CHAPTER 7

THE CENTERS OF ELLIPTICAL GALAXIES

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ABSTRACT

The properties of distant quasars and the stellar kinematics of nearby galaxies independently suggest that many, perhaps most, galaxies contain central black holes of mass $10^6-10^9 M_\odot$. The distribution of black-hole masses as a function of galaxy luminosity and type, and the influence of the black holes on the structure of the central regions of galaxies are important unsolved problems.

7.1 INTRODUCTION

Most astronomers believe that quasars are active galactic nuclei (AGNs), and that the power source for AGNs is accretion onto a massive black hole (BH). The supporting arguments (Rees 1984, Blandford et al. 1991) include the high efficiency of gravitational energy release through disk accretion onto a BH compared to other power sources; the rapid variability of some AGNs, which implies a compact source; and the apparent superluminal expansion in some radio sources, implying relativistic outflow which is most naturally produced in a relativistic potential well. Moreover most other plausible power sources eventually evolve into BHs so these objects are likely to be present even if they were not the power source.

The comoving density of quasars is a strong function of redshift, declining by a factor of $10^2-10^3$ from $z = 2$ to the present (Hartwick and Schade 1990). Thus many local galaxies should contain “dead quasars”—massive central BHs that show no sign of activity because they are starved of fuel.

These simple arguments suggest several unsolved problems: Are massive black
holes present in the centers of nearby galaxies? What is the distribution of black-hole masses as a function of galaxy luminosity and type? How are the structure and dynamics of galaxies in their central regions related to the central black hole?

### 7.1.1 Black Holes and Quasars

The local energy density in quasar light is (Chokshi and Turner 1992)

\[ u = 1.3 \times 10^{-15} \text{ erg cm}^{-3}. \quad (7.1) \]

If this energy is produced by burning fuel with an assumed efficiency \( \epsilon \equiv \Delta E / (\Delta Mc^2) \), then the mean mass density of dead quasars must be at least (Soltan 1982, Chokshi and Turner 1992)

\[ \rho_* = \frac{u}{\epsilon c^2} = 2.2 \times 10^5 \left( \frac{0.1}{\epsilon} \right) \text{M}_\odot \text{Mpc}^{-3}, \quad (7.2) \]

assuming that most of the fuel is accreted onto the BH, and that the universe is homogeneous and transparent.

The mass of a dead quasar may be written

\[ M_* = \frac{L_Q \tau}{\epsilon c^2} = 7 \times 10^8 \text{M}_\odot \left( \frac{L_Q}{10^{12} \text{L}_\odot} \right) \left( \frac{\tau}{10^9 \text{y}} \right) \left( \frac{0.1}{\epsilon} \right), \quad (7.3) \]

where \( L_Q \) is the quasar luminosity and \( \tau \) is its lifetime. An upper limit to the lifetime is the evolution timescale for the quasar population as a whole, \( \sim 10^9 \text{y} \); however, upper limits to BH masses in nearby galaxies and direct estimates of the BH masses in AGNs both suggest that the typical masses of dead quasars are \( M_* = 10^7 \)–\( 10^8 \text{M}_\odot \) (Haehnelt and Rees 1993), so that equation (7.3) suggests that the lifetime of an individual quasar is only \( 10^7 \)–\( 10^8 \text{y} \).

To focus the discussion, let us adopt a “strawman” model in which a fraction \( f \) of all galaxies contain a central BH and the BH mass is proportional to the galaxy luminosity. Thus \( M_* = \Upsilon L \) where \( \Upsilon \) is the (black hole) mass to (galaxy) light ratio. The luminosity density of galaxies is \( j = 1.5 \times 10^8 \text{L}_\odot \text{Mpc}^{-3} \) in the blue band (Efstathiou et al. 1988; I assume a Hubble constant \( H_0 = 80 \text{ km s}^{-1} \text{Mpc}^{-1} \)); thus

\[ \Upsilon = \frac{\rho_*}{f j} = 0.0015 \left( \frac{0.1}{\epsilon} \right) \frac{\text{M}_\odot}{\text{L}_\odot}. \quad (7.4) \]

A second estimate of \( \Upsilon \) comes from dividing the typical dead quasar mass derived above, \( M_* \approx 10^{7.5} \text{M}_\odot \), by the typical luminosity of a bright galaxy, \( L \approx 10^{10} \text{L}_\odot \), to get \( \Upsilon \approx 10^{-2.5} \). If this estimate is to be consistent with equation (7.4) then \( f \) cannot be far from unity; in other words most or all galaxies must contain massive central BHs (Haehnelt and Rees 1993). A possible concern with ubiquitous
central BHs is the absence of significant non-stellar radiation from most nearby galaxies with claimed BHs (Fabian and Canizares 1988, Rees 1990, Kormendy and Richstone 1995); however, Narayan et al. (1995) have argued persuasively that the required low accretion efficiency is a natural consequence of advection-dominated accretion flows.

