DARK MATTER MODELING FOR FLYBY

Stephen L. Adler
Institute for Advanced Study, Princeton

SOME FLYBYS GAIN VELOCITY (ANOMALOUS DRAG)
SOME FLYBYS LOSE VELOCITY (NORMAL DRAG)

MODEL ASSUMES TWO SPECIES OF DARK MATTER ORBITING EARTH

- INELASTIC, EXOTHERMIC SCATTER

\[ D(m_2) + N(m_1) \rightarrow D'(m_2') + N(m_1) \quad m_2 > m_2' \]

CAN GIVE VELOCITY GAIN IF N(m_1) SCATTERS FORWARD

- ELASTIC SCATTER

\[ D''(m_2) + N(m_1) \rightarrow D''(m_2') + N(m_1) \]

give (NORMAL DRAG)
VELOCITY CHANGE FORMULAS

\( \hat{u}_i = \text{VELOCITY OF INITIAL NUCLEON (IN SPACECRAFT)} \)
\( \hat{u}_2 = \text{VELOCITY OF INCIDENT DARK MATTER} \)

\( \langle S \hat{v} \rangle = \text{CROSS-SECTION AVERAGED NUCLEON VELOCITY CHANGE} \)
\( \text{(ASSUME NO AZIMUTHAL ANGLE DEPENDENCE)} \)

\( \langle S \hat{v} \rangle = \frac{\hat{u}_i - \hat{u}_2}{1 - \hat{u}_i \cdot \hat{u}_2} A_i \)
\( A_i = \left[ \frac{2 (m_2 - m_0') m_2'}{m_2 (m_2 + m_0')} \right]^{1/2} \cos \langle \cos \theta \rangle_i \)

ELASTIC

\( \langle S \hat{v} \rangle = - (\hat{u}_i - \hat{u}_2) A_e \)
\( A_e = \left( \frac{m_2''}{m_1 + m_2''} \right) \left[ 1 - \langle \cos \theta \rangle_e \right] \)

WILL LUMP CROSS SECTION X DENSITY INTO THE PARAMETERS \( A_i, A_e \)
DARK MATTER ORBITAL GEOMETRY

ASSUME AXIAL SYMMETRY AROUND EARTH ROTATION AXIS

SIMPLEST MODEL - DISK IN EQUATORIAL PLANE - DOES NOT WORK - "NEAR" GETS SMALLEST VELOCITY CHANGE

NEXT SIMPLEST MODEL -

CIRCULAR ORBIT AT ANGLE θ TO EARTH AXIS;

RADIUS r

E, L CONSERVED, BUT ORBIT WILL PRECEDE BECAUSE OF EARTH QUADRUPOLE - WILL FILL SHELL OBTAINED BY AVERAGING ORBIT OVER AZIMUTHAL ANGLE AROUND EARTH AXIS

\[ \rho \propto \frac{1}{\sqrt{1 - \sin^2 \theta}} \]
DARK MATTER VELOCITY $\vec{U}_2$

For given $\lambda$, $\nu$, $\tau$, there are two $\vec{U}_2$ values, one for an up-going segment and one for a down-going segment.

$$\vec{U}_2 = \sqrt{\frac{GM_\odot}{r}} \left( C_\pm \vec{\nu}_\parallel + d_\pm \vec{\nu}_\perp \right)$$

$$C_\pm = \frac{\lambda \cos \nu}{\sqrt{\lambda^2 - \nu^2}}$$
$$d_\pm = \pm \frac{\sqrt{\lambda^2 \sin^2 \nu - \nu^2}}{\sqrt{\lambda^2 - \nu^2}}$$
$$C_\pm^2 + d_\pm^2 = 1$$

$\vec{\nu}_\parallel$ parallel to equatorial plane
$\vec{\nu}_\perp$ perpendicular to $\vec{U}$ and $\vec{\nu}_\parallel$
FLYBY ORBITAL GEOMETRY

WORK IN FLYBY ORBIT PLANE

\[ \lambda = \frac{v}{1 + e \cos \theta} \]

\[ \hat{v} = (x, y, 0) \quad x = \lambda \cos \theta \]
\[ y = \lambda \sin \theta \]

FLYBY HYPERBOLIC ORBIT

EARTH AXIS AS SEEN FROM FLYBY ORBIT PLANE

\[ z' = \text{EARTH ROTATION AXIS (CALLED THIS Z BEFORE)} \]

POLAR ANGLES \( I, \omega \)

- FIRST, EXPRESS DARK MATTER ORBIT ON EARTH-FIXED AXES
- THEN, REWRITE ON FLYBY PLANE AXES
# Flyby Parameters on Flyby Plane Axes

(And Intrinsic Parameters $\nu$, $\alpha$)

