alpha \beta} \phi^{x \alpha \beta} \right) \]

\[ \Rightarrow g_{1,2} \text{ order } g^2 \text{ or higher} \]
**DISCUSSION**

**THREE NOVEL INGREDIENTS**

- **Boson-Fermion Balance without Full SUSY**
- **Canceling Anomalies Between Spin $\frac{1}{2}$ and Spin $\frac{3}{2}$**
- **Breaking Gauge Symmetry with Scalar Representation with Nonzero U(1) Generator: Modular Ground State**
  
  \[ \text{Mod } 5, \text{ not Mod } 1 \text{ or Mod } 15 \]

**PROMISING FEATURES**

- **Three Families**
- **Route to Flipped SU(5)**
- **SU(8) \rightarrow SU(3)_{\text{family}} \times SU(5) \times U(1)/\mathbb{Z}_5 \rightarrow SM** **Using Scalar 56**
  
  \[ SM \rightarrow \text{Electromagnetic U(1)} \text{ Using Higgs Generated by Dynamical Symmetry Breaking from Most Attractive Channels} \]

- **Vanishing Bare Yukawa Couplings**
  
  **Zero Gauge Coupling SUSYs**
EXPERIMENTAL SIGNATURES

- **STERILE NEUTRINOS** (*generic*)

- **Higgs**, if $O(2)$ chiral symmetry broken first by condensates giving spin $3/2$ a mass

**OPEN ISSUES**
- **Is $O(12)$ massless?** (*needed? to get massless $U(1)$*)
- **Choice of $mod(s)$ representative — discrete analog of vacuum alignment condition?**

**DYNAMICAL ISSUES**
- **Running couplings**
  - Extra fermion states beyond usual families
  - Extra families
  - Proton decay
  - Masses and mixings of SM
  - CP violation
  - Flavor changing neutral currents
  - Monopoles

**FURTHER UNIFICATION**
- "$SU(8)$ gravity multiplet" 128 boson and fermion helicities
- "$SU(8)$ matter multiplet" 112 """
- Match numbers of half-integer, integer roots of $E(8)$
- Is there a connection to the $E(8)$ root lattice?
APPENDIX: MODULAR GROUND STATE

(AN XV: 1409.1180)

FOR SU(8) BROKEN BY RANK THREE ANTISYMMETRIC TENSOR

\[ N^A_m = (1,1) (0) + (8,1) (0) + (1, 24) (0) + (3, 3) (-8) + (5, 5) (8) \]

\[ \varphi^{(15)} = (1,1) (-15) + (1, 10) (9) + (3, 10) (1) + (5, 5) (-7) \]

\( g \) ARE UNI GENERATOR VALUES DEFINED BY

\[ [G, (m,n)] = g (m,n) \]

\[ G = \text{Diag} (-5, -5, -5, 3, 3, 3, 3, 3) \]

\( \Xi \equiv (1,1) \) OF \( \varphi \) HAS \( g = -15 \) \( \Rightarrow \) GROUND STATE AND

STATE BASIS SHOULD HAVE MODULO 15 PERIODICITY

IN UNI GENERATOR \( g \), WHERE \( 0 \leq 15 \)
PROOF

\[ \langle G, \bar{\Phi} \rangle = -15 \bar{\Phi} \]

\[ \Rightarrow 0 = \langle g' \mid \langle G, \bar{\Phi} \rangle \mid g' \rangle = -15 \langle g' \mid \bar{\Phi} \mid g' \rangle \]

To get symmetry breaking, ground state must be superposition of at least two U(1) generator values differing by 15.

Clustering \( \Rightarrow \) need an infinite periodic sum, like QCD \( \theta \) vacuum.

\[ |0_2, \omega \rangle_p = \sum_{n=-\infty}^{\infty} (\frac{\rho}{2\pi})^{1/2} e^{i\rho_n \omega \cdot 1\rho_n} \]

State basis is, for \( k = 0, \ldots, p-1 \)

\[ |k_1, \omega \rangle_p = \sum_{n=-\infty}^{\infty} (\frac{\rho}{2\pi})^{1/2} e^{i(k+p_n)\omega \cdot 1(k+p_n)} \]

\[ G \{ k_1, \omega \}^p = -i \frac{2}{\Delta \omega} |k_1, \omega \rangle_p \]

\( G \text{ generates rotations} \)

IN \( \omega: \bar{\Phi} = 1\bar{\Phi} e^{-15i\omega} \)
**Properties of State Basis**

**Modulo $p$ Periodicity**

\[ |k + ps, w\rangle_p = |k, w\rangle_p \quad \text{any integer } s \]

**Clustering**

\[ |k a + k b, w\rangle_p \propto |k a, w\rangle_p |k b, w\rangle_p \quad \text{widely separated } a, b \]

**Angular Periodicity**

\[ |k, w + 2\pi m/p\rangle_p = e^{2\pi i m k/p} |k, w\rangle_p \]

Higgs potential minimized over orbifold with conical singularity at origin

\[ 0 \leq |\xi| < \infty \]

\[ 0 \leq \omega \leq 2\pi/p \]
SPECIAL CASES

\( p = 1 \quad 0 \leq \omega \leq \pi \quad U(1) \text{ completely broken} \)

\( \left( \frac{U(1)}{\mathbb{Z}_1} \Rightarrow \text{no conserved U(1) charge} \right) \)

\( \text{Classical Higgs calculation} \)

\( p = 15 \quad \text{residual} \quad \frac{U(1)}{\mathbb{Z}_{15}} \)

Related to Abelian model of Krauss & Wilczek

\( \left( \gamma \text{ charge} \quad \phi \text{ charge} \right) \)

\( \chi \text{ condenses} \Rightarrow \text{residual} \quad \frac{U(1)}{\mathbb{Z}_5} \)

\( p = 5 \quad \text{residual} \quad \frac{U(1)}{\mathbb{Z}_5} \quad \text{new possibilities not in literature} \)

\( p = 3 \quad \frac{U(1)}{\mathbb{Z}_3} \quad \text{mod 5 needed for SU(8) family model} \)
UL1) GENERATOR MASS

FOR POTENTIAL MINIMUM AT $|\Xi| = 0$, UL1) GAUGE BOSON GETS MASS PROPORTIONAL TO

$$[G, \Xi]^\dagger [G, \Xi] = (15)^2 \rho^2$$

WHEN $G$ ACTS AS $-15$

BANKS [NUCL. PHYS. B 323, 90 (1989)] POINTS OUT THAT
FOR $\nu \geq 5$, TERMS IN LOW ENERGY EFFECTIVE ACTION
WITH UL1)/$\mathbb{Z}_5$ SYMMETRY ARE "IRRELEVANT": MASS POWER SUPPRESSED

⇒ LOW ENERGY ACTION HAS FULL UL1) GLOBAL SYMMETRY
⇒ $G$ ACTS AS $0$
SUGGESTS THAT EFFECTIVE OR "DRESSED" MASS OF
UL1)/$\mathbb{Z}_5$ GAUGE BOSON MAY BE $0$