Detection of a significant sample of these exotic objects—or proof that they are not present—would enhance our understanding of both AGNs and the central regions of all galaxies.

7.1.2 The Sphere of Influence

In the remainder of this article I discuss the dynamical interactions between a massive central BH and the surrounding galaxy. Most of these interactions only require a massive dark object, which need not be a black hole. Thus the term "BH" henceforth refers to any such object although strictly we should use a different acronym such as MDO (Kormendy and Richstone 1995).

The radius \( r_h \) of the dynamical sphere of influence of a central BH is found by equating the potential energy from the BH, \( GM_*/r \), to the kinetic energy of the stars, \( \frac{3}{2} \sigma^2 \), where \( \sigma \) is the line-of-sight velocity dispersion. Neglecting factors of order unity,

\[
r_h \equiv \frac{GM_*}{\sigma^2}, \quad \theta_h \equiv \frac{r_h}{d} = 0.9 \left( \frac{M_*}{10^8 M_\odot} \right) \left( \frac{100 \text{ km s}^{-1}}{\sigma} \right)^2 \left( \frac{10 \text{ Mpc}}{d} \right),
\]

(7.5)

where \( d \) is the distance to the galaxy. It is natural to expect that the presence of a BH should be reflected in the photometric and kinematic behavior of the galaxy near \( r_h \) (Peebles 1972).

The number of galaxies in which the BH sphere of influence exceeds some limiting resolution \( \theta \) can be estimated using the Faber-Jackson law, \( \sigma(L) \approx \sigma^*(L/L^*)^{0.25} \), and the Schechter (1976) luminosity function, which states that the number of galaxies per unit volume with luminosity in the range \([L, L + dL]\) is

\[
\phi(L)dL = \phi^*(L/L^*)^\alpha \exp(-L/L^*)dL/L^*.
\]

(7.6)

Taking \( \sigma^* = 220 \text{ km s}^{-1}, \phi^* = 0.008 \text{ Mpc}^{-3}, L^* = 1.8 \times 10^{10} L_\odot, \alpha = -1.07 \) (Efstathiou et al. 1988), and assuming once again that a fraction \( f \) of all galaxies host BHs with mass \( M_* = \gamma L \) yields the estimate

\[
N(\theta_h > \theta) = \frac{4\pi}{3} f \int_0^\infty dL \phi(L) \left[ \frac{G\gamma L}{\sigma^2(L)\theta} \right]^3 \simeq 0.03 f \left( \frac{\gamma}{0.003} \right)^3 \left( \frac{1\arcsec}{\theta} \right)^3.
\]

(7.7)
Thus the number of galaxies in which the sphere of influence is resolved is a strong function of the resolution. Ground-based observations (FWHM $\gtrsim 0''3$) are expected to resolve $\theta_h$ in at best a handful of galaxies, and in most of these $\theta_h$ will be close to the resolution limit. The Hubble Space Telescope (HST; FWHM $\lesssim 0''1$) should resolve $\theta_h$ in a much larger sample, of order $10^2$ galaxies—which of course is one reason why it was built.

### 7.1.3 Cores and Cusps

Understanding the central structure of a galaxy without a central BH is a prerequisite for investigating the effects of central BHs. A modest initial assumption is that all physical variables vary smoothly near the center and hence can be expanded in Taylor series in a Cartesian coordinate system with origin at the center (as in the Sun, planets, globular clusters, etc.). Then in a spherical galaxy the luminosity density may be written $j = j_0 + j_1 r^2 + O(r^4)$—terms that are odd powers of $r$ vanish because $r = (x^2 + y^2 + z^2)^{1/2}$ is not a smooth function of the Cartesian coordinates near the center—and the surface brightness may be written

$$I(R) = I_0 + I_1 R^2 + O(R^4),$$  \hspace{1cm} (7.8)

where $r$ is the radius and $R$ is the projected radius. A galaxy satisfying (7.8) can be said to have an “analytic core” and its “core radius” $R_c$ is defined by the relation $I(R_c) = \frac{1}{2} I(0)$ (e.g., Richstone and Tremaine 1986). Note that $d \log I / d \log R \to 0$ as $R \to 0$ is not sufficient to ensure an analytic core; both the Hubble-Reynolds law $I(R) = I_0 a^2 / (R + a)^2$ and de Vaucouleurs’ law $I(R) = I_0 \exp(-kR^{1/4})$ satisfy this constraint, but have singular luminosity density as $r \to 0$ because they do not satisfy (7.8).

We shall see below that few if any galaxies have analytic cores; thus the term “core” must have a broader meaning to be useful. Following Lauer et al. (1995), I use “core” to mean a region around the center in which the surface-brightness profile slope $|d \log I / d \log R|$ is markedly smaller than at larger radii, usually less than 0.3. The transition from steep outer slope to shallow inner slope occurs at the “break radius”, which is a generalization of the core radius: the radius of maximum curvature in a $\log I$-$\log R$ plot, that is, the radius at which $|d^2 \log I(R) / d(\log R)^2|$ is maximized. The term “cusp” denotes a region in which the logarithmic slope of the surface-brightness profile is constant and non-zero at all radii exceeding the resolution limit. Cores can (and generally do) have shallow cusps.

