Comments on Alfven propulsion (Steve Adler, IAS)

Conversion of the Drell, Foley, Ruderman formulas to SI notation:

Their equations (6), (8), (9), (10) for $V$, $P$, $I$, and $Z$ become respectively, with $\mu_0$ the vacuum permeability:

\begin{align*}
V &= v_c B_0 M \\
\frac{P}{\mu_0} &= \frac{ML v_a^2}{v_c} B_0^2 \\
I &= \frac{L}{\mu_0} v_a B_0 \\
Z &= \frac{M}{L} v_a \mu_0
\end{align*} (6-10)

I have corrected an apparent factor of 2 error in their Eqs. (8-10), since I think that the energy density should be $h^2/8\pi$ rather than $h^2/4\pi$, so I replaced the $h^2/4\pi$ in their (8) by $h^2/2\mu_0$. The SI formula of Eq. (10) above reproduces their $Z = \frac{23}{\Omega}$ in the example on page 174 of their article, since with $M \approx L$, $v_a = 2 \times 10^7m/s$, and $\mu_0 = 1.256 \times 10^{-6}mkg/(s^2A^2)$, one gets $Z = 0.25m^2kg/(s^3A^2) = 0.25\Omega$ (with $\Omega$=ohm).

For the NEAR satellite solar panels, the ratio $\frac{M}{L}$ will be smaller than 1 if $M$ is the panel width, and much smaller than 1 if $M$ is the panel thickness. Since the altitude is higher than 1600m, and since the Alfven velocity $v_a$ increases with decreasing ionic density (see the discussion following their Eq. (4)), one will have $v_a > 10^7m/s$; scaling from their example, one has

$$Z = \frac{M}{L} v_a \mu_0^2$$

since $v_a < c = 3 \times 10^8m/s$, this gives

$$12.5 \frac{M}{L} \Omega < Z < 375 \frac{M}{L} \Omega$$ (I)

For a solar cell of voltage $V$, internal series resistance $R_S$ and insulation resistance $R_I$ between the cell elements and the plasma, the power that can be developed in Alfven propulsion when $N$ cells in a square array are wired in parallel will be

$$P = \frac{V^2}{(Z/N^{1/2} + (R_S + R_I)/N)}$$ (II)

Here $Z$ is the impedance from Eq. (10) evaluated with the dimension $L$ of the individual cell; since a square array will have side length $LN^{1/2}$, the $Z$ value for the array will be $Z/N^{1/2}$. Similarly, for $N$ cells wired in parallel, the resistances $R_S$ and $R_I$ of an individual cell are divided by $N$ in an array. Note that Eq. (II) has the correct invariance property if each cell is imagined to be cut into four sub-cells wired in parallel: the cell side is decreased by a factor of 2, and so $Z$ increases by a factor of 2, but since $N$ is increased by a factor of 4, this increase in $Z$ is compensated by the factor $1/N^{1/2}$. Similarly, the resistances of each subcell are increased by a factor of 4 (one expects the series resistance and the insulation resistance, per unit area, to be constant), but this is compensated by the factor $1/N$.

To make a rough estimate, from advertisements on the internet I find typical polycrystalline solar cell parameters $V \sim 7$volt, $R_S \sim 150\Omega$, $L \sim 5cm$, and from pictures it looks like $M/L \sim 1/12.5$ as a rough order
of magnitude. Then Eq. (I) gives \( 1 \Omega < Z < 30 \Omega \). Taking the lowest \( Z \) value in this range, Eq. (II) gives an upper bound on \( P \). Assuming that \( N \approx 10^4 \), this becomes

\[
P < \frac{49}{(2.5 \times 10^{-2} + 10^{-4} R_I)^{\text{watt}}},
\]

with the insulation resistance \( R_I \) in ohms. So the possible power going into Alfven propulsion depends crucially on the quality of the solar cell insulation resistance \( R_I \). For \( R_I \approx 10^5 \) ohms, one could get 5 watts of propulsion power, of the order needed for the flyby anomalies. But good glass insulation should be much better; for a \( R_I \) of 10 megohms, one gets a bound on the Alfven propulsion power of 0.05 watt, too small to be of interest.