Beyond the MSSM (BMSSM)

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Based on
M. Dine, N.S., and S. Thomas, to appear
Assume

• The LHC (or the Tevatron) will discover some of the particles in the MSSM.
• These include some or all of the 5 massive Higgs particles of the MSSM.
• No particle outside the MSSM will be discovered.
The Higgs potential

The generic two Higgs doublet potential depends on 13 real parameters:

- The coefficients of the three quadratic terms can be taken to be real

\[ m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_{ud}^2 (H_u H_d + c.c.). \]

- The 10 quartic terms may lead to CP violation.
- The minimum of the potential is parameterized as

\[ |\langle H_u \rangle| = v \sin \beta \]

\[ |\langle H_d \rangle| = v \cos \beta. \]
The MSSM Higgs potential

• The tree level MSSM potential depends only on the 3 coefficients of the quadratic terms. All the quartic terms are determined by the gauge couplings.

• The potential is \textbf{CP invariant}, and the spectrum is
  – a light Higgs \( h \)
  – a CP even Higgs and a CP odd Higgs \( H, A \)
  – a charged Higgs \( H^\pm \)

• It is convenient to express the 3 independent parameters in terms of \( v, m_A, \tan \beta \).

• For simplicity we take \( \tan \beta \gg 1 \) with fixed \( m_A \). Soon we will physically motivate this choice.
The tree level Higgs spectrum

\[ m_h^2 = M_Z^2 - \mathcal{O}(\cot^2 \beta) \]
\[ m_H^2 = M_A^2 + \mathcal{O}(\cot^2 \beta) \]
\[ m_{H^\pm}^2 = M_A^2 + M_W^2 \]

- The corrections to the first relation are negative, and therefore \( m_h \leq M_Z \).
- Since the quartic couplings are small, \( m_h^2 \approx M_Z^2 \ll v^2 \).

- The second relation reflects a \( U(1) \) symmetry of the potential for large \( \tan \beta \).

- The last relation is independent of \( \tan \beta \). It reflects an \( SU(2) \) custodial symmetry of the scalar potential for \( g = 0 \).
The lightest Higgs mass

- The LEPII bound
  \[ m_h \gtrsim 114 \text{GeV} \]
  already violates the first mass relation \( m_h \leq M_Z \).

- To avoid a contradiction we need both large \( \tan \beta \) and large radiative corrections.

- Intuitively, large \( \tan \beta \) means:
  - The electroweak breaking is mostly due to \( \langle H_u \rangle \approx v \).
  - The light Higgs \( h \) is predominantly from \( H_u \).
  - The four massive Higgses \( H^\pm, H, A \) are predominantly from \( H_d \).
Role of radiative corrections

- The radiative corrections depend on the two stop masses $m_{\tilde{t}_L}, m_{\tilde{t}_R}$ and on the trilinear coupling (A-term)
  \[ A_t \lambda_t \tilde{t}_L H_u \tilde{t}_R^c, \]
  where $\lambda_t$ is the top Yukawa coupling. (There is also some dependence on the bottom sector.)
- Consistency with the LEP bound is achieved either with heavy stops,
  \[ m_{\tilde{t}_L}, m_{\tilde{t}_R} \sim 600 - 1000 \text{ GeV} \]
  or with large A-terms,
  \[ A_t \sim 2m_{\tilde{t}}. \]
- Large A-terms are hard to achieve in specific models of supersymmetry breaking, and are fine tuned in the UV.
The problem with large stop mass

- With large stop mass the radiative corrections to the quadratic terms in the potential need to be fine tuned.
  - Intuitively, the superpartners make the theory natural and they should not be too heavy.
  - More quantitatively,

\[ m^2 = m_0^2 - \frac{6\lambda_t^2}{16\pi^2} (2m_\tilde{t}^2 + |A_t|^2) \ln(\Lambda/m_\tilde{t}). \]

For small A-terms and high cutoff \( \Lambda \), this amounts to roughly 1% fine tuning in the UV theory.

- This problem is known as the SUSY little hierarchy problem.
Corrections to the MSSM

• Assume that there is new physics beyond the MSSM at a scale $M$, much above the electroweak scale $\mu$ and the scale of the SUSY breaking terms $m_{SUSY}$

$$\epsilon \sim \frac{m_{SUSY}}{M} \sim \frac{\mu}{M} \ll 1$$

• The corrections to the MSSM can be parameterized by operators suppressed by inverse powers of $M$; i.e. by powers of $\epsilon$.