<table>
<thead>
<tr>
<th></th>
<th>GL-5</th>
<th>GL-II</th>
<th>NEAR</th>
<th>CASSINI</th>
<th>ROSETTA</th>
<th>MESSENGER</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I(^\circ)$</td>
<td>142.9</td>
<td>138.7</td>
<td>108.0</td>
<td>25.4</td>
<td>199.9</td>
<td>133.1</td>
</tr>
<tr>
<td>$\alpha(^\circ)$</td>
<td>-45.1</td>
<td>-197.9</td>
<td>-55.1</td>
<td>-158.4</td>
<td>-53.1</td>
<td>0.0</td>
</tr>
<tr>
<td>$\nu$ (km)</td>
<td>25,973</td>
<td>22,155</td>
<td>19,498</td>
<td>51,689</td>
<td>19,262</td>
<td>20,563</td>
</tr>
<tr>
<td>$e$</td>
<td>2.473</td>
<td>2.319</td>
<td>1.914</td>
<td>5.851</td>
<td>1.312</td>
<td>1.360</td>
</tr>
</tbody>
</table>

\[ c = 1 + \frac{2 V_0^2}{V_c^2 - V_0^2} \quad \nu = \frac{4 GM_\odot}{V_0^2} \left[ \left( \frac{V_0^2}{V_c^2 - V_0^2} \right)^2 + \frac{V_0^2}{V_c^2 - V_0^2} \right] \]

\( \alpha \) requires attention to \( \pm \) in arc-cosine; above values reproduce the $\delta_2$, $\delta_0$, $\alpha_2 - \alpha_0$ values in Anderson et al PRL.
WHERE THINGS ARE NOW

PROGRAM INTEGRATES OVER ORBIT OF FLYBY TO CALCULATE \( \vec{V}_{\text{in}} \cdot \vec{S} \cdot \vec{V}_{\text{in}} \)

HAVE TAKEN A QUICK LOOK AT AN 8 PARAMETER MODEL (NO SYSTEMATIC OPTIMIZATION OVER PARAMETERS)

INELASTIC SCATTERERS \( \alpha_1 \quad \gamma_1 \quad \Delta_1 \leq \lambda \leq \beta_1 \)

ELASTIC SCATTERERS \( \alpha_e \quad \gamma_e \quad \Delta_e \leq \lambda \leq \beta_e \)

EXAMPLE:
\( \alpha_1 = 0.99 \quad \gamma_1 = 1.69 \text{ rad} \quad \frac{1}{2} (\alpha_1 + \beta_1) = 34,740 \text{ km} \)
\( \beta_1 - \alpha_1 = 10,000 \text{ km} \)
\( \alpha_e = 0.076 \quad \gamma_e = 1.79 \text{ rad} \quad \frac{1}{2} (\alpha_e + \beta_e) = 30,000 \text{ km} \)
\( \beta_e - \alpha_e = 10,000 \text{ km} \)

ASSUME just \( \frac{1}{\lambda} \) FROM JACOBIAN NO DENSITY PROFILE \( a \leq \lambda \leq b \)

NO AVERAGING OVER DIFFERENT \( \gamma \) VALUES

NO DEPARTURE FROM CIRCULAR ORBIT VELOCITY
-8-

**GET FROM THESE PARAMETERS**

<table>
<thead>
<tr>
<th>GLL-I</th>
<th>GLL-II</th>
<th>NEAR</th>
<th>CASSINI</th>
<th>ROSSETTA</th>
<th>MESSENGER</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S \nu_{\infty} \exp$</td>
<td>3.92</td>
<td>-9.6</td>
<td>13.5</td>
<td>-2</td>
<td>1.8</td>
</tr>
<tr>
<td>($\text{mm/s}^2$)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$S \nu_{\infty} \text{ model}$

|        | 3.0  | -18  | 12   | -33  | 0.7  | 3.4  |

**CAN GET QUALITATIVE SIGNS RIGHT**

**MAGNITUDES NOT GOOD**

$$
\chi^2 = \sum_{k=1}^{6} \left[ \frac{S \nu_{\infty} \exp (k) - S \nu_{\infty} \text{ model} (k)}{\sigma(k)} \right]^2 = 1.3 \times 10^5 \text{ LOUSY!}
$$

- Don't know whether a systematic search of parameter space will give a better fit
- Can make the model more elaborate
KEY THINGS NEEDED FROM FLYBY REANALYSIS

- REAL EFFECT (NEW PHYSICS)
  OR ARTIFACT?

- \( \theta_{13} \) VALUES

- \( \theta_{24} \) VALUES

\( \delta = \begin{cases} 0.01 & \text{MESSENGER} \\ 0.03 & \text{ROSETTA} \end{cases} \)

GIVES THEM A VERY BIG WEIGHT IN CHI-SQUARE FOR FIT

ALSO TO LOOK FOR,

- GLAST, PAMELA RESULTS ON DARK MATTER SEARCHES
- LHC SEARCH FOR SUPERSYMMETRY

HEAVY, SELF-ANNIHILATING SUSY DARK MATTER DOESN'T OBEY FLYBY MODEL CONSTRAINTS