EARTH ORBITING DARK MATTER AS A POSSIBLE EXPLANATION OF THE FLYBY ANOMALIES

Stephen L. Adler
Institute for Advanced Study

• EXPERIMENTAL DATA
  Anderson et al. PRL 100 091102 (2008)

• POSSIBLE EXPLANATIONS

• DARK MATTER SCATTERING?

• BOUNDING THE MASS OF EARTH ORBITING DARK MATTER

• DARK MATTER MODELING

• SPACECRAFT CALORIMETRY AS A TEST
  "white paper" for decadal review on space science

• OUTLOOK
EXPERIMENTAL DATA

Anderson et al.
PRL 100, 091102 (2008)

Anderson & Nite
Physics Today Oct 2009
pp 76-77

INCOMING ASYMPTOTE

EARTH

OUTGOING ASYMPTOTE

OUTGOING VELOCITY EXTRAPOLATED FROM INCOMING VELOCITY DOES NOT AGREE WITH MEASURED VALUE

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>GALILEO I</th>
<th>GALILEO II</th>
<th>NEAR EARTH</th>
<th>ROSETTA</th>
<th>MESSENGER</th>
</tr>
</thead>
<tbody>
<tr>
<td>DATE</td>
<td>12/18/96</td>
<td>12/18/96</td>
<td>8/18/99</td>
<td>3/14/05</td>
<td>8/12/05</td>
</tr>
<tr>
<td>ΔV∞ (m/s)</td>
<td>3.92</td>
<td>-9.6</td>
<td>17.96</td>
<td>-2</td>
<td>1.80</td>
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<tr>
<td>&lt;V∞ (m/s)</td>
<td>0.3</td>
<td>1.0</td>
<td>0.01</td>
<td>0.03</td>
<td>0.01</td>
</tr>
</tbody>
</table>
Anderson et al. Fit (Empirical - No Theoretical Basis)

\[ \frac{\Delta V}{V} = \frac{1}{2} \frac{\Delta E}{E} = K (\cos S_x - \cos S_0) \]

\( |S|_0 \) = Incoming, outgoing Declination

\( \gamma \) = Latitude Analog in Celestial Coordinate System

\[ K = \frac{2 \omega_E R_E}{c} = 3.099 \times 10^{-6} \]

\( \omega_E \) = Earth Angular Rotation Velocity

\( R_E \) = Earth Radius \( \approx 6,371 \) km

- Declination dependence indicates that maximal symmetry of any mechanism can be axial symmetry around Earth rotation axis
- Spherical symmetry cannot give a declination dependence
MAGNITUDE AS AN ACCELERATION

NEAR FLYBY: \( \Delta V_{\text{flyby}} = 13.96 \text{ mm/s} \approx 1.3 \times 10^{-2} \text{ m/s} \)
OVER \( \Delta t \approx 4 \text{ hours} \approx 10^4 \text{ s} \)
\( \Rightarrow \) ACCELERATION \( \approx 10^{-6} \text{ m/s}^2 \)

SOME COMPARISONS:

PIONEER ANOMALOUS ACCELERATION \( \approx 8.7 \times 10^{-10} \text{ m/s}^2 \)
MOND CUT-OFF ACCELERATION \( \approx 10^{-10} \text{ m/s}^2 \)
COSMOLOGICAL SCALE \( \rho_c \approx 2.3 \times 10^{-18} \text{ g/cm}^3 \approx 3 \times 10^8 \text{ m}^3 = 7 \times 10^{-10} \text{ m}^3 \)

FLYBY ANOMALY, INTERPRETED AS AN ACCELERATION,
IS 3 ORDERS OF MAGNITUDE LARGER THAN THESE
POSSIBLE EXPLANATIONS

FOUR POSSIBILITIES:

- EFFECT IS AN ARTIFACT - SOME ESSENTIAL CONVENTIONAL PHYSICS HAS BEEN OMITTED FROM THE ORBITAL CALCULATION
- THERMAL EFFECTS (WHICH MAY EXPLAIN MOST OF THE PIONEER ANOMALY) ARE TOO SMALL TO EXPLAIN FLIGHT (BEAN WORKSHOP REPORTS)
- OTHER POSSIBILITIES UNDER STUDY
- NEW ELECTROMAGNETIC PHYSICS
  - QED TESTED TO HIGH PRECISION
  - PVLS - DIDN'T FIND AXIONS, BUT SHOWED VACUUM LINEAR TO 1 IN 10^18 IN 0.5 TEILLA FIELD
- NEW GRAVITATIONAL PHYSICS
  - PPN DEVIATIONS FROM GENERAL RELATIVITY AT LEAST A FACTOR OF 100 TOO SMALL (SEE MEMO ON S.L.A. WEB PAGE TALKS & MEMO SECTION)
Explanations - continued

So a gravitational explanation must be outside the PPN framework of metric theories that obey the equivalence principle - but equiv. prin. tested to high precision

- MOND or cosmology: accelerations too small

- DARK MATTER

   Cosmology ⇒ only ~4% of the mass-energy density of the universe is baryonic matter

   ~23% is gravitationally attractive "Dark Matter"

   ~73% is gravitationally repulsive "Dark Energy"

Little is known about dark matter:
- Bosonic or Fermionic?
- Self-annihilating?
- Mass (60) ?
- Non-gravitational interactions?
DARK MATTER SCATTERING?

DARK MATTER CAN BE GRAVITATIONALLY BOUND ON DIFFERENT SCALES

- GALACTIC HALO DARK MATTER
  MASS DENSITY \( \rho_m \approx 0.3 \text{ GeV/}c^2 \text{ cm}^{-3} \)

- SOLAR SYSTEM-BOUND DARK MATTER?
  \( \rho_m < 10^5 \text{ GeV/}c^2 \text{ cm}^{-3} \)
  FROM STUDY OF PLANETARY ORBITS

- EARTH-BOUND DARK MATTER?
  CAN SCATTERING FROM EARTH-BOUND DARK MATTER EXPLAIN THE FLYBY ANOMALIES?
- 8 -

SCATTERING - CONTINUED

- **ELASTIC SCATTERING** - GIVES NORMAL DRAG VELOCITY DECREASE

- **INELASTIC EXOTHERMATIC SCATTERING**

\[ D(m_2) + N(m_1) \rightarrow D'(m_2') + N(m_1) \quad m_2 > m_2' \]

CAN GIVE A VELOCITY INCREASE - WILL GIVE DETAILED FORMULAS

- TO GIVE OBSERVED VELOCITY ANOMALIES, NEED

  \[ \sigma_{m} \geq 10^{-16} \text{ GeV/cm}^2 \text{ cm}^{-1} \]

  CROSS SECTION

  \[ \sigma_m \geq 10^{-20} \text{ GeV/cm}^2 \text{ cm}^{-1} \]

  \[ \sigma_{m} \sim 10^{-33} \text{ cm}^2 \quad \sigma_{m, \text{ ELASTIC}} \sim 10^{-17} \text{ GeV/cm}^2 \text{ cm}^{-3} \]

  \[ \sigma_{m, \text{ INELASTIC}} \sim 10^{13} \text{ GeV/cm}^2 \text{ cm}^{-3} \]

  ( INELASTIC / ELASTIC \sim C/V_{ORBITAL} \sim 10^4 )
PLACING DIRECT LIMITS ON THE MASS OF EARTH-BOUND DARK MATTER

Can set a direct limit on the total earth-bound dark matter mass lying between moon orbit radius \( \approx 384,000 \) km.

LAGEOS Geodetic
Satellite orbit radius \( \approx 12,300 \) km.

