The Role of Magnetic Reconnection in Angular Momentum Transport

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with Jeremy Goodman

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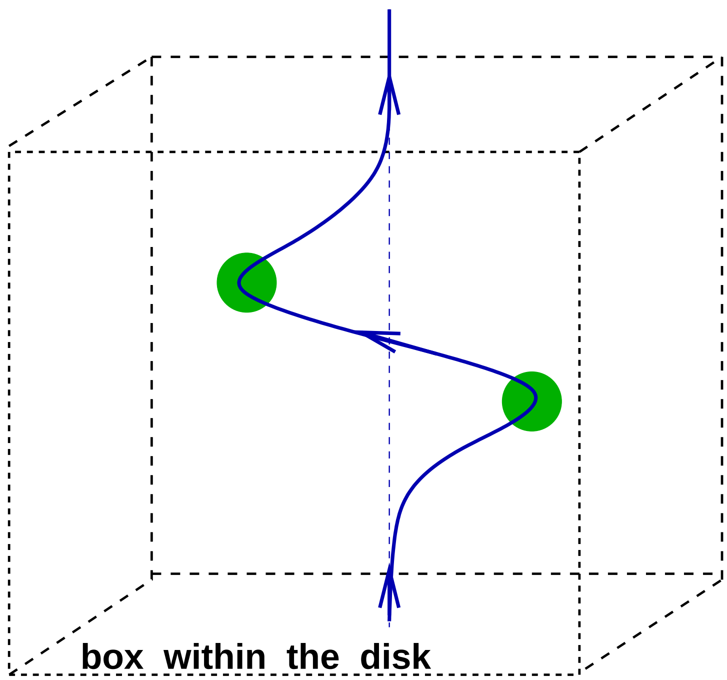
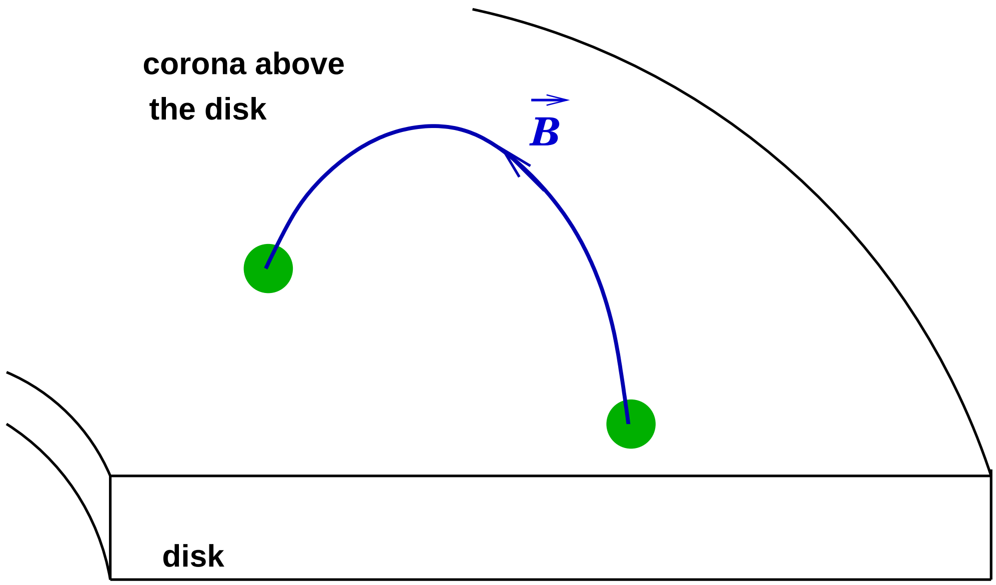
Disk Magneto-Rotational Instability

Disk MRI:



MRI mechanism: angular momentum transfer (AMT) by magnetic fields **within the disk**.

Coronal MRI



Angular momentum transfer by coronal magnetic loops.

