General Gauge Mediation

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Based on Meade, NS and Shih
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The LHC is around the corner
What will the LHC find?

• My personal prejudice is that the LHC will discover something we have not yet thought of.

• Among the known suggestion I view supersymmetry as the most conservative and most conventional possibility for LHC physics. It is also the most concrete one.

• We need to understand:
  – How is supersymmetry broken?
  – How is the information about supersymmetry breaking mediated to the MSSM?
  – Predict the soft breaking terms.
SUSY Breaking mediation

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Minimal gauge mediation – models with messengers [Dine, Nelson, Nir, Shirman, …]

\[ W = X \Phi^2 + \ldots \]

\[ \langle X \rangle = M + \theta^2 F \]

\( X \) couples to the SUSY breaking sector. Its vev is the only source of SUSY breaking. It can be treated classically.

\( \Phi \) are messengers in a real representation of the MSSM gauge group

The messengers’ spectrum is not SUSY. Their coupling to the MSSM gauge fields feeds SUSY breaking to the rest of the MSSM.
Properties of minimal gauge mediation

- Very simple – calculable (perturbative)
- Very predictive (too predictive?)
  - Gaugino masses arise at one loop
    \[ m_{\lambda_r} \sim \alpha_r \frac{F}{M} \quad r = 1, 2, 3 \]
  - Sfermion mass squares arise at two loops (8 graphs).
    \[ m_{\tilde{f}}^2 \sim \frac{F^2}{M^2} \sum_{r=1}^{3} \alpha_r^2 c_r^2 \]
    \[ m_{\tilde{f}} \sim m_{\lambda} \sim \alpha \frac{F}{M} \]
\[ m_{\tilde{f}} \sim m_\lambda \sim \alpha \frac{F}{M} \]

- Flavor universality – no FCNC
- Colored superpartners are heavier than non-colored ones
- Relations between gaugino masses and sfermion masses
- Small A-terms
- Hard to generate \( \mu \sim B \sim m_\lambda \)
- Gravitino LSP
- Bino or stau are NLSP
- …
Original gauge mediation models – direct mediation

[Dine, Fischler, Nappi, Ovrut, Alvarez-Gaume, Claudson, Wise…]

• Start with the O’Raifeartaigh model

\[ W = X(\Phi^2 - F) + MY\Phi \]

• Let \( Y, \Phi \) be in a real representation of the MSSM gauge group. (Need to extend it to break its R-symmetry.)

• Similar to the previous case but:
  – The spontaneous SUSY breaking mechanism is manifest (explicit).
  – The messengers participate in SUSY breaking – more economical.
Direct mediation of dynamical supersymmetry breaking

- Dynamical SUSY breaking is more natural [Witten].
- Combine DSB with direct mediation [Affleck, Dine, NS].
- Messengers participate in SUSY breaking or might not even be well defined (strongly coupled messengers).
- More elegant, but:
  - Landau poles in MSSM
  - R-symmetry problem
  - Complicated models
  - Hard to compute
- These difficulties are made easier or even avoided using metastable DSB [… Intriligator, NS, Shih …].
Goals

• Understand how to couple a strongly coupled “hidden sector” to the MSSM (early work by Luty).
• Find a formalism which simultaneously deals with all gauge mediation models.
• Find general predictions of gauge mediation.
Definition: gauge mediation

- In the limit $\alpha_r \rightarrow 0$ the theory decouples to two sectors.

  - The hidden sector includes
    - the SUSY breaking sector
    - messengers if they exist
    - other particles outside the MSSM

- For small $\alpha_r$ the gauge fields of the MSSM couple to the hidden sector and communicate SUSY breaking.
The hidden sector

• It is characterized by a scale $M$.
• It is supersymmetric at short distance but breaks SUSY at long distance, of order $1/M$.

• It has a global symmetry $G$.
• A subgroup of it $H \subseteq G$ includes (part of) the MSSM gauge symmetry

$$H \subseteq SU(3) \times SU(2) \times U(1)$$

Example: $G = H = SU(3) \times SU(2) \times U(1)$

• When we include the MSSM interactions $\alpha_r \neq 0$ the two sectors are coupled via these gauge fields.
The currents

- All the hidden sector information we’ll need is captured by the global symmetry currents and their correlation functions.

