QCD, Strings and Black holes

The large N limit of Field Theories and Gravity

Juan Maldacena
Field Theory = Gravity theory

Gauge Theories
QCD

Latin

Plan
QCD, Strings, the large N limit
Supersymmetric QCD

\[ \downarrow N \text{ large} \]
Gravitational theory in 10 dimensions
Calculations \rightarrow Correlation functions
Quark-antiquark potential
Black holes
Strings and Strong Interactions

Before 60s \(\rightarrow\) proton, neutron \(\rightarrow\) elementary
During 60s \(\rightarrow\) many new strongly interacting particles
Many had higher spins \(s = 2, 3, 4 \ldots\)
All these particles \(\rightarrow\) different oscillation modes of a string.

This model explained “Regge trajectories”

Rotating String model

\[m^2 \sim T J_{\text{max}} + \text{const}\]

From E. Klempt  hep-ex/0101031
Strong Interactions from Quantum ChromoDynamics

Experiments at higher energies revealed quarks and gluons

Electrodynamics

- Photon
- Electron

Gauge group
- U(1)

Chromodynamics (QCD)

- 3 colors (charges)
- They interact exchanging gluons

Glueons carry color charge, so they interact among themselves

3 x 3 matrices
Coupling constant decreases at high energy

\[ g \longrightarrow 0 \quad \text{at high energies} \quad \longrightarrow \quad \text{QCD is easier to study at high energies} \]

Hard to study at low energies

Indeed, at low energies we expect to see confinement

Flux tubes of color field = glue

At low energies we have something that looks like a string

Can we have an effective theory in terms of strings?

Large N limit

Take N colors instead of 3, \( \text{SU}(N) \)

Gross, Politzer, Wilczek

t’ Hooft ‘74
Large $N$ and strings

Gluon: color and anti-color

Open strings $\rightarrow$ mesons
Closed strings $\rightarrow$ glueballs

Looks like a string theory, but…
1. Simplest action = Area

\[
\text{Not consistent in } D=4 \quad (D=26?) \\
\downarrow \text{generate} \\
\text{At least one more dimension (thickness)}
\]

Polyakov

2. Strings theories always contain a state with m=0, spin =2: a Graviton.

For this reason strings are commonly used to study quantum gravity

Scherk-Schwarz
Yoneya

We combine these two problems into a solution. We will look for a 5 dimensional theory that contains gravity. We have to find an appropriate 5 dimensional curved spacetime.
Most supersymmetric QCD

Supersymmetry

Bosons $\rightarrow$ Fermions

Gluon $\rightarrow$ Gluino

Many supersymmetries

B1 $\leftrightarrow$ F1
B2 $\leftrightarrow$ F2

Maximum 4 supersymmetries, $N = 4$ Super Yang Mills

$A_\mu$ Vector boson $\quad$ spin = 1
$\Psi_\alpha$ 4 fermions (gluinons) $\quad$ spin = 1/2
$\Phi^I$ 6 scalars $\quad$ spin = 0

SO(6) symmetry

All NxN matrices

Susy might be present in the real world but spontaneously broken at low energies.

We study this case because it is simpler.
Similar in spirit to QCD

Difference: most SUSY QCD is scale invariant

Classical electromagnetism is scale invariant
\[ V = \frac{1}{r} \]
QCD is scale invariant classically but not quantum mechanically, \( g(E) \)

Most susy QCD is scale invariant even quantum mechanically

Symmetry group

Lorentz + translations + scale transformations + other

The string should move in a space of the form

\[ ds^2 = R^2 \ w^2 (z) \left( dx_{3+1}^2 + dz^2 \right) \]

\[ \text{redshift factor } = \text{warp factor } \sim \text{gravitational potential} \]

Demanding that the metric is symmetric under scale transformations
\[ x \rightarrow \lambda \ x \ , \ \text{we find that } w(z) = \frac{1}{z} \]
This metric is called anti-de-sitter space. It has constant negative curvature, with a radius of curvature given by $R$.

\[ ds^2 = R^2 \left( \frac{dx_{3+1}^2 + dz^2}{z^2} \right) \]
Anti de Sitter space

Solution of Einstein’s equations with negative cosmological constant.