• The suppression of an operator is not merely by its dimension. It is by its “effective dimension” (examples below).
Leading corrections to the MSSM

- There are **only two operators** at order $\epsilon$

\[ O_1 = \frac{1}{M} \int d^2 \theta (H_u H_d)^2 \]

\[ O_2 = \frac{m_{SUSY}}{M} (H_u H_d)^2 = \frac{m_{SUSY}}{M} \int d^2 \theta \theta^2 (H_u H_d)^2 \]

- The operator $O_1$ is a higher dimension supersymmetric operator.
- The operator $O_2$ represents (hard) supersymmetry breaking.
- Both operators can lead to CP violation.
The first operator

\[ \mathcal{O}_1 = \frac{1}{M} \int d^2 \theta (H_u H_d)^2 \]

• Using the MSSM term \( \mu H_u H_d \), it corrects the scalar potential by

\[ 2\epsilon_1 (H_u^2 H_u^* H_d + H_d^2 H_d^* H_u) + c.c. \]

\[ \epsilon_1 \equiv \frac{\mu}{M} \]

• It contributes also to the charginos and neutralinos masses and to their couplings.

• Note, this operator is of dimension four but its effective dimension is five – it is suppressed by one power of \( M \).
The second operator

\[ \mathcal{O}_2 = \frac{m_{SUSY}}{M} (H_u H_d)^2 = \frac{m_{SUSY}}{M} \int d^2 \theta \theta^2 (H_u H_d)^2 \]

- It corrects only the quartic terms of the potential by

\[ \epsilon_2 (H_u H_d)^2 + c.c. \]

\[ \epsilon_2 \equiv \frac{m_{SUSY}}{M} \]

- Note, this operator is also of dimension four but effective dimension five.
Leading corrections to Higgs masses

\[ \delta m_h^2 \approx 16v^2 \cot \beta \, \text{Re} \, \epsilon_1 + \mathcal{O}(\epsilon_{1,2} \cot^2 \beta) \]

\[ \delta m_H^2 = 4v^2 \text{Re} \, \epsilon_2 + \mathcal{O}(\epsilon_{1,2} \cot \beta) \]

\[ \delta m_{H\pm}^2 = 2v^2 \text{Re} \, \epsilon_2 \]

\[ \delta m_A^2 = 0 \]

Recall, we express the masses in terms of \( m_A \).

• For large \( \tan \beta \)
  – The leading order corrections are independent of \( \epsilon_1, \text{Im} \epsilon_2 \).
  – They over-determine one real number, \( \text{Re} \, \epsilon_2 \).
  – The light Higgs mass is not corrected at leading order.
• The corrections to \( m_{H\pm} \) are independent of \( \tan \beta \).
Corrections to the light Higgs mass

The order $\epsilon$ correction to $m_h$ is suppressed for $\cot \beta \ll 1$.

Yet, we can have light stops ($\sim 300 \ GeV$) and small A-terms (hence no little hierarchy problem), and be consistent with the LEPII bound $m_h \gtrsim 114 GeV$.

This can be achieved in various ways, e.g.

- Use the order $\epsilon$ correction with $\tan \beta \sim 10$, $\epsilon_1 \gtrsim .06$.
- Continue to order $\epsilon^2$, where there are several operators leading to $\delta m_h^2 = v^2 \epsilon_3^2$ and use $\epsilon_3 \gtrsim .3$.

We conclude that the SUSY little hierarchy problem can be avoided with $M \sim 1 - 5 \ TeV$. 
What is the new physics?

It is easy to find microscopic models which lead to such new terms:

- Add an $SU(2)$ singlet (or an $SU(2)$ triplet) $S$ with couplings
  \[ \int d^2 \theta (M S^2 + S H_u H_d) \]

- Add $SU(2)$ triplets $T^\pm$ with couplings
  \[ \int d^2 \theta (M T^+ T^- + T^+ H_u^2 + T^- H_d^2) \]

- Add $U(1)'$ gauge fields
- Have a strongly coupled Higgs sector
Consequences

• The **SUSY little hierarchy problem** can be avoided by allowing corrections to the MSSM. Equivalently, the little hierarchy problem should be interpreted as a pointer to **new physics**.
  – Various existing solutions to the little hierarchy problem fit an effective action framework.

• There could be **measurable deviations from MSSM relations** at the LHC. These could point to new higher energy physics.
  – A systematic organization of the corrections in terms of operators will over-determine their coefficients (or alternatively will bound them).
An optimistic scenario

- The LHC discovers SUSY.
- A light stop (~300 GeV) is discovered, and hence there is no little hierarchy problem.
- With such a light stop the radiative corrections cannot lift the light Higgs mass to the desired value (assuming no large A-terms).
- Similarly, the (radiatively corrected) mass relations of the heavy Higgses are not satisfied.

- Hence, there must be new physics in the TeV range. It can be parameterized by our operators.
- There is a rationale for building the next machine to explore this new physics.