Much of our intuition about the structure of the centers of stellar systems is based on models with analytic cores. For contrast, let us assume that the stellar density near the center varies as a power-law in radius,

$$\rho(r) = \rho_0 (r_0 / r)^k,$$  \hspace{1cm} (7.9)
where $0 \leq k < 3$ (the second constraint ensures that the enclosed mass is finite). This density distribution produces a surface-brightness cusp, $I(R) \propto R^{1-k}$, for $k > 1$, while for $0 < k < 1$ there is no surface-brightness cusp but the core is not analytic. The mass within radius $r$ is

$$M(r) = 4\pi \int_0^r r^2 \rho(r) dr = \frac{4\pi \rho_0 r_0^k r^{3-k}}{3-k}. \quad (7.10)$$

For simplicity I assume that the velocity-dispersion tensor is isotropic (although similar anisotropic models exist, and exhibit even richer behavior). The velocity dispersion $\sigma(r)$ is found by integrating the equation of hydrostatic equilibrium,

$$\frac{d}{dr} [\rho(r) \sigma^2(r)] = -\frac{GM(r)\rho(r)}{r^2}, \quad (7.11)$$

to yield

$$\sigma^2(r) = \frac{1}{\rho(r)} \int_r^{r_{\text{max}}} \frac{GM(r)\rho(r)}{r^2} dr = \frac{4\pi G \rho_0 r_0^k r^k}{3-k} \int_r^{r_{\text{max}}} r^{1-2k} dr, \quad (7.12)$$

where $r_{\text{max}} \gg r$ is a measure of the "edge" of the system.

For $k < 1$ the integral is dominated by radii near $r_{\text{max}}$ and

$$\sigma^2(r) = \frac{2\pi G \rho_0 r_0^k r_{\text{max}}^{2-k}}{(3-k)(1-k)} r^k. \quad (7.13)$$

In the limit $k \to 0$ the velocity dispersion is constant, and proportional to the square of the size of the constant-density core; this is a crude version of King's celebrated formula (Richstone and Tremaine 1986)

$$\sigma^2 = \frac{4}{9} \pi G \rho_0 R_c^2. \quad (7.14)$$

For $0 < k < 1$ the velocity dispersion decreases as $r \to 0$ but still is determined by $r_{\text{max}}$ (because the pressure $\rho \sigma^2$ is dominated by stars with apocenters near $r_{\text{max}}$, while the density is dominated by local stars).

For $k > 1$ the integral in equation (7.12) is dominated by radii near $r$ so that

$$\sigma^2(r) = \frac{2\pi G \rho_0 r_0^k r^{2-k}}{(3-k)(k-1)} = \frac{2\pi G \rho(r) r^2}{(3-k)(k-1)}. \quad (7.15)$$

In this case the velocity dispersion is determined locally (i.e. there is no dependence on $r_{\text{max}}$); the dispersion decreases as $r \to 0$ for $k < 2$ but grows for $2 < k < 3$.

This interesting behavior is described in more detail by Dehnen (1993) and Tremaine et al. (1994), who construct finite spherical systems in which the density near the center obeys equation (7.9).
7.2 Photometry

There are now over 60 elliptical galaxies and spiral bulges with HST photometry (Crane et al. 1993, Jaffe et al. 1994, Lauer et al. 1995, Faber et al. 1996). Their surface-brightness profiles can be divided into two classes:

1. "Core" galaxies exhibit a well-resolved core. The slope of the surface-brightness profile within the core is \(|d \log I/d \log R| < 0.3\) but most observed slopes are significantly different from zero; in other words few if any of the galaxies contain an analytic core and the luminosity density is growing with decreasing radius at the innermost measured point. Core galaxies are bright, \(M_V \lesssim -20\). Examples include M87 and several cD galaxies.

2. "Power-law" galaxies have no detectable core. Their surface-brightness profiles are approximate power laws, with \(d \log I/d \log R \simeq -0.8 \pm 0.3\), to the smallest resolvable radius. Power-law galaxies are generally fainter than core galaxies \((M_V \gtrsim -22)\) but their luminosity density near the center is higher (Fig. 7.1). Examples of power-law galaxies are M32 and the Galaxy.

This morphology suggests several unsolved problems:

*Do the power-law galaxies contain unresolved cores?* There is at least one (weak) argument that power-law galaxies have negligible cores. The Galactic bulge is a relatively bright \((M_V \simeq -18.3;\) Kent et al. 1991) power-law galaxy, but near-infrared maps of the Galactic center show that its core radius is only \(0.15 \pm 0.05\) pc (Eckart et al. 1993). This is much smaller than one would expect from extrapolating the core radius-luminosity correlation observed for bright galaxies. If power-law galaxies have negligible cores, why do some galaxies have cores, while others do not?