For a circular orbit, measuring radius \( R \) and period \( T \) gives GM of attracting body:

\[
GM = \frac{9 \pi^2 R^3}{T^2}
\]

LAGEOS tracking gives \( GM@ \)

Earth mass \( M@ \) includes dark matter within the LAGEOS orbit.
DIRECT LIMITS - LUNAR ORBITERS, EROS TRACKING,

LUNAR LASER RANGING

1. LUNAR ORBITERS GIVE $6 M_m$

2. MORE ACCURATE $M_m$ FROM TRACKING EROS ASTEROID
   GIVES
   \[ R \oplus M_m = \frac{6 M_\oplus + 6 \Delta M_\oplus}{6 M_m} \]
   \[ = \frac{6 M_\oplus}{6 M_m} (1 + \frac{\Delta M_\oplus}{M_\oplus}) \]
   \[ s = \frac{\Delta M_\oplus}{M_\oplus} \]

3. LUNAR LASER RANGING GIVES COMBINED $GM_{\text{tot}}$ OF
   EARTH-MOON SYSTEM:
   \[ GM_{\text{tot}} = 6 M_\oplus + 6 M_m + 6 M_{\text{dm}} \]
   MASS OF DARK MATTER LYING BETWEEN MOON AND
   LAGEDI ORBITS
DIRECT LIMIT - FINAL FORMULA

COMBINING FORMULAS

\[ G M_{\text{Tot}} - G M_\oplus - \frac{6 M_\oplus}{R_\oplus \rho_m} = G M_{dm} + \frac{M_m}{M_\oplus} \delta M_\oplus \]

\[ = G M_{dm} + 0.0123 \delta M_\oplus > G M_{dm} \]

\[ = G M_{dm} \left[ 1 + 0(0.01) \right] \text{ IF } \delta M_\oplus \sim N_{dm} \]

NUMERICAL (ALL CONVERTED TO GARYCENTRIC DYNAMICAL TIME)

\( \text{LAGEOS} \Rightarrow G M_\oplus = 398,600.4356 \pm 0.0008 \; \text{km}^3 \text{s}^{-2} \)

\( \text{LUNAR RANGING} \Rightarrow G M_{\text{Combined}} = 403,503.2357 \pm 0.0014 \; \text{km}^3 \text{s}^{-2} \)

\( \text{ERTS} \Rightarrow R \rho_m = 81,300 \; \text{km} \pm 0.0006 \text{os} \)

\( \text{LAGEOS} \; G M_\oplus \Rightarrow G M_m = 4902.8000 \pm 0.0003 \; \text{km}^3 \text{s}^{-2} \)
**DIRECT LIMITS - RESULTS**

Combining these gives

\[ G M_{dm} \leq (0.0001 \pm 0.0016) \text{ km}^3 \text{ s}^{-2} \times (0.3 \pm 0.4) \times 10^{-9} \text{ GM}_\odot \]

\[ \Rightarrow \text{ DOMINANT ERROR FROM LUNAR LASER RANGING} \]

so \( M_{dm} \leq 9 \times 10^{-9} \text{ M}_\odot \)

If this bound were attained, and the mass were uniformly distributed below the moon's orbit, the density would be

\[ \rho_m \approx 6 \times 10^{10} \text{ GeV/}c^2 \text{ cm}^{-3} \Rightarrow \text{ GALACTIC HALO DENSITY AND LIMIT ON SUN-BOUND DARK MATTER} \]

If confined to a narrow band, \( \rho_m \) for earth-bound dark matter could be even higher:

For a shell of radius \( R \) and thickness \( D \),

\[ \rho_m \leq 2 \times 10^{10} \text{ GeV/}c^2 \text{ cm}^{-3} \left( \frac{389,000 \text{ km}}{R^2 D} \right) \]
DARK MATTER MODELING FOR FLYBY

SOME FLYBYS GAIN VELOCITY (ANOMALOUS DRAG)

SOME FLYBYS LOSE VELOCITY (NORMAL DRAG)

MODEL ASSUMES TWO SPECIES OF DARK MATTER ORBITING EARTH

- INELASTIC, EXOTHERMIC SCATTER
  \[ D(m_2) + N(m_1) \rightarrow D'(m'_2) + N(m_1), \quad m_2 > m'_2 \]
  CAN GIVE VELOCITY GAIN IF \( N(m_1) \) SCATTERS FORWARD

