1. If the encounter hypothesis for planet formation is correct, roughly how many planetary systems should there be in the Galaxy? Assume that an encounter between stars with periastron distance \(< 2R_\odot\) is needed to make planets, and don’t forget gravitational focusing. You may approximate the Galaxy as a slab of \(N = 10^{11}\) solar-type stars with number density \(n = 0.1\,\text{pc}^{-3}\), mass \(1\,\text{M}_\odot\), and rms velocity \(50\,\text{km}\,\text{s}^{-1}\). Your answer need only be correct to within an order of magnitude.

2. The minimum solar nebula is usually assumed to have the surface-density distribution

\[ \Sigma(R) = \Sigma_0 \left(\frac{1\,\text{AU}}{R}\right)^k. \]

Derive estimates for \(\Sigma_0\) and \(k\) from the properties of the planets.

3. Assuming that the gaseous protoplanetary disk is isothermal, show that its density can be written in the form

\[ \rho(R, z) = \rho_0(R) \exp\left(-\frac{z^2}{2h^2}\right), \]

where \(z\) is the height above the midplane, and derive the scale height \(h\) in terms of the solar mass \(M_\odot\) and the temperature \(T\).

4. What is the approximate mean free path of a molecule in the minimum solar nebula at 1 AU? What is the geometric optical depth of the disk, assuming that 0.5% of the mass is in dust grains of density \(\rho_p = 3\,\text{g}\,\text{cm}^{-3}\) and radius \(r = 0.2\mu\)? (The geometric optical depth is the optical depth that the disk would have if the cross-section were \(\pi r^2\).)

5. If we can approximate a planet as a black-body heated by radiation from its parent star, then the planetary surface temperature should be \(T \propto r^{-b}\) where \(r\) is the distance of the planet from the star. What is the exponent \(b\)?

6. How do we know that Jupiter has a rock-ice core?