- Assume for simplicity that the global symmetry is $U(1)$.
- The conserved current is in real a supermultiplet $\mathcal{J}(x, \theta, \bar{\theta})$ satisfying the conservation equation

$$D^2 \mathcal{J} = 0.$$ 

- In components

$$\mathcal{J} = J + i\theta j - i\bar{\theta} \bar{j} - \theta \sigma^\mu \bar{\theta} j_\mu + \ldots$$

The ellipses represent terms which are determined by the lower components, and

$$\partial_\mu j^\mu = 0.$$
Current correlation functions

\[ \mathcal{J} = J + i\theta j - i\bar{\theta} \bar{j} - \theta \sigma^\mu \bar{\theta} j_\mu + \ldots \]

Lorentz invariance and current conservation determine the nonzero two point functions:

\[
\begin{align*}
\langle J(p)J(-p) \rangle &= C_0(p^2) \\
\langle j_\alpha(p)\bar{j}\dot{\alpha}(-p) \rangle &= -\sigma^\mu_{\alpha\dot{\alpha}} p_\mu C_{1/2}(p^2) \\
\langle j_\mu(p)j_\nu(-p) \rangle &= -(p^2 \eta_{\mu\nu} - p_\mu p_\nu) C_1(p^2) \\
\langle j_\alpha(p)j_\beta(-p) \rangle &= \epsilon_{\alpha\beta} M B(p^2)
\end{align*}
\]
\[ \langle J(p)J(-p) \rangle = C_0(p^2) \]
\[ \langle j_\alpha(p)\bar{j}_\dot{\alpha}(-p) \rangle = -\sigma_{\alpha\dot{\alpha}}^\mu p_\mu C_{\frac{1}{2}}(p^2) \]
\[ \langle j_\mu(p)j_\nu(-p) \rangle = -(p^2 \eta_{\mu\nu} - p_\mu p_\nu)C_1(p^2) \]
\[ \langle j_\alpha(p)j_\beta(-p) \rangle = \epsilon_{\alpha\beta} MB(p^2) \]

The coefficient functions are dimensionless functions of \( p^2 \). The dimensions are fixed with the scale \( M \).

\( C_{a=0,\frac{1}{2},1}(p^2) \) are real and \( B(p^2) \) is complex.

\( B(p^2) \) breaks the R-symmetry.
Properties of the current correlators

• All the two point functions are finite in position space.
• The Fourier transform to momentum space can have logarithmic divergences (see below).
• If SUSY is unbroken, \( C_0 = C_{\frac{1}{2}} = C_1 \)
  \[ B = 0. \]
• If SUSY is broken, these relations are violated.
• But since SUSY is unbroken at short distance, they are satisfied at high momentum. Therefore,
  \[ C_a = c \log\left( \frac{\Lambda^2}{p^2} \right) + \text{finite} = c \log\left( \frac{\Lambda^2}{M^2} \right) + \text{finite} \]
  with the same \( c \) for \( a = 0, \frac{1}{2}, 1 \) and \( B = \text{finite} \).
Couple to the MSSM gauge fields

- We now turn on \( \alpha_r \neq 0 \) – we couple the hidden sector to the MSSM gauge fields.
- Assume, for simplicity, only \( U(1) \)

\[
\mathcal{L} = 2g \int d^4 \theta \mathcal{J} \mathcal{V} + \ldots \\
= g(JD + \lambda j + \bar{\lambda} \bar{j} + j^\mu V_\mu) + \ldots
\]

The ellipses represent higher order terms including contact terms which are needed for gauge invariance.

- We integrate out the hidden sector exactly, but expand to lowest order in \( g \).
Focus on a single $U(1)$

- Expanding to second order in $g$ we need the exact current two point functions in the hidden sector theory.

- $C_a(p^2)$ correct the kinetic terms of the gauge multiplets:

\[
\frac{1}{2}g^2 \left[ C_0(p^2) D^2 - C_1 \left( p^2 \right) i \lambda \sigma^\mu \partial_\mu \bar{\lambda} - \frac{1}{4} C_1(p^2) F_{\mu \nu}^2 \right].
\]

- $B(p^2)$ generates a gluino bilinear term

\[
- \frac{1}{2}g^2 M B(p^2) \lambda \lambda.
\]
Tree effects in this Lagrangian

- \( C_a = c \log(\frac{\Lambda^2}{M^2}) + \text{finite} \)
  means that the beta function jumps as we cross the threshold at \( M \)
  \[ b_{\text{high}} = b_{\text{low}} - 16\pi^2 c. \]

- The term \( -\frac{1}{2}g^2 M B \lambda \lambda \) which originates from

leads to gaugino mass

\[ m_\lambda = g^2 M B(p = 0). \]
One-loop effects

Using

$$\frac{1}{2} g^2 \left[ C_0(p^2) D^2 - C_{1/2}(p^2) i \lambda \sigma^\mu \partial_\mu \bar{\lambda} - \frac{1}{4} C_1(p^2) F^2_{\mu \nu} \right]$$

the four one-loop graphs

lead to sfermion masses (for a sfermion of charge one)

$$m^2_{\tilde{f}} = -\alpha^2 \int dp^2 \left[ C_0(p^2) - 4C_{1/2}(p^2) + 3C_1(p^2) \right] .$$
Sfermion masses

\[ m^2_{\tilde{f}} = -\alpha^2 \int dp^2 \left[ C_0(p^2) - 4C_{\frac{1}{2}}(p^2) + 3C_1(p^2) \right] \]

- The logarithmic divergences in \( C_\alpha \) cancel.
- This integral over the momentum converges. (Otherwise we would have needed counter-terms which cannot be present in a theory with spontaneous SUSY breaking.)
- The typical momentum in the integral is of order \( M \). Therefore this effect cannot be computed in the low energy theory with \( p \ll M \).
More generally, for the MSSM gauge group

- We have independent functions labeled by $r = 1, 2, 3$ for the three factors of

$$SU(3) \times SU(2) \times U(1).$$

- In order to preserve gauge coupling unification, the thresholds $C_{a}^{r}(p = 0)$ should be (approximately) independent.