- ELASTIC SCATTER
  \[ D''(m_2) + N(m_1) \rightarrow D''(m'_2) + N(m_1), \quad \text{GIVE NORMAL DRAG} \]
VELOCITY CHANGE FORMULAS

\[ \hat{u}_1 = \text{VELOCITY OF INITIAL NUCLEON (IN SPACECRAFT)} \]
\[ \hat{u}_2 = \text{VELOCITY OF INCIDENT DARK MATTER} \]

\[ \langle \\delta \hat{v} \rangle = \text{CROSS-SECTION AVERAGED NUCLEON VELOCITY CHANGE} \]
\( \text{ASSUME NO AZIMUTHAL ANGLE DEPENDENCE} \)

INELASTIC

\[ \langle \\delta \hat{v} \rangle = \frac{\hat{u}_1 - \hat{u}_2}{|\hat{u}_1 - \hat{u}_2|} A_i \]
\[ A_i = \left[ \frac{2 \left( m_1 - m_2 \right) m_2}{m_1 \left( m_1 + m_2 \right)} \right]^{1/4} \cdot \langle \cos \theta \rangle_i \]

ELASTIC

\[ \langle \\delta \hat{v} \rangle = - \left( \hat{u}_1 - \hat{u}_2 \right) A_E \]
\[ A_E = \left( \frac{m_2}{m_1 + m_2} \right)^{1/2} \left[ 1 - \langle \cos \theta \rangle_E \right] \]

WILL COMP CROSS SECTION X DENSITY INTO THE
PARAMETERS A_i, A_E
CHANGE IN OUTGOING SPACECRAFT VELOCITY

\[ \delta \mathbf{F} = \int d^3u_2 \langle \delta \mathbf{v} \rangle \left| \mathbf{u}_1 - \mathbf{u}_2 \right| \sigma (\mathbf{x}, \mathbf{u}_2) \]

\[ \mathbf{u}_i = \frac{d\mathbf{x}}{dt} \]

INTEGRATING \( \delta \mathbf{F} \cdot d\mathbf{x} \) ALONG TRAJECTORY

\[ \delta \int_{t_i}^{t_f} (\mathbf{\dot{v}}_f - \mathbf{\dot{v}}_i) = \mathbf{\dot{v}}_f \cdot \int_{t_i}^{t_f} d\mathbf{x} \left( \frac{d\mathbf{u}_i}{dt} \right) \cdot \delta \mathbf{F} \]

\[ = \int_{t_i}^{t_f} dt \int d^3u_2 \frac{d\mathbf{u}_2}{dt} \sigma (\mathbf{x}, \mathbf{u}_2) \left| \mathbf{u}_1 - \mathbf{u}_2 \right| \]
CROSS SECTION AND SCATTERING-ANGLE AVERAGES

\[ W = \text{CENTER OF MASS SCATTERING ENERGY} \]

\[ \frac{W}{(m_1 + m_2)c^2} = 1 + \frac{m_1 m_2}{2 (m_1 + m_2)^2} \left( \frac{\tilde{u}_1 - \tilde{u}_2}{c^2} \right)^4 \]

For \( m_2 \leq m_1 \) and non-relativistic \( \tilde{u}_1, \tilde{u}_2 \), \( W \approx 1 \)

Cross section dominated by lowest partial wave

**ELASTIC**

5-wave dominated \( \sigma \approx \sigma_{el} = \text{constant} \)

\[ \langle \cos \theta \rangle_{el} = 0 \quad \text{1} - \langle \cos \theta \rangle_{el} = 1 \]

**INELASTIC**

Entrance channel momentum \( k = \frac{m_1 m_2}{m_1 + m_2} (\tilde{u}_1 - \tilde{u}_2) \)

\[ \frac{d\sigma}{d\Omega} = A_{inel} k^{-1} + B_{inel} \frac{3}{4\pi} \cos \theta + \ldots \text{near threshold} \]

\( \sigma \approx A_{inel} k^{-1} \quad \langle \cos \theta \rangle = B_{inel} / (A_{inel} k^{-1}) \)

\( \sigma \langle \cos \theta \rangle_x = B_{inel} = \text{constant} \)
DARK MATTER ORBITAL GEOMETRY

Assume axial symmetry around Earth rotation axis.