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Coronal vs. Disk MRI

Q: Which mechanism (disk MRI or coronal MRI) is the **D**ominant **A**ngular **M**omentum **T**ransport **P**rocess (**DAMTP :)**)

- Magnetic stress in the disk:

$$G_{\text{disk}} \sim 2\pi r \ < B_r B_\phi >_{\text{disk}} H \sim B_{\text{disk}}^2 H$$

- Magnetic stress in the corona:

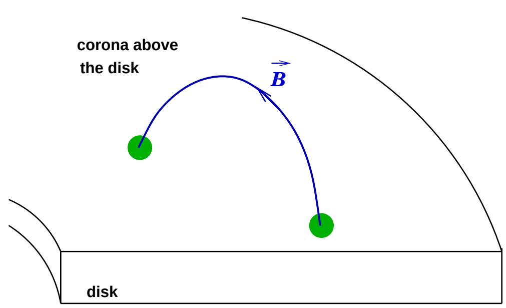
$$G_{\text{cor}} \sim 2\pi r \ < B_r B_\phi >_{\text{cor}} H_B \sim B_{\text{cor}}^2 H_B$$

- Thus,

$$\frac{G_{\text{cor}}}{G_{\text{disk}}} \sim \frac{B_{\text{cor}}^2}{B_{\text{disk}}^2} \frac{H_B}{H}$$

- Two AMT ingredients:
  (1) Characteristic magnetic field strength (B_{cor} or B_{disk});
  (2) Magnetic scale height (H_B or H).

Coronal MRI dominates if $H_B \gg H$, while $B_{\text{cor}} \sim B_{\text{disk}}$.



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Coronal Magnetic Field B_{cor}:

Flux Emergence from MRI-active disk

- Typical coronal magnetic field strength, B_{cor}, and hence the overall coronal power and torque depend on the normalization of $F(L, \theta)$.

- This, in turn, depends on the rate and form of buoyant **flux emergence** from the disk.

- 3D MHD simulations of MRI-turbulent stratified disks
  (e.g., Brandenburg et al. 1995; Stone et al. 1996; Miller & Stone 2000; Hirose et al. 2007; Blaes et al. 2007)

- **Key Question:**
  What controls flux emergence in a stratified MRI-turbulent disk ?

- **Suggestion:**
  Competition between **Parker** and other secondary instabilities.

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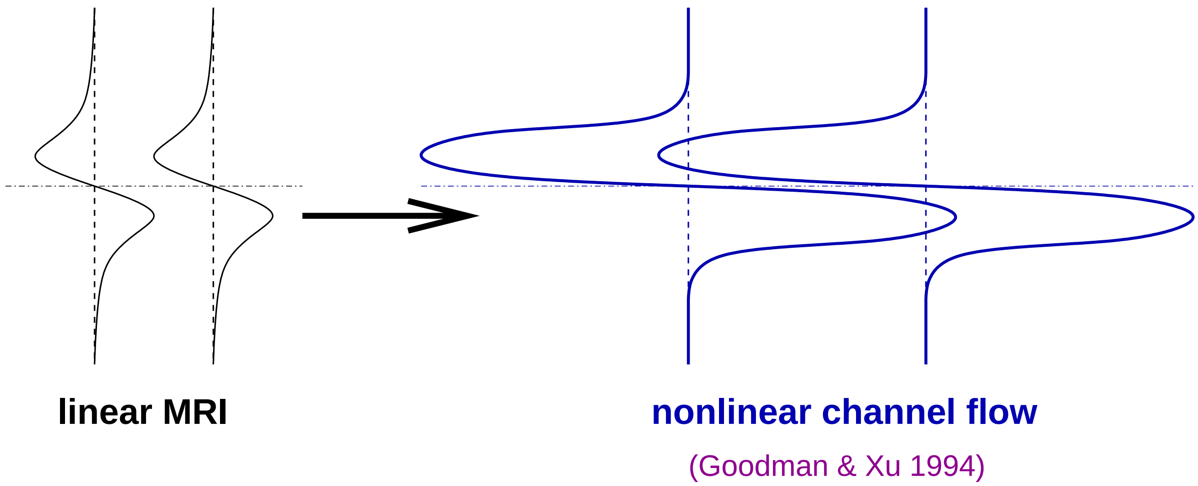
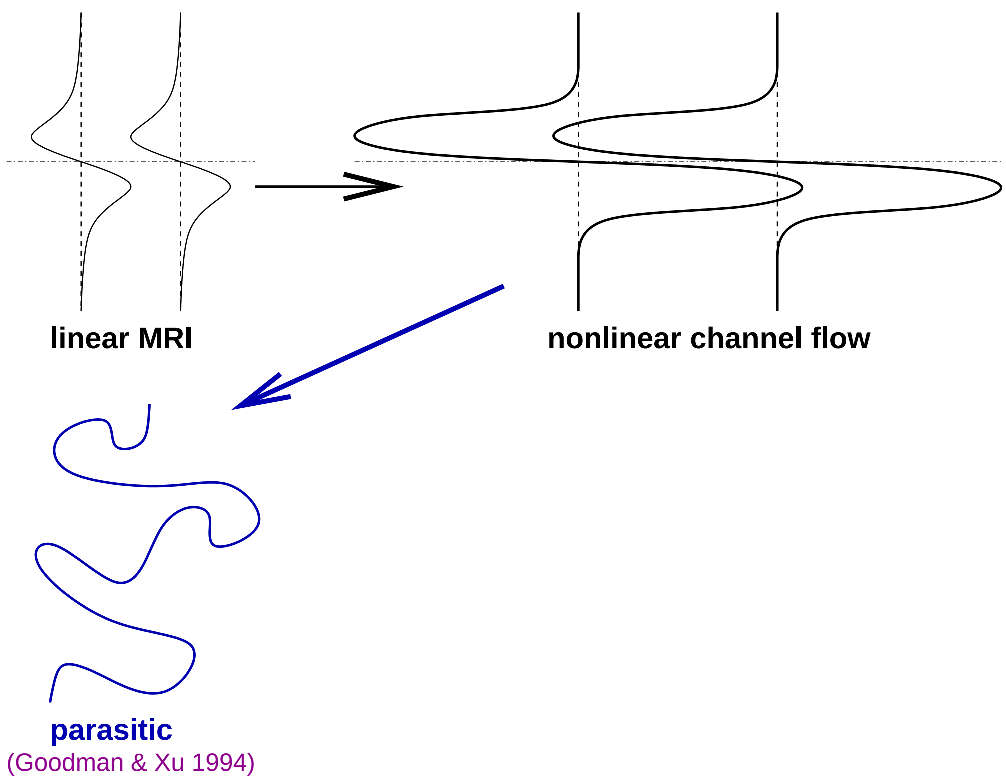
Nonlinear Fate of MRI Channel Flows