Dissipationless collapse or merging cannot increase the maximum phase-space density (Carlberg 1986, Tremaine et al. 1986); thus galaxies produced by either process must have analytic cores if the initial phase-space distribution is itself analytic. Then *why are there no analytic cores?* There are several possible explanations: (i) The initial distribution may contain dense, cold regions with very high phase-space density—perhaps compact bulges—that collect at the center of the galaxy during collapse or merger (Hernquist et al. 1993); (ii) The central density may be enhanced by gas infall and subsequent star formation, or other dissipative processes such as viscous evolution of a gaseous disk (Kormendy and Sanders 1992, Mihos and Hernquist 1994). Unfortunately, numerical simulations cannot confirm this explanation because they do not have either sufficiently high spatial resolution or reliable models of gas dynamics and star formation. (iii) There may be a central BH, in which case the velocity dispersion near the center diverges so
that high spatial density need not imply high phase-space density (see § 7.2.1 for a specific model).

Galaxy mergers are common in most models of galaxy formation and offer a natural way to form many ellipticals from disk galaxies (Toomre 1977, Wielen 1990, Barnes and Hernquist 1992). Suppose that a faint, power-law galaxy merges with a bright core galaxy. The density near the center in the faint galaxy may be 100 times higher than in the bright galaxy (Fig. 7.1). Thus tidal forces during the merger should not disrupt the central part of the fainter galaxy, which should spiral intact to the center of the bright galaxy, and remain as a dense lump in the middle of the core. Why are no such structures seen? One possible answer is that the merger rate is so low that a remnant of this kind is not expected in our sample. The merger rate probably is not strongly dependent on the mass of the smaller galaxy—there are more small galaxies but their orbital decay from dynamical friction is slower—but nevertheless is quite uncertain: (i) Toomre (1977) estimated that roughly 10%
of giant galaxies have undergone major mergers, based on the frequency of tidal tails and the assumption that most ellipticals are made by mergers; (ii) Tóth and Ostriker (1992) argue that most spiral galaxies cannot accrete more than a few percent of their disk mass without excessive thickening of the disk (this limit may be too stringent, since it neglects the excitation of bending waves in the disk which are subsequently damped by the halo); (iii) Lacey and Cole (1993) estimate that the fraction of giant galaxies that have consumed a companion is between 4% and 60% depending on the eccentricity of the companion orbit. Given these estimates, the fraction of core galaxies that have merged with a power-law galaxy could be small enough that no such systems are present in our sample, although such low rates are difficult to reconcile with the extensive observational evidence for recent mergers in ellipticals, such as kinematically decoupled cores and shells. A second possible answer to this unsolved problem is that the smaller galaxy may indeed be disrupted by time-varying tidal forces. Weinberg (1996) has argued that tides from the larger galaxy can disrupt a satellite galaxy before it merges if the secondary/primary mass

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**Figure 7.2:** Angular radius of the sphere of influence of a central BH (eq. 7.5) as a function of absolute magnitude for a sample of elliptical galaxies and spiral bulges (Faber et al. 1996). The BH mass is assumed to be \( M_\bullet = \Upsilon L \) where \( L \) is the galaxy luminosity and \( \Upsilon = 0.003 \). Open circles denote core galaxies and filled circles denote power-law galaxies.
ratio exceeds $\sim 0.01$. A third possibility, perhaps the most appealing, is that the smaller galaxy is disrupted by a central BH in the large galaxy.

We may estimate the angular radius $\theta_h$ of the BH sphere of influence (eq. 7.5) in any galaxy if, following § 7.1.1, we assume that every galaxy contains a central BH of mass $M_\bullet = \Upsilon L$, where $\Upsilon \simeq 0.003$. A dozen galaxies in the sample shown in Figure 7.2 have $\theta_h > 0.1''$ and yet in most of these galaxies there is no feature in the photometric profile that might be identified with this transition radius (an exception is M31, where the edge of the nucleus coincides with $\theta_h \simeq 0''.5$). Why is there no evidence of central BHs in the photometry? Perhaps (i) central BHs are only found in a small fraction of galaxies, or (ii) their masses are smaller than we have assumed; or, what is more likely, (iii) the response of the galaxy to a central BH does not generate a clear feature at $\theta_h$, a possibility which is discussed further in the next subsection.

### 7.2.1 The Peebles-Young Model

The Peebles-Young or adiabatic model describes the effect of a central BH on the surrounding galaxy, based on the plausible (but quite possibly wrong) assumptions that the galaxy initially has a spherical analytic core, and that the BH grows slowly compared to the characteristic orbital time of stars in the core, \( \lesssim 10^6 \) yr in (say) the central 100 pc. Thus it might apply, for example, if the BH is formed by accretion of material from a viscous disk formed at kpc scales during the initial collapse of the galaxy (e.g., Haehnelt and Rees 1993).