De Sitter → solution with positive cosmological constant, accelerated expanding universe

Boundary is $S^3 \times \text{time}$

Spatial cross section of AdS = hyperbolic space
Spatial section of AdS = Hyperbolic space
R = radius of curvature

Energies of particles in AdS are quantized, particles feel as if they were in a gravitational potential well, they cannot escape to infinity.

Qualitatively AdS is like a box of size $R$.

Boundary is $S^3 \times \text{Time}$

The Field theory is defined on the boundary of AdS.
Building up the Dictionary

Graviton $\rightarrow$ stress tensor

$T_{\mu\nu}(x)\, T_{\mu\nu}(y)\, T_{\mu\nu}(z)$

$\Rightarrow$ Probability amplitude that gravitons go between given points on the boundary

Other operators

Other fields (particles) propagating in AdS.

Mass of the particle $\rightarrow$ scaling dimension of the operator

$$\Delta = 2 + \sqrt{4 + (mR)^2}$$
Most supersymmetric QCD

We expected to have string theory on AdS. Supersymmetry $\rightarrow$ D=10 superstring theory on $\text{AdS}^5 \times (\text{something})^5$

SO(6) symmetry $\downarrow$

$\text{S}^5$

Type IIB superstrings on $\text{AdS}^5 \times \text{S}^5$

5-form field strength $F = \text{generalized magnetic field} \rightarrow \text{quantized}$

$$\int_{\text{S}^5} F = N$$

Similar solution in 4 dimensional gravity + electromagnetism: $\text{AdS}^2 \times \text{S}^2$, with a flux of magnetic field on $\text{S}^2$:

Start with a charged, extremal black hole $\rightarrow$ near horizon geometry
**String Theory**

**Free strings**

String

Tension = \( T = \frac{1}{l_s^2} \), \( l_s \) = string length

Relativistic, so \( T = \frac{\text{mass}}{\text{unit length}} \)

Excitations along a stretched string travel at the speed of light

**Closed strings**

Can oscillate → Normal modes → Quantized energy levels

Mass of the object = total energy

M=0 states include a graviton (a spin 2 particle)

First massive state has \( M^2 \sim T \)
String Interactions
Splitting and joining

Simplest case: Flat 10 dimensions and supersymmetric

Precise rules, finite results, constrained mathematical structure

At low energies, energies smaller than the mass of the first massive string state

Gravity theory

Radius of curvature $\gg$ string length $\Rightarrow$ gravity is a good approximation

( Incorporates gauge interactions $\Rightarrow$ Unification )
Particle theory = gravity theory

Most supersymmetry QCD theory = String theory on AdS$_5 \times$ S$^5$

N colors

Radius of curvature

\[ R_{S^5} = R_{AdS_5} = \left( g_{YM}^2 N \right)^{1/4} l_s \]

Duality:

\[ g^2 N \text{ is small} \quad \rightarrow \quad \text{perturbation theory is easy – gravity is bad} \]

\[ g^2 N \text{ is large} \quad \rightarrow \quad \text{gravity is good – perturbation theory is hard} \]

Strings made with gluons become fundamental strings.
Where Do the Extra Dimensions Come From?

3+1 $\rightarrow$ AdS$_5$ $\rightarrow$ radial dimension

Strings live here

Gluons live here

Interior

Z

Boundary
What about the $S^5$?