- Non-linear development of MRI: channel flows — exact nonlinear MHD solutions *(Goodman & Xu 1994)*.



Nonlinear Fate of MRI Channel Flows

- Non-linear development of MRI: channel flows — exact nonlinear MHD solutions
- Eventual fate of channel flows: disruption by secondary instabilies:
  - parasitic instabilities (e.g., KH) of *Goodman & Xu (1994)*, $\Rightarrow$ 3D MHD turbulent cascade;



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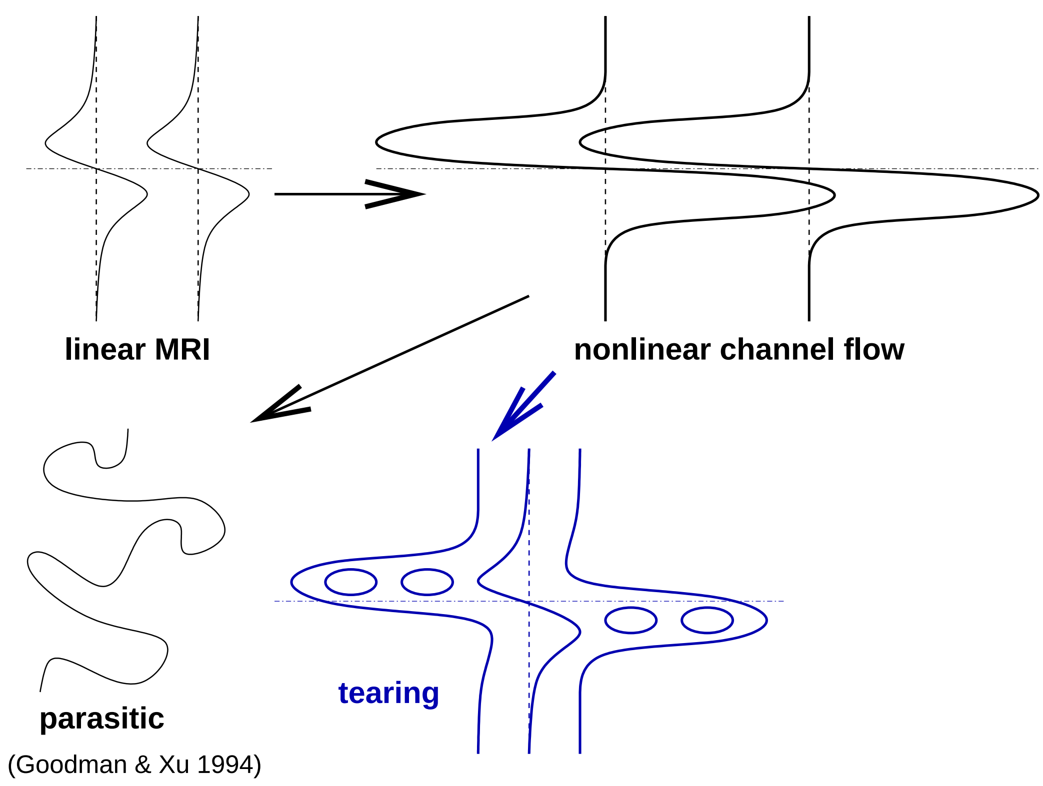
  - resistivity $\Rightarrow$ tearing mode;