Simplest model - disk in equatorial plane - does not work - "near" gets smallest velocity change.

Next simplest model -

Circular orbit at angle \( \psi \) to Earth axis; radius \( r \).

\( E, L_z \) conserved, but orbit will precess because of Earth quadrupole - will fill shell obtained by averaging orbit over azimuthal angle around Earth axis.

Get this:

Density \( \rho \) is given by

\[
\rho \propto \frac{1}{\sqrt{(r \sin \psi)^2 - x^2}}
\]

\[\text{Jacobian} \]

\[\text{Weighting function} \]

\[W(r, \psi)\]
Dark Matter Velocity $\vec{u}_2$

For given $\lambda, \psi, t$, there are two $\vec{u}_2$ values, one for an up-going segment and one for a down-going segment.

\[
\vec{u}_2 = \sqrt{\frac{GM_0}{\lambda}} \left( C_\pm \vec{u}_n \pm d_\pm \vec{u}_\perp \right)
\]

$C_\pm = \frac{\lambda \cos \psi}{\sqrt{\lambda^2 - \xi^2}} \quad d_\pm = \pm \frac{\sqrt{\lambda^2 \sin^2 \psi - \xi^2}}{\sqrt{\lambda^2 - \xi^2}}$

$C_\pm^2 + d_\pm^2 = 1$

$\vec{u}_n$ parallel to equatorial plane $\vec{u}_\perp$ perpendicular to $\vec{u}$ and $\vec{u}_n$
**Simple Choice of \( W(y, \psi) \)**

**Single Tilt Angle \( \psi_i, \psi_e \) for Inelastic, Elastic Shells**

**Gaussian \( y \) Profile**

\[
W_i(y, \psi) = k_i e^{-\frac{(y - \psi_i)^2}{b_i^2}} \delta(\psi - \psi_i) \quad \text{Inelastic}
\]

\[
W_e(y, \psi) = k_e e^{-\frac{(y - \psi_e)^2}{b_e^2}} \delta(\psi - \psi_e) \quad \text{Elastic}
\]

\[
N = \text{Total Number of Particles in Shell} = \int d^3 x \int d^3 u_z \rho(x, u_z) = \pi^\frac{3}{2} k_e b_e \delta \quad l = i, e
\]

**Density Times Cross Section Parameters:**

\[
\rho_i = N_i B_{\text{incl}} k_i
\]

\[
\rho_e = N_6 B_{\text{el}} k_e
\]
FLYBY ORBITAL GEOMETRY

WORK IN FLYBY ORBIT PLANE

\[ \lambda = \frac{\gamma}{1 + \epsilon \cos \theta_0} \]

\[ \vec{v} = (x_0, y_0, \theta) \]

\[ x_0 = \lambda \epsilon \cos \theta_0 \]

\[ y_0 = \lambda \epsilon \sin \theta_0 \]

FLYBY HYPERBOLIC ORBIT

EARTH AXIS AS SEEN FROM FLYBY ORBIT PLANE

\[ \vec{z} = \text{EARTH ROTATION AXIS} \]

POLAR ANGLES \( \lambda, \nu \)

**POINTS**

1. EXPRESS DARK MATTER ORBIT ON EARTH-FIXED AXES
2. THEN, REWRITE ON FLYBY PLANE AXES TO GET

\[ \hat{z}, \hat{\mu}, \hat{\lambda} \]
## Flyby Parameters on Flyby Plane Axes

(And Intrinsic Parameters $\theta, \alpha$)

