Symmetries Come and Go

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Common in physics
• Today’s signal is tomorrow’s background
• Today’s new deep concepts are tomorrow’s derived consequences

Examples:
• The periodic table ← Quantum Mechanics ← ???
• Kepler’s laws ← Newton’s gravity ← General relativity ← String theory?
• Many others

This talk: color and gauge symmetry
Physicists love symmetries

- Crystallography
- Lorentz
- Flavor $SU(2), SU(3)$
  - Consequence of light quarks. Quarks are deep.
- Color
  - Gauge symmetry is deep
Gauge symmetry is deep

• Largest symmetry (a group for each point in spacetime)
• Useful in making the theory manifestly Lorentz invariant, unitary and local (and hence causal)
• Appears in
  • Maxwell theory, the Standard Model
  • General Relativity
  • Many condensed matter systems
  • Deep mathematics (fiber bundles)
But

• Because of Gauss law the Hilbert space is gauge invariant. (More precisely, it is invariant under small gauge transformation; large gauge transformations are central.)

• Hence: gauge symmetry is not a symmetry.
  • It does not act on anything.

• A better phrase is gauge redundancy.
Gauge symmetry can appear trivial

• Start with an arbitrary system and consider some transformation, say a $U(1)$ phase rotation on some fields. It is not a symmetry.

• Introduce a Stueckelberg field $\phi(x)$, which transforms under the $U(1)$ by a shift.

• Next, multiply every non-invariant term by an appropriate phase $e^{i\phi(x)}$, such that the system has a local $U(1)$ gauge symmetry.

• Clearly, this is not a fundamental symmetry.
Gauge symmetries cannot break

- Not a symmetry and hence cannot break
- For spontaneous symmetry breaking we need an infinite number of degrees of freedom transforming under the symmetry. Not here.
- This is the deep reason there is no massless Goldstone boson.
- For weakly coupled systems (like Landau-Ginsburg theory of superconductivity, or the weak interactions) the language of spontaneous symmetry breaking is perfectly appropriate and extremely useful [Stueckelberg, Anderson, Brout, Englert, Higgs, ...].
Global symmetries can emerge as accidental symmetries at long distance. Then they are approximate.

Exact gauge symmetries can be emergent.
Examples of emergent gauge symmetry

• The example with the added field $\phi(x)$ above.
• Some $\sigma$-models can be described using gauge fields (e.g. the $CP^N$ $\sigma$-model) and then they become dynamical.
  – This is common in condensed matter physics.
• Simple dualities
  – In 3d a compact scalar is dual to Maxwell theory, whose gauge symmetry is emergent.
  – In 4d Maxwell theory is dual to a magnetic Maxwell theory.
Duality in interacting field theories

\( N = 4 \) supersymmetry

- This is a scale invariant theory characterized by a gauge group \( G \) and a complex coupling constant

\[ \tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2} \]

for each factor in \( G \).

- For simply laced \( G \) the theory with \( \tau \) is the same as with \( \tau + 1 \) (shift \( \theta \) by \( 2\pi \)) and the same as with \(-1/\tau\) (generating \( SL(2, \mathbb{Z}) \))...
Duality in interacting field theories
\( N = 4 \) supersymmetry

- The duality is an exact equivalence of theories.
  - Same spectrum of states
  - Same spectrum of operators
  - Same correlation functions
- \( \tau \to -1/\tau \) maps strong to week coupling.
- More technical:
  - This ignores certain global issues.
  - Some modifications for non-simply laced \( G \).
Duality in interacting field theories $\mathcal{N} = 4$ supersymmetry

- The gauge symmetry of the dual description is emergent!
- Which of the two gauge symmetries is fundamental?
- Which set of gluons are elementary?
- Perhaps neither gauge symmetry is fundamental.
Interacting gauge theories

Start at short distance with a gauge group $G$. Depending on the details we end up at long distance with:

- IR freedom – a free theory based on $G$ (same theory)
- A nontrivial fixed point. Interacting conformal field theory – no notion of particles.
- An approximately free (IR free) theory of bound states
- An empty theory – gap (possibly topological order)

All these options are realized in QCD for various numbers of flavors. (The approximately free theory is a theory of pions.)
Duality in interacting field theories

$N = 1$ supersymmetry

Here there is also a dual description based on another gauge theory with gauge group $\tilde{G}$ (magnetic theory).

- When the original theory (electric) is IR free the dual theory is strongly coupled.
- When the electric theory flows to a non-trivial fixed point so is the magnetic theory. The two theories are in the same universality class...
Duality in interacting field theories

\( N = 1 \) supersymmetry

Electric theory

\( G \)

Magnetic theory

\( \tilde{G} \)

Non-trivial IR fixed point
Duality in interacting field theories

$N = 1$ supersymmetry

A third option:

Electric theory

Based on $G$

Approximately free theory (IR free)

Based on $\tilde{G}$
Duality in interacting field theories

$N = 1$ supersymmetry

In the **UV** an asymptotically free theory based on $G$
In the **IR** an IR free theory based on $\tilde{G}$

At low energies QCD has pions. This theory has a non-Abelian gauge theory.

- The gauge fields of $\tilde{G}$ are composite.
- Their gauge symmetry is emergent.
- There is no ambiguity in the IR gauge symmetry – approximately free massless gauge fields.
Duality in interacting field theories

\( N = 1 \) supersymmetry

In all these cases

• As the original electric theory becomes more strongly coupled, the magnetic theory becomes more weakly coupled.

• When the electric theory confines the magnetic theory exhibits spontaneous gauge symmetry breaking (meaningful because it is weakly coupled).

• Clear physical demonstration of dynamical properties of gauge theories.
Many more examples of emergent gauge symmetries

- Many known examples based on different
  - gauge groups and matter representations
  - spacetime dimensions
  - amount of supersymmetry
- They exhibit rich physical phenomena.
- They lead to interesting mathematics (many applications).
- Duality and emergent gauge symmetry are ubiquitous.
Emergent general covariance and emergent spacetime

• So far we discussed duality between two field theories
  • String-string duality
    – T-duality
    – S-duality
    – U-duality
  • String-fields duality
    – Matrix models for low dimensional string theories
    – BFSS M(atrix) model
    – AdS/CFT
    – More generally gauge-gravity duality
Conclusions

• Gauge symmetries can come and go.
• They can emerge.
• It is often convenient to use them to make the description manifestly Lorentz invariant, unitary and local.
  – But there can be different such descriptions.
• Gauge symmetry is not fundamental.
• Look for a formulation of field theory that makes the duality manifest.
• We should not be surprised by duality!