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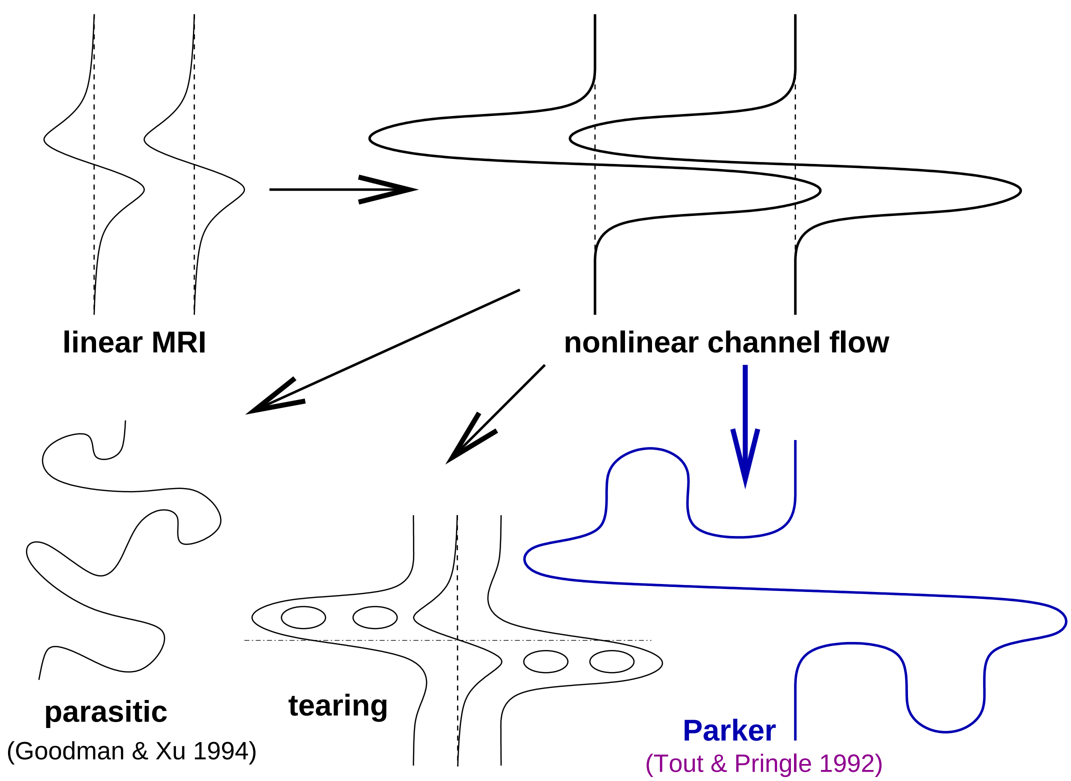
  - resistivity ⇒ tearing mode;
  - gravitational stratification ⇒ Parker instability ⇒ **flux emergence ⇒ magnetized corona.**