Given these assumptions, the predicted density distribution close to the BH is easy to derive. The stars that end up here are initially on low-energy orbits. This region of phase space has approximately constant phase-space density $f_0$ in the initial analytic core. Since phase-space density is conserved as the BH grows adiabatically, the final density of stars bound to the hole is

$$\rho(r) = \int_{E<0} f_0 \, dv = \frac{4\pi f_0}{3} v_m^3,$$  \hspace{1cm} (7.16)

where $v_m = (2GM_\bullet/r)^{1/2}$ is the escape speed from the BH. If the initial phase-space density is Maxwellian, then $f_0 = \rho_0/(2\pi\sigma^2)^{3/2}$ where $\rho_0$ is the initial central density and $\sigma$ is the one-dimensional velocity dispersion; thus (Peebles 1972)

$$\rho(r) = \frac{4\rho_0}{3\sqrt{\pi}} \left( \frac{r_h}{r} \right)^{3/2} \quad \text{when } r \ll r_h,$$  \hspace{1cm} (7.17)

which implies a cusp in surface brightness,

$$I(r) \propto r^{-\gamma} \quad \text{when } r \ll r_h,$$  \hspace{1cm} (7.18)
where $\gamma = \frac{1}{2}$. This result is more robust than its derivation, since other mechanisms of BH formation might also preserve the phase-space density in the core.

Numerical solutions for the Peebles-Young model (Young 1980, Quinlan et al. 1995) confirm that the surface brightness is accurately described by (7.18) with $\gamma = \frac{1}{2}$ at sufficiently small radii. However, the transition to this asymptotic slope is very slow: at radii $\gtrsim 0.01 R_c$, where $R_c$ is the core radius of the initial analytic core, the surface-brightness profile shows a smooth transition from the unperturbed profile at $r \gg R_c$ to an approximate power law with slope $\gamma$ at $0.01 R_c \lesssim r \lesssim R_c$. The slope $\gamma$ varies from 0 to $\frac{1}{2}$ depending on the ratio of the BH mass to the core mass; the asymptotic slope of $\frac{1}{2}$ is only approached at much smaller radii, $r \ll 0.01 R_c$. There is no clear break in the surface-brightness profile near $r_h$ unless $r_h \ll 0.01 R_c$. Since even HST resolves only a limited range inside the break or core radius (the largest break radii of nearby galaxies are $\lesssim 4''$) no sharp feature at $r_h$ should be expected, and the power-law profiles revealed by HST inside the break radius of core galaxies are therefore consistent with Peebles-Young models.

The power-law galaxies have $\gamma \simeq 0.8 \pm 0.3$, significantly greater than the asymptotic slope $\gamma = \frac{1}{2}$ predicted by the arguments above. Nevertheless even these profiles could be explained by the Peebles-Young model, in at least two ways: (i) Quinlan et al. (1995) have shown that the adiabatic growth of a central BH in galaxies with non-analytic cores can produce extended power-law surface-brightness profiles with slopes as large as $\gamma = \frac{3}{4}$; even initial cores that are almost indistinguishable in their initial surface-brightness profile from analytic cores (such as $I(R) = I_0 + I_1 R^2 + I_1' R^2 \log R + \cdots$) can have $\gamma$ as large as 1. (ii) Even if the core is analytic, power-law profiles with $\gamma \simeq 1$ can also result if the BH mass exceeds the core mass, a plausible supposition since low-luminosity galaxies should have small core masses.

Thus, if all galaxies contain central BHs, the Peebles-Young model can reproduce the main features of the photometric profiles of both core and power-law galaxies; the difference between the two types might simply reflect the relative mass of the initial core and the BH. The converse, however, is not correct: the match between the observed profiles and Peebles-Young models does not imply that massive central BHs are present, since gas dynamics and star formation may generate similar structures. To distinguish these alternatives we need high-resolution kinematic data: if the velocity-dispersion tensor is isotropic, the dispersion should rise as we approach the center if a BH is present, and fall if it is not.

The effects of slow growth of a central BH in an axisymmetric core have not been investigated; triaxial cores are discussed briefly in § 7.4.
7.3 Kinematic Evidence for Central Black Holes

The strongest evidence for a nearby massive BH comes from the Sbc galaxy NGC 4258 (Miyoshi et al. 1995, Moran et al. 1995). Water masers have been detected in an edge-on disk \( \sim 0.2 \) pc from the center of the galaxy. The disk is perpendicular to the radio jet seen at much larger distances, as is expected if the jet emerges from the axis of the disk. The rotation curve of the disk is symmetric and Keplerian; the velocities and centripetal accelerations of the masers imply a central mass of \( 3.6 \times 10^7 M_\odot \) and incidentally provide the best available distance estimate for the galaxy, \( 6.4 \pm 0.9 \) Mpc. Unfortunately, bright maser disks with the favorable geometry found in NGC 4258 are rare.

