IS THE PHYSICAL METRIC A REAL NUMBER?

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REFS:

S.L.A. QUANTUM THEORY AS AN EMERGENT PHENOMENON
CAMBRIDGE, 2009

S.L.A. INCORPORATING GRAVITY INTO TRACE DYNAMICS:
THE INDUCED GRAVITATIONAL ACTION
arXiv: 1306.0482

S.L.A. & F.M. RAMAZANOĞLU
SPHERICALLY SYMMETRIC SOLUTIONS ARISING FROM
TRACE DYNAMICS MODIFICATIONS TO GRAVITATION
arXiv: 1308.1448

S.L.A. GRAVITATION AND THE NOISE NEEDED IN
OBJECTIVE REDUCTION MODELS
arXiv: 1401.0353
Trace dynamics background (Cambridge book)

- Start from a matrix theory with global unitary invariance
  - Conserved quantities:
    - Trace Hamiltonian $\mathcal{H}$
    - Operator $\mathcal{E} = \sum_{\eta B} \{ q_\eta, p_\eta \} - \sum_{\eta F} \{ q_\eta, p_\eta \}$

- Statistical mechanics
  - Canonical ensemble
    - $\rho = \mathcal{Z}^{-1} \exp \left[ - \mathcal{E} \right]$

- Equipartition (small $\tau$)
  - Effective quantum theory for averages
    - $\langle \mathcal{E} \rangle_{\mathcal{H}} = h c / \epsilon \tau$

- Frame dependence through $\mathcal{H}$
  - Rotational symmetry $\Rightarrow$ CMB rest frame
INCORPORATE A CLASSICAL BACKGROUND METRIC (1306.0482)

CORRECTIONS TO CLASSICAL GRAVITATIONAL ACTION
FROM PRE-QUANTUM MATRIX FIELDS

- ASSUME MASSLESS FIELDS ⇔ WEYL SCALING INVARIANCE
- THREE SPACE GENERAL COORDINATE INVARIANCE

\[ \Delta S = \int d^4x \sqrt{g} \left[ \bar{A} + \text{METRIC DERIVATIVE TERMS} \right] \]

- FOR ROBERTSON-WALKER COSMOLOGY, \( g_{00} = 1 \)

\[ \bar{A} \text{ TERM LOOKS LIKE COSMOLOGICAL CONSTANT ("DARK ENERGY")} \]

SO IDENTIFY \( A_0 = -\Lambda/8\pi G \) [\( (1, -1, -1, -1) \) METRIC]

- THEN CAN USE \( \Delta S \) TO STUDY EFFECTS ON OTHER GEOMETRIES
SPHERICALLY SYMMETRIC METRIC (1308.1448)

\[ S_{\text{total}} = \frac{1}{16\pi G} \int d^3x \sqrt{g} \mathcal{R} - \frac{\Lambda}{8\pi G} \int d^3x \sqrt{g} \frac{g}{g_{00}} \]

ANALYTIC AND NUMERICAL RESULTS

- \( g_{00} \) non-vanishing for finite values of the polar radius - behavior changes within \( 10^{17} \) M/M\(_{\odot}\) (cm of nominal horizon)
  so no horizon

- \( g_{00} \) in polar coordinates has a \textbf{square root branch point} near the nominal horizon
  \[ g_{00} = x + C (x - 2)^{1/2} \quad x = \Lambda^{1/2} r \]
  complex \( g_{00} \) for \( x < 2 \) cusp at \( x = 2 \)
Cusp a coordinate singularity

\[ R_{\mu\nu} R^{\mu\nu} = \frac{12 \Lambda^2}{a^4} - \frac{9 \Lambda^2}{a^2} (x-a)^{-12} + \ldots \]

\[ \frac{d}{dx} (x-a)^{-12} \propto (x-a)^{-11} \text{ singular} \]

But for \( D = \) proper distance

\[ \frac{d}{dD} (R_{\mu\nu} R^{\mu\nu}) \text{ finite at } x = a \]

Ricci scalar \( R = 0 \)

\[ g_{00} \text{ real in isotropic coordinates} \]

\[ \text{No horizon} \]

Physical singularity at cosmological distances

Artifact of static assumption?
Noise needed in objective reduction models (1401.0353 and Bell volume)

Ghirardi–Rimini–Weber–Pearle CSL model

Continuous spontaneous localization

Add anti-Hermitian noise to Hamiltonian

Two requirements

- **State vector normalization:** Unit norm preserved in time

- **No faster than light signaling:** Noise averaged density matrix has linear evolution

⇒ **Unique form**, can prove probabilities obey Born rule, Lüders rule for degenerate systems
EXPERIMENTAL CONSTRAINTS $\Rightarrow$ MASS-PORPORTIONAL NOISE COUPLING

CSL EXTENDED TO NON-WHITE NOISES

$$\frac{d|\Psi(t)|}{dt} = \left[ -iH + \sqrt{\varpi} \int d^3x \ M(x) \overline{\Phi}(\vec{x},\vec{a}) + O \right] |\Psi(t)|$$

$\uparrow$ NONLINEAR TERMS

$M(x) = \sum_{n} m_n \delta^3(x - \vec{r}_n)$

$\overline{\Phi}(\vec{x},\vec{a}) = \text{CLASSICAL NOISE FIELD}$

$\mathbb{E} [ \overline{\Phi}(\vec{x},\vec{a})] = 0 \quad \mathbb{E} [ \overline{\Phi}(\vec{x},\vec{a}) \overline{\Phi}(\vec{y},\vec{b})] = D(\vec{x} - \vec{y}, t - t')$

$\varpi = \text{COUPLING}$

$D = \text{NOISE AUTOCORRELATOR}$
WHAT IS THE PHYSICAL ORIGIN OF THE NOISE?

**Suppose**

\[ S_{\mu\nu} = \bar{g}_{\mu\nu} + \phi_{\mu\nu} \]

\( \bar{g}_{\mu\nu} \) **REAL SPACE-TIME METRIC**

\[ (ds)^2 = \bar{g}_{\mu\nu} \, dx^\mu \, dx^\nu \]

\( \phi_{\mu\nu} \) **AN IRREDUCIBLY COMPLEX FLUCTUATION**

\[ \epsilon \left[ \phi_{\mu\nu} \right] = 0 \]

\[ \epsilon \left[ \phi_{00} \left( x, t_1 \right) \phi_{00}^* \left( \tilde{x}, t_2 \right) \right] = 0 \left( x - \tilde{x}, t_1 - t_2 \right) \]

\[ \epsilon \left[ \phi_{00} \left( x, t_1 \right) \phi_{00} \left( \tilde{x}, t_2 \right) \right] = 0 \left( x - \tilde{x}, t_1 - t_2 \right) \]

\[ S_{\text{MATTER INTERACTION}} = -\frac{1}{2} \int d^5x \sqrt{\bar{g}} \, T^{\mu\nu} \phi_{\mu\nu} \]

\[ = -S_{\text{H INTERACTION}} \]
When $T^{00}$ dominates, get interaction term

$$\delta \mathcal{H}_{\text{interaction}} = \frac{1}{2} \int \sqrt{g} \ T^{00} \phi_{00}$$

$\text{Im}(\phi_{00})$ gives a coupling $\sqrt{g} \phi$ in CCL equation

This suggests:

State vector reduction is driven by

A complex number valued "spacetime foam"

Could come from a complex minimum of the gravitational effective action

$$\Gamma(S_{\text{grav}}): \quad \delta \mathcal{P} = 0 \at \eta_{\mu \nu} + \phi_{\mu \nu}$$