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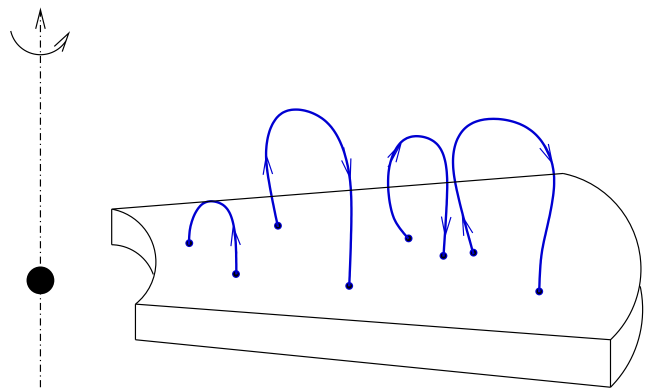
Coronal Magnetic Scale Height



- What is the characteristic **magnetic scale-height** H_B ?

<u>*More generally:*</u>

- what is the distribution of magnetic energy density $\bar{B}^2(z)/8\pi$ with height z above the disk ? (for $H \ll z \ll R$)
- If there is a characteristic magnetic scale H_B, how large is it compared with H and R ? What determines it ?
- Or, if $\bar{B}^2(z)/8\pi$ is a power law, what is the power-law exponent?
- What is the distribution of **magnetic dissipation** with height ?

<u>*Even more generally:*</u>

- What is the distribution F of coronal magnetic loops in sizes, field strengths, etc. ? Is there **inverse cascade** of coronal loops ?

- How does $\bar{B}^2(z)/8\pi$ control **coronal AMT ?**
- What is the role of **magnetic reconnection ?**

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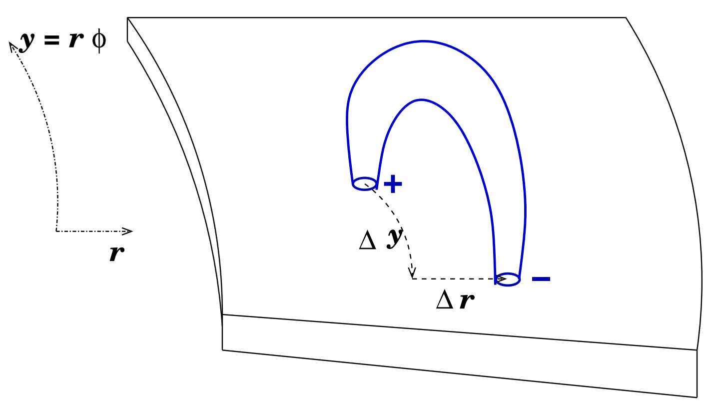
APPROACH

Uzdensky & Goodman 2008; arXiv:0803.0337

- **Goal:** build statistical description of force-free coronal magnetic field above turbulent accretion disk.

- **Program** *(see also Tout & Pringle 1996)*:

  - Represent corona by ensemble of elementary magnetic loops characterized by footpoint separations: Δr, $\Delta y = r\Delta\phi$.
  - Introduce **loop distribution function $F(\Delta r, \Delta y)$.**
  - Derive equations of motion for loops in this parameter space.
  - Derive the **Loop Kinetic Equation** for F.
  - Obtain a **Statistical Steady State**.



- **Scales of Interest:**

  - temporal: $\Omega^{-1} \ll \tau \ll T_{\rm accr}$
  - spatial: $H \ll l \ll R$

PHYSICS OF THE ADC

Processes governing individual loop evolution:

- stressing:

(increasing magnetic energy and magnetic scale-height H_B)

- **emergence** (and submergence) of loops into corona
- stretching by **Keplerian shear**
- **random footpoint walk** due to disk turbulence

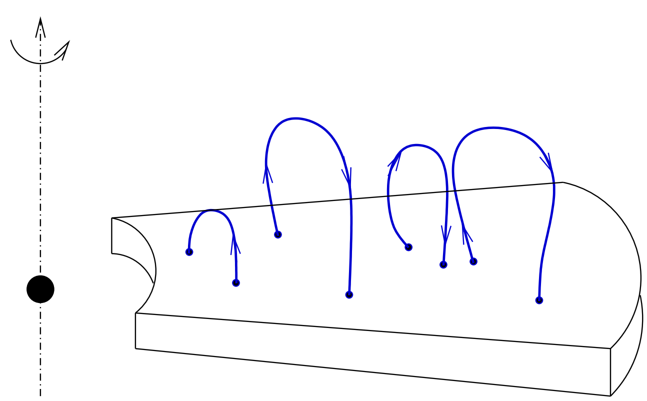
- relaxation:

(dissipating magnetic energy and bringing the field closer to potential)

- **reconnection** between loops (manifested as flares)
  $\Rightarrow$ **inverse cascade:** formation of larger structures.
  (Tout & Pringle 1996)

Overall, a magnetically-active corona can be described as

A BOILING MAGNETIC FOAM !



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THE LOOP KINETIC EQUATION

- Final Result: **Loop Kinetic Equation** for the distribution function F:

$$\frac{\partial F}{\partial t}(\mathcal{A},t) = \underbrace{S(\mathcal{A})}_{(1)} + \underbrace{(D_r \frac{\partial^2}{\partial \Delta r^2} + D_y \frac{\partial^2}{\partial \Delta y^2})\, F(\mathcal{A})}_{(2)}$$

$$- \underbrace{\frac{\partial}{\partial \Delta y}(F \dot{\Delta y})}_{(3)} + \underbrace{\dot{F}_{\text{rec}}(\mathcal{A})}_{(4)}$$

- (1) **Flux Emergence** ⇒ source term $S(\mathcal{A})$
- (2) **Random footpoint motions** ⇒ diffusion term
- (3) **Keplerian shear** ⇒ advection term $-\partial_{\Delta y}(F\dot{\Delta y})$, with $\dot{\Delta y} = -3/2\,\Omega\,\Delta r$.
- (4) **Reconnection** between loops ⇒ $\dot{F}_{\text{rec}}$

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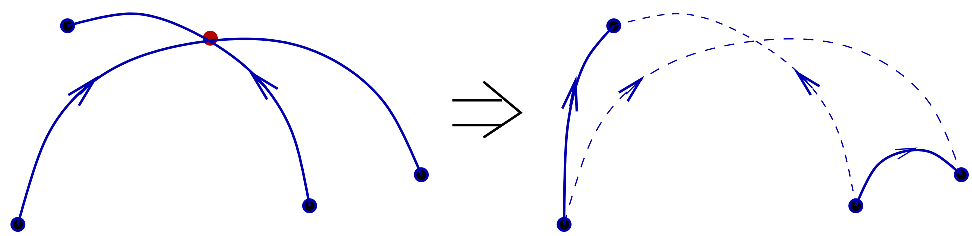
Reconnection as a Collision Integral

- Reconnection acts as a **binary collisions** between particles in a gas:



- Footpoint positions are preserved during interaction $\Rightarrow$ **reconnection rules** (similar to conservation laws).

- Reconnection is represented by a non-linear **integral operator** $\sim$ collision integral in Boltzmann's eqn: *(c.f., Tout & Pringle 1996)*

$$\dot{F}_{\rm rec}(\mathcal{A}) = \dot{F}_{{\rm coll},-}(\mathcal{A}) + \dot{F}_{{\rm coll},+}(\mathcal{A})$$