- Related to the 6 scalars
- $S^5 \rightarrow$ other manifolds = Most susy QCD $\rightarrow$ less susy QCD.
- Large number of examples

Klebanov, Witten, Gauntlett, Martelli, Sparks, Hannany, Franco, Benvenutti, Tachikawa, Yau …..
Quark anti quark potential

\[ V = \text{potential} = \text{proper length of the string in AdS} \]

\[ V \approx -\frac{\sqrt{g^2 N}}{L} \]

Weak coupling result:

\[ V \approx -\frac{g^2 N}{L} \]
Confining Theories

Add masses to scalars and fermions $\rightarrow$ pure Yang Mills at low energies $\rightarrow$ confining theory. There are many concrete examples.
At strong coupling $\rightarrow$ gravity solution is a good description.

String at $z_0$ has finite tension from the point of view of the boundary theory.
Graviton in the interior $\rightarrow$ massive spin=2 particle in the boundary theory = glueball.

Baryons $\rightarrow$ D-branes
Checking the conjecture

- Energies of string states in AdS = Dimensions of operators in gauge theory
- Direct computation in the gauge theory
- Planar diagrams $\rightarrow$ “spin” chains
- Connections to “integrable” systems that also appear in condensed matter.
- Works nicely in a special, large charge, limit
- Not yet done in general, active research area…

Berenstein, J.M, Nastase, Minahan, Zarembo, Staudacher, Beisert,…. 
What can we learn about gravity from the field theory?

• Useful for understanding quantum aspects of black holes
Black holes

Gravitational collapse leads to black holes

Classically nothing can escape once it crosses the event horizon

Quantum mechanics implies that black holes emit thermal radiation. (Hawking)

\[ T \approx \frac{1}{r_s} \approx \frac{1}{G_N M} \]

Black holes evaporate

Evaporation time

\[ \tau = \tau_{\text{universe}} \left( \frac{M}{10^{12} \text{ Kg}} \right)^3 \]

Temperature is related to entropy

\[ dM = T \, dS \quad S = \frac{\text{Area of the horizon}}{4 \, L_{\text{Planck}}^2} \] (Hawking-Bekenstein)

What is the statistical interpretation of this entropy?
Black holes in AdS

Thermal configurations in AdS.

Entropy:

$S_{\text{GRAVITY}} = \text{Area of the horizon} = S_{\text{FIELD THEORY}} = \log[\text{Number of states}]$

Evolution: Unitary

(these calculations are easier in the AdS$_3$ case)
Confining Theories and Black Holes

Low temperatures

Confinement

High temperatures

Deconfinement = black hole (black brane)

Gravitational potential

\[ w(z_0) > 0 \]

boundary

\[ z=0 \]

\[ z=z_0 \]

Gravitational potential

\[ w(z) \]
Black holes in the Laboratory

High energy collision → produces a black hole =

droplet of deconfined phase ~
quark gluon plasma.

Black hole → Very low shear viscosity →
similar to what is observed at RHIC:
“the most perfect fluid”

QCD → 5d string theory

Kovtun, Son, Starinets, Policastro

Very rough model, we do not yet know the precise string theory
Quantum theories of gravity should be **holographic**.

All physics in a region $\rightarrow$ described in terms of a number of q-bits which is smaller than the area (not volume) of the region (in Planck units).

The gauge theory gravity duality $\rightarrow$ concrete realization $\rightarrow$ gives a non-perturbative definition of quantum gravity in AdS.
Emergent space time

Spacetime: like the fermi surface, only defined in the classical limit

Lin, Lunin, J.M.
Conclusions

- Gravity and particle physics are “unified”
  Usual: Quantum gravity $\rightarrow$ particle physics.
  New: Particle physics $\rightarrow$ quantum gravity.
Most elementary particles $\rightarrow$ quantum gravity $\rightarrow$
particles physics of the real world
- Black holes and confinement are related
- Emergent space-time
- Tool to do computations in gauge theories.
- Tool to do computations in gravity.
Future

Field theory:
- Theories closer to the theory of strong interactions
- Solve large N QCD

Gravity:
- Quantum gravity in other spacetimes
- Understand cosmological singularities