<table>
<thead>
<tr>
<th></th>
<th>C/U-I</th>
<th>C/U-II</th>
<th>NEAR</th>
<th>CASSINI</th>
<th>ROSETTA</th>
<th>MESSENGER</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I^{(0)}$ (°)</td>
<td>192.9</td>
<td>138.7</td>
<td>108.0</td>
<td>25.4</td>
<td>199.9</td>
<td>133.1</td>
</tr>
<tr>
<td>$\alpha^{(0)}$ (°)</td>
<td>-45.1</td>
<td>-197.4</td>
<td>-55.1</td>
<td>-158.4</td>
<td>-53.1</td>
<td>0.0</td>
</tr>
<tr>
<td>$\rho$ (km)</td>
<td>25,973</td>
<td>22,155</td>
<td>19,498</td>
<td>51,689</td>
<td>19,262</td>
<td>20,563</td>
</tr>
<tr>
<td>$e$</td>
<td>2.473</td>
<td>2.319</td>
<td>1.814</td>
<td>5.851</td>
<td>1.312</td>
<td>1.360</td>
</tr>
</tbody>
</table>

$$e = 1 + \frac{2V_\infty^2}{V_x^2 - V_\infty^2} \quad \rho = \frac{4GM_{\odot}}{V_x^2} \left[ \left( \frac{V_x^2 - V_\infty^2}{V_x^2 - V_\infty^2} \right)^2 + \frac{V_\infty^2}{V_x^2 - V_\infty^2} \right]$$

$\alpha$ requires attention to $\pm$ in AR/CURINE; above values reproduce the $I^*, \rho, \alpha_x - \alpha_0$ values in ANDERSON et al. 2015
NUMERICAL RESULTS

MINIMIZE LIKELIHOOD FUNCTION

\[ \chi^2 = \sum_{k=1}^{N} \left( \frac{S_{V_{ks\text{th}}} - S_{V_{ks\text{A}}}}{\sigma_{V_{ks\text{A}}}} \right)^2 \]

\[ S_{V_{ks\text{th}}} = \rho_z S_{V_{ksz}} + \rho_e S_{V_{kse}} \text{ linear in } \rho_z, \rho_e \text{; } \chi^2 \text{ quadratic} \]

\[ \Rightarrow \text{MINIMIZATION WITH RESPECT TO } \rho_z, \rho_e \text{ CAN BE DONE ALGEBRAICALLY} \]

REDUCES PARAMETER SPACE FOR NUMERICAL SEARCH TO 6 PARAMETERS: \( \psi_z, \lambda z, \delta_z, \psi_e, \lambda e, \delta_e \)

- WITH SMOOTHED JACOBIAN, DO COARSE MESH SURVEY TO IDENTIFY REGIONS WITH LOW \( \chi^2 \) TO USE AS STARTS FOR MINIMIZATION
- WITH ORIGINAL JACOBIAN, AND ADAPTIVE INTEGRATION, DO MINIMIZATION USING CERN PROGRAM "MINUIT"
<table>
<thead>
<tr>
<th>Parameters</th>
<th>Illustrative</th>
<th>Good Fit (2d from Preprint)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( S V_A ) (( \text{mm/s} ))</td>
<td>( 3.92 )</td>
<td>(-4.6)</td>
</tr>
<tr>
<td>( C_A ) (( \text{mm/s} ))</td>
<td>( 0.3 )</td>
<td>( 1.0 )</td>
</tr>
<tr>
<td>( \text{Fit 2d} )</td>
<td>( 0.51 )</td>
<td>( 3.90 )</td>
</tr>
</tbody>
</table>

**Parameters**

\[
\begin{align*}
10^6 \times \beta_c & (\text{km}) \\
10 \times \beta_c & (\text{km}) \\
\Psi_c & (\text{rad}) \\
\Psi_e & (\text{rad}) \\
R_i & (\text{km}) \\
D_1 & (\text{km}) \\
R_e & (\text{km}) \\
D_2 & (\text{km})
\end{align*}
\]

\[
\begin{align*}
1.000 & \\
0.288 & \\
1.372 & \\
0.392 & \\
34520 & \\
30030 & \\
29370 & \\
6678 &
\end{align*}
\]