M87 is a bright, nearby AGN galaxy and has long been regarded as the best site to prospect for a massive BH. The evidence from measurements of the spatial and velocity distribution of the stars remains inconclusive (Kormendy and Richstone 1995). However, HST observations reveal a disk of ionized gas which is approximately perpendicular to the well-known optical jet and which appears to be in circular Keplerian rotation at \( \sim 20 \) pc from the center; the inferred central mass is \( M_\bullet = (2.4 \pm 0.7) \times 10^9 M_\odot \) (Harms et al. 1994). This finding supports (weakly) the Peebles-Young model, which fits the photometric profile (Lauer et al. 1992) accurately if the central BH mass \( M_\bullet = (2.6 \pm 0.5) \times 10^9 M_\odot \) (Young et al. 1978), in surprisingly good agreement with the measurement from the gas disk. However, it would be rash to invoke this agreement as an argument for the existence of massive BHs without similar comparisons for other galaxies.

Kormendy and Richstone (1995) review the strong stellar-dynamical evidence for central BHs in six other nearby galaxies: M31, M32, NGC 3115, NGC 3377, NGC 4594, and the Galaxy. They argue that there is evidence for BHs in a fraction \( f \approx 0.2 \) of the galaxies surveyed, and that the fraction of galaxies that actually contain BHs is substantially higher because detection is difficult. They suggest that BH mass correlates with luminosity of the elliptical galaxy or the spiral bulge (not the spiral disk), although the correlation is partly a selection effect—small BHs can only be detected in small galaxies. The median ratio of BH mass to the elliptical or bulge luminosity for the eight galaxies is 0.013. Thus the kinematic evidence from nearby galaxies is roughly consistent with the strawman model of § 7.1.1, which assumes that \( f \approx 1 \) and that the ratio of BH mass to total luminosity is \( \Upsilon \approx 0.003 \).

This research area should advance rapidly in the next few years. Higher resolution kinematic data on the central regions of nearby galaxies will be provided by ground-based telescopes and by the next-generation STIS on HST. High signal-to-noise spectra can yield the complete line-of-sight velocity profile rather than just the mean velocity and dispersion. Several groups are developing improved modeling techniques that generate (i) axisymmetric solutions of the collisionless
Boltzmann and Poisson equations, with (ii) distribution functions depending on all three integrals of motion, that (iii) predict the complete line-of-sight velocity profile.

A challenging unsolved problem is what is the demography of BHS in galaxies, that is, the probability \( p(M_*, L, T) \) that a galaxy of luminosity \( L \) and type \( T \) contains a central BH of mass \( M_* \). The problem is difficult because a large sample of reliable BH detections is needed, and because of strong selection effects: maser disks or emission-line disks are rare, and the detection of a BH from stellar kinematics is a strong function of the galaxy’s photometric profile, velocity dispersion, etc. For example, the existing data suggest that the BH mass is proportional to the bulge luminosity. This correlation probably reflects in part the influence of selection effects on a much broader black-hole mass distribution. A tight correlation between BH mass and bulge luminosity would be difficult to reconcile with the belief that many elliptical galaxies are formed by the merger of disk galaxies (Toomre 1977, Wielen 1990, Barnes and Hernquist 1992), since mergers can convert galactic disks to bulges without any corresponding change in the BH mass.

Finally, we stress once again the distinction between massive dark objects (MDOS) and BHS. MDOS are systems with mass-to-light ratios much higher than normal stellar populations, which may be either relativistic (i.e. BHS) or non-relativistic (e.g., clusters of neutron stars or brown dwarfs). The observations described in this section provide evidence for MDOS; we believe the MDOS are BHS only because of indirect arguments such as those given in § 7.1.1, Kormendy and Richstone (1995), and Maoz (1995).

The kinematic evidence that BHS are guilty of lurking in the centers of nearby galaxies is strong by astronomical standards but perhaps weak by legal ones (or perhaps not; Gastwirth 1992 reproduces a poll of judges showing that the confidence level they assigned to the legal standard “beyond a reasonable doubt” spanned the surprisingly low range 75–95%). Undoubtedly a sharp lawyer could persuade a jury to acquit many individual suspects, and efforts to do so are worthwhile. However, the consequences of erroneous conviction are less severe in astronomy than in law; therefore we should explore the implications of massive BHS for galactic structure whether or not the debate over their existence is fully resolved.

### 7.4 Physical Processes

In this section I review some of the physical processes that operate in the region \( r \gtrsim 1 \text{ pc} \) that is now accessible to HST in nearby galaxies.
7.4. Physical Processes

Relaxation. The high stellar densities in Figure 7.1 imply that the relaxation time from star-star encounters is relatively short near the centers of ellipticals. The relaxation times at 10 pc for the galaxies in Figure 7.1 range from $10^{14} \text{ y}$ to $10^{11.5} \text{ y}$ (Faber et al. 1996), too long to be interesting. However, as the density is generally still rising at 10 pc, relaxation and stellar collisions are likely to be important at smaller radii—they are certainly important in several nearby systems such as our own Galaxy (Phinney 1989) and M33 (Kormendy and McClure 1993).