- In particular, the coefficients of the logarithmic divergence, $c^{F}$, should be independent of $r$. 

• The gaugino masses

\[ m_{\lambda_r} = g^2 M B^r (p = 0) \]

are in general unrelated to each other. This fact is independent of preserving unification. (Is there a CP problem?)

• The sfermion masses

\[ m_{\tilde{f}}^2 = \sum_{r=1}^{3} \alpha_r^2 c_2^r(f) A_r \]

\[ A_r = - \int dp^2 \left[ C_0^r (p^2) - 4C_{1/2}^r (p^2) + 3C_1^r (p^2) \right] \]

depend on the Casimirs of the representation of \( f \) under the factor labeled by \( r \), and on the gauge coupling \( \alpha_r \).

• The sfermion masses are in general unrelated to the gaugino masses.
\[ m_{\tilde{f}}^2 = \sum_{r=1}^{3} \alpha_r^2 c_r^2(f) A_r \]

- All the dependence on the hidden sector is through the three real numbers \( A_r \).
- 5 sfermion masses are expressed in terms of 3 constants. Hence there must be two linear relations between them – sum rules:

\[
\text{Tr} \ (B - L)m_{\tilde{f}}^2 = 0 \\
\text{Tr} \ Y m_{\tilde{f}}^2 = 0.
\]

These are valid at the scale \( M \) and should be renormalized down.
Possible FI terms

• Consider the one point function in the hidden sector

\[ \langle J \rangle = \zeta. \]

• If it is nonzero, it leads to sfermion masses

\[ m_{\tilde{f}}^2 = g_1^2 Y_f \zeta + \ldots \]

(\( Y_f \) is the hypercharge of \( f \)) through the diagram

• This contribution is not positive definite, and might destabilize the vacuum.
\[ m_f^2 = g_1^2 Y_f \zeta \]

- \( \langle J \rangle = \zeta = 0 \) can be guaranteed with an unbroken discrete symmetry of the hidden sector which maps \( J \rightarrow -J \) (messenger parity).
The $\mu/B\mu$ problem

- Why are

$$\mu \sim B \sim m_\lambda \sim m_{\tilde{f}}?$$

- One possibility is to “put $\mu$ by hand.” This is technically natural but aesthetically unnatural.

- Alternatively, we need a direct coupling $h$ between the hidden sector and the Higgs fields.

- Therefore, we extend the definition of gauge mediation – the MSSM and the hidden sector are decoupled when

$$\alpha_r, \ h \rightarrow 0.$$
A possible couplings of the hidden sector to the Higgs fields

\[ h \int d^2 \theta A H_u H_d \]

\[ h \langle A \rangle = \mu + \theta^2 B \mu \]

- If \( A \) is a composite operator (its dimension is larger than one), then \( h \) is suppressed by an inverse power of a high scale.

- Alternatively, \( A \) can be an elementary singlet. Then we need a symmetry to prevent a large tadpole.
Alternative coupling

\[ \int d^2 \theta \left( h_u \mathcal{O}_u H_u + h_d \mathcal{O}_d H_d \right) \]

- \( \mathcal{O}_{u,d} \) can be bilinears of the elementary fields – no need for elementary singlets (no dangerous tadpoles).

- Compute \( \mu, B \) using correlation functions like \( \langle \mathcal{O}_u \mathcal{O}_d \rangle \).

- Unfortunately, typically \( \mu \sim h_u h_d M \ll B \sim M \).
  (This is the known \( \mu \) problem of gauge mediation.) However, there might be a symmetry reason or a dynamical reason ensuring \( \mu \sim B \).
Other consequences of these couplings

- The nontrivial dynamics of the hidden sector leads to multi-point functions; e.g. \[ \langle O_u O_d O_u O_d \rangle. \]

- These generate higher dimension operators beyond the MSSM like

\[
\frac{(h_u h_d)^2}{M} \int d^2 \theta (H_u H_d)^2.
\]

- Such operators can solve the little hierarchy problem – lift the Higgs mass [Dine, NS, Thomas].
Conclusions

- Formalism for dealing with *dynamical direct mediation models*
- Generic predictions of gauge mediation
  - Sfermion degeneracy (no FCNC)
  - Two mass relations
  - Small A-terms
  - $\mu/B\mu$ are challenging
  - Gravitino LSP
- Specific to models with messengers
  - Relations between gaugino masses
  - Relations between gaugino and sfermion masses
  - Large hierarchies between different sfermion masses
  - Bino or stau NLSP