**Overfitting?**

- With only inelastic, cannot fit near \text{& Messenger} \( R_h = 0.61 \times 10^5 \)
- With full model, fitting to 5 of 6 flybys gives qualitatively reasonable value for \text{6}\text{th}.

\[
\begin{align*}
3.71 & \\
-4.6 & \\
16.03 & \\
-2.7 & \\
1.62 & \\
0.12 &
\end{align*}
\]

- Not just fitting a wiggly curve through data points.
IMPLICATION OF DARK MATTER MASS BOUND

LUNAR LASER RANGING $\Rightarrow$ $6(M_\odot + M_m + M_{DM})$
LAGEOS $\Rightarrow$ $6M_\odot$  LUNAR ORBITER $\Rightarrow$ $6M_m$
ERS TRACKING

COMBINING $\Rightarrow$ $M_{DM} < 4 \times 10^{-9} M_\odot \sim 1.4 \times 10^{33}$ eV/c^2

Fit 2 ds $\rho_e D_e = 0.00304$ km^2
$\rho_e D_e = 19.2$ km

USE $M_e = m_e N_e = 4\pi \frac{\rho_e D_e}{\rho_e D_e} m_e / \sigma t$

$M_i = m_i N_i = 4\pi \frac{\rho_i D_i}{\rho_i D_i} m_i / \sigma i t$

$M_e \leq 4 \times 10^{-9} M_\odot \Rightarrow \sigma e_t \geq 9.4 \times 10^{-31}$ cm^2

$M_i \leq 4 \times 10^{-9} M_\odot \Rightarrow \sigma incr \geq 1.5 \times 10^{-34}$ cm^2
TEMPERATURE CHANGE

FLYBY VELOCITY CHANGE USED

\[
\langle \Delta v \rangle = \frac{\int_0^\pi d\theta \sin \theta |\langle \Delta v \rangle|^2}{\int_0^\pi d\theta \sin \theta |\langle \Delta v \rangle|^2}
\]

FLUCTUATION

\[
\langle (\Delta v - \langle \Delta v \rangle)^2 \rangle
\]

GIVES A TEMPERATURE INCREASE

\[
\delta T_{\text{smooth}} = \frac{m_1}{2k_B} \langle (\Delta v - \langle \Delta v \rangle)^2 \rangle
\]

\[
\frac{dT}{dt} = \int d^3 u_2 \delta T \quad \text{s.t.c.} \quad \langle \Delta v_1 - \Delta v_2 \rangle \propto |\langle \Delta v_1 \rangle|
\]

\[
T_e - T_i = \int_{k_0}^{k_e} dt \frac{dT}{dt} \quad \text{SIMPLE MODIFICATION TO PROGRAMS}
\]

DEFINE:

\[
\frac{\Delta n_{\text{inel}}}{\Delta n_{\text{inel}}} = R_{\text{inel}} \frac{m_2}{c} \quad S_{\text{inel}} = R_{\text{inel}} \left[ \frac{2(m_2^2 - m_2) m_2^2}{m_2} \right]^{1/2}
\]

GET THEN:

\[
\frac{\Delta T}{T_{0K}} = \left( \frac{m_2 c^2}{M_{\text{eV}}} \right) \left[ T_e + S_{\text{inel}} T_i \right]
\]

FIND:

\[
T_e \sim (0.3 - 0.9) \times 10^5 \quad T_i \sim (0.2 - 0.7) \times 10^4
\]
KEY THINGS NEEDED FROM FLYBY REANALYSIS

- REAL EFFECT (NEW PHYSICS) OR ARTIFACT?
- SU4 VALUES
- \( C \) VALUES \( C = 0.01 \) MESSENGER, \( C = 0.03 \) ROSSETA GIVES THEM A VERY BIG WEIGHT IN \( \chi^2 \)
- FLYBY ORBITAL PLANE PARAMETERS
  - EARTH AXU POLAR ANGLES I & AZIMUTH \( \alpha \)

ALSO LOOK FOR

- RESULTS FROM DIRECT DARK MATTER SEARCHES
- LHC SEARCH FOR SUPERSYMMETRY