In globular clusters, energy is transported outwards by relaxation, since the velocity dispersion $\sigma$ decreases outwards (i.e. energy flows from "hot" to "cold" regions), leading eventually to core collapse. In contrast, dynamical models of ellipticals that match the photometry (assuming spherical symmetry, isotropic velocity-dispersion tensor, and constant mass-to-light ratio) show that outside the sphere of influence of the central BH, the velocity dispersion generally increases outwards in the central region. Within the BH sphere of influence, the dispersion decreases outwards. Thus, if relaxation is important, energy is expected to flow towards the transition radius $r_h$ (eq. 7.5) from both larger and smaller radii. The consequences of this flow have not been explored, although Quinlan (1995) has examined the relaxation-driven evolution of elliptical galaxy models without central BHs.

Our understanding of relaxation in stellar systems is not necessarily complete. The usual estimate of the relaxation time, based on binary encounters between stars, has only been confirmed by N-body experiments for $N < 4 \times 10^3$ over times $< 10^3 t_{\text{dyn}}$, where $t_{\text{dyn}} = \tau/\sigma$ is the dynamical time (Farouki and Salpeter 1994). The dynamical time at 10 pc is $10^5 (100 \text{ km s}^{-1}/\sigma) \text{ y}$ so the age of the galaxy is of order $10^5 t_{\text{dyn}}$ at 10 pc; the luminosity inside 10 pc ranges from $3 \times 10^4$ to $4 \times 10^7 \text{L}_\odot$ for the galaxies in Figure 7.1, so the effective value of $N$ is $10^5$ to $10^6$; these values are far outside the range in which our concepts of relaxation have been tested. Other relaxation mechanisms may operate more quickly than binary encounters in some cases: (i) Weinberg (1993) has demonstrated that collective interactions can enhance the relaxation rate from binary encounters by a substantial factor, although he was not able to estimate the enhancement factor for realistic stellar systems. (ii) Angular-momentum relaxation may be enhanced in potentials where two or more of the fundamental frequencies are nearly degenerate, such as the near-Kepler potential close to a central BH (Ostriker 1974, Rauch and Tremaine 1996).

Should the relaxation time be shorter than the age of the galaxy, it can be shown that the stellar surface brightness near the central BH should have a power-law cusp, described by equation (7.18) with $\gamma = \frac{3}{4}$ (Bahcall and Wolf 1976).
**Globular Clusters.** The relaxation rate of stars of mass \( m_\ast \) due to a population of objects with number density \( n \) and mass \( m \gtrsim m_\ast \) is proportional to \( nm^2 \). In an elliptical galaxy, the number density of globular clusters relative to stars is \( n_{gc}/n_\ast \sim 10^{-7.5} \) (Harris 1991), and the typical cluster mass is \( m_{gc} \sim 10^5 m_\ast \). Thus relaxation from globular cluster encounters is more important than relaxation from stellar encounters, by a factor of \( \sim 10^{2.5} \). Over most of a typical galaxy, neither process is significant, but near the center star-globular encounters could be important. The encounters heat up and isotropize the stellar population, while draining energy from the globular cluster orbits (dynamical friction). The clusters spiral in to the center of the galaxy where they are disrupted by tidal forces (from the stellar distribution, a central \( \rm{BH} \), and other clusters). Tremaine et al.'s (1975) suggestion that the nucleus of M31 was formed by cluster inspiral fails to explain its rapid rotation (van den Bergh 1991) and high metallicity.

A major uncertainty is the number of globulars near the center. In most galaxies, the surface number density of globulars is flat at radii \( \lesssim 2 \) kpc, while the surface density of stars continues to rise to much smaller radii; thus at radii \( \lesssim 1 \) kpc the spatial density of globulars is probably flat or even decreasing—perhaps to zero near the center. This deficit may be primordial or may arise through a dynamical process that preferentially destroys clusters at small radii; in the latter case the clusters may have an important influence on the galactic center before they disappear. The most promising dynamical process is tidal disruption of clusters on stochastic orbits that pass too close to a central \( \rm{BH} \) (Ostriker et al. 1989).

**Triaxiality.** Dissipationless collapse usually produces triaxial stellar systems and observations such as minor-axis rotation suggest that elliptical galaxies may be triaxial (de Zeeuw and Franx 1991)—although the central regions are probably less so than the outer parts because of gas infall and dissipation. A central \( \rm{BH} \) or density cusp has important consequences for triaxial models (Norman et al. 1985, Gerhard and Binney 1985). Regular box orbits are the "backbone" that supports triaxiality; these orbits pass arbitrarily close to the center and if they are scattered by a central feature they become chaotic so the backbone dissolves. More precisely, the family of regular box orbits is replaced by centrifilphilic stochastic orbits and centrophobic regular "boxlets" (Miralda-Escudé and Schwarzschild 1989), both of which are less effective supports for triaxiality. Merritt and Fridman (1995) have extended Gerhard and Binney's two-dimensional orbit calculations to three dimensions and find that even weak cusps can destroy strongly triaxial models over the lifetime of a galaxy (\( \sim 10^9 \) dynamical times at 100 pc). Thus power-law galaxies, and perhaps many core galaxies, are likely to be axisymmetric near their centers.
Off-Center Structures. The nucleus of the nearby spiral galaxy M31 is offset from the kinematic and photometric center of the bulge. The offset was revealed by the Stratoscope II balloon-borne telescope (Light et al. 1974), but a detailed picture of the nucleus was only obtained two decades later by HST, which showed that the nucleus contains two separate components, separated by 0\'\'49 (Lauer et al. 1993). The component with the lower surface brightness coincides with the center of the bulge, while the brighter, off-center component is the nuclear core measured by Stratoscope. The offset is unlikely to be an artifact of irregular dust obscuration as there are no color gradients or far-infrared emission; the nucleus is unlikely to be binary since the binary orbit would decay by dynamical friction in $\lesssim 10^8$ y. A more promising possibility is that the nucleus contains an eccentric stellar disk orbiting a central BH (Tremaine 1995); the dynamics and evolution of such disks is another largely unsolved problem.

Lauer et al. (1995) find that roughly 15% of elliptical galaxies and spiral bulges observed with HST show lopsided structure (i.e. the bright isophotes do not share a common center). Most of these are core galaxies, which is perhaps not surprising since the restoring force $GM(r)/r^2$ approaches zero near the center in galaxies with relatively flat cores ($k < 1$ in the notation of eq. (7.9)), but diverges near the center when the density profile is steep.

Possible explanations for lopsided structure include (i) irregular dust obscuration; (ii) an eccentric stellar disk surrounding a central BH; (iii) a binary BH, formed by the merger of two galaxies containing central BHs (Begelman et al. 1980) or a steady trickle of BHs from the galactic halo (Xu and Ostriker 1994); (iv) a collective oscillation of the central part of the galaxy. A result that bears on (iv) is Weinberg’s (1991) discovery that lopsided modes in spherical stellar systems with flat cores are very weakly damped; thus if an arbitrary spectrum of modes is excited the lopsided modes will persist much longer than the others.

An unsolved problem is how far does a “central” BH wander from the center of the galaxy? Dynamical friction drags orbiting BHs towards the center but when does the inspiral stop? This issue was addressed by Bahcall and Wolf (1976), who estimated the equilibrium separation of a BH from the center of a globular cluster with an analytic core. The BH is in thermal equilibrium with the core stars, which have “temperature” $kT = m\sigma^2/2$, where $m$ is the stellar mass and $\sigma$ is the one-dimensional velocity dispersion (which is constant within the core). In the center of a spherical core with constant density $\rho_0$, the potential is $U(r) = \frac{2}{3}\pi G \rho_0 r^2$; equipartition implies that $M_* \langle U(r) \rangle = \frac{3}{2} kT$, so that the mean-square separation of the BH from the center is

$$\langle r^2 \rangle = \frac{9m\sigma^2}{4\pi G \rho_0 M_*} = \frac{m}{M_*} R_e^2,$$  

(7.19)
where $R_c$ is the core radius and we have used King's formula (7.14). This simple argument does not apply to galaxies since they do not have flat cores with constant velocity dispersion. The Brownian motion of BHs near the centers of more realistic galaxy models has been investigated by Quinlan (1995), who finds quite different scaling laws.

Progress in resolving many of these unsolved problems will be driven by large-scale $N$-body simulations, which are rapidly advancing in power and sophistication. Sigurdsson et al. (1995) now run simulations with $N = 10^6$--$10^8$ on massively parallel machines, using a self-consistent field method—which unfortunately cannot follow relaxation-driven evolution because it does not compute forces between individual stars. Simulations that actually calculate the force between every star pair are slower, but now benefit from the special-purpose GRAPE computers (Ebisuzaki et al. 1993), which use hard-wired parallel pipelines to speed up the force calculations.

7.5 Summary

Over the past twenty years, most of the discussion of BHs in the centers of elliptical galaxies has focused on whether or not they exist. This issue has become less controversial as the evidence for BHs accumulates. Some of the important remaining unsolved problems include:

- What is the distribution of BH masses as a function of galaxy luminosity and type? In particular, why does the BH mass appear to be roughly proportional to the bulge luminosity?

- What determines the distribution of stars near the center of an elliptical galaxy? In particular, why are there no analytic cores? Do the power-law galaxies contain unresolved cores? If not, why do some galaxies have cores, while others do not? How are these questions related to the presence of a central BH?

- Is there any reliable signature of a central BH in the stellar photometry?

- Under what conditions do eccentric stellar disks form and persist around central BHs? Are such disks the explanation for the double nuclei and lopsided structures seen at the centers of some galaxies?

- How do the low-density cores in giant galaxies survive mergers with small dense companion galaxies?
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BIBLIOGRAPHIC NOTES

The following references provide an introduction to many of the issues discussed in this article.


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