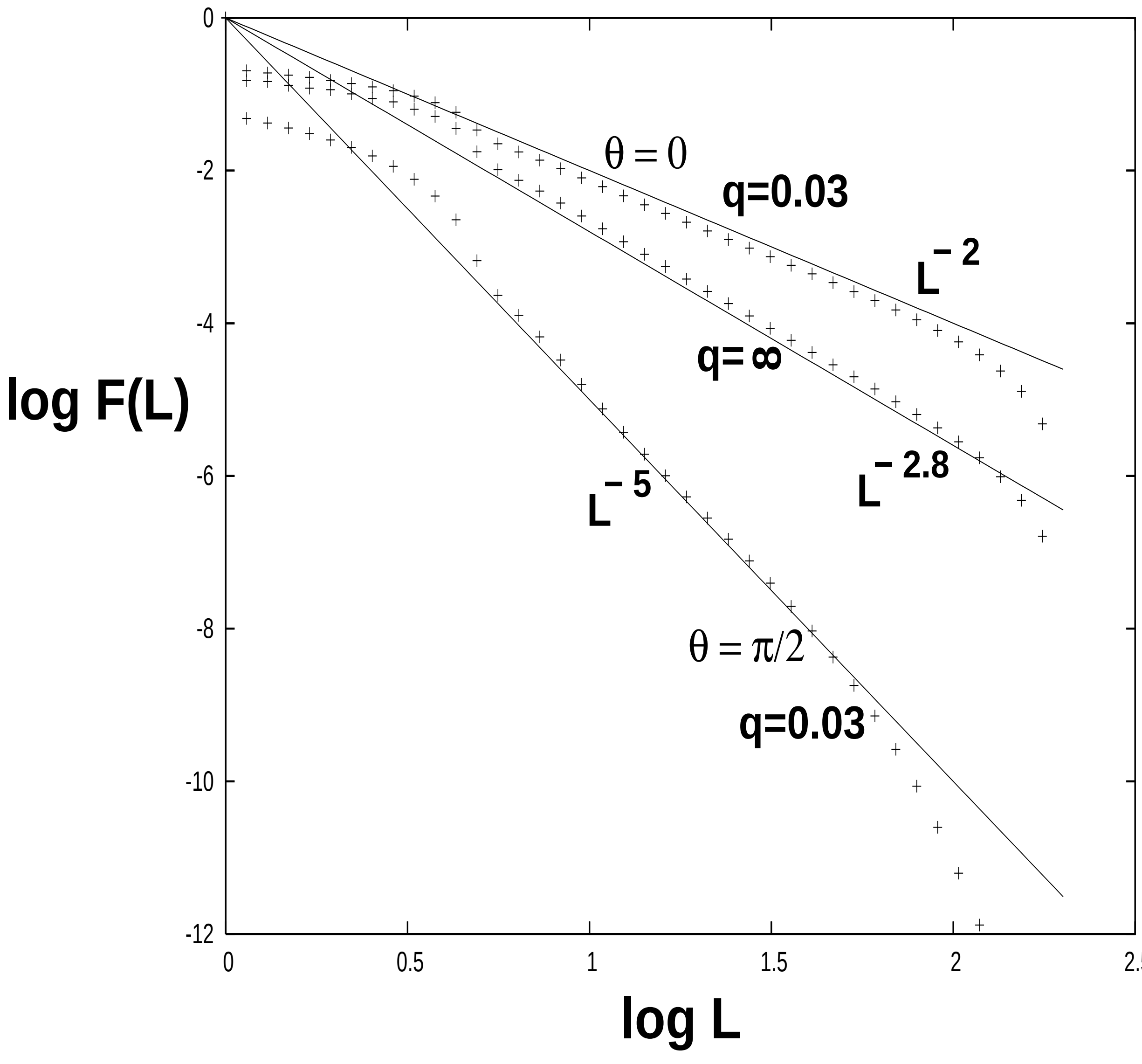
$$\begin{aligned} \dot{F}_{{\rm coll},-}(\mathcal{A}) &= -\int d\mathcal{B}\, Q_{\mathcal{AB}}\, F(\mathcal{A})\, F(\mathcal{B}) \\ \dot{F}_{{\rm coll},+}(\mathcal{A}) &= \frac{1}{2}\int\int d\mathcal{C}\, d\mathcal{D}\, Q_{\mathcal{CD}\to\mathcal{A}}\, F(\mathcal{C})\, F(\mathcal{D}) \end{aligned}$$

$Q_{\mathcal{AB}} = q\,\Omega\,\sigma_{AB}$ is the reconnection-event rate.

Results: Statistical Steady State

projected loop length: $L = \sqrt{\Delta r^2 + \Delta y^2}$
orientation angle with respect to ϕ-direction: θ
$q = \infty$ – no Keplerian shear; $q \ll 1$ – strong shear

(semi-circular loops)



$$F(L, \theta) \sim L^{-\alpha(\theta)}$$

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CORONA ENERGETICS

- Energy associated with a single loop:

$$\mathcal{E}(L) = 2\,E_{\rm magn}(L) = \frac{\Delta\Psi}{4\pi}\int\limits_0^L B_x(z=0;L')\,dL'$$
$$= \frac{\Delta\Psi}{4\pi}\int\limits_0^L B_{\rm top}(L')\,dL'$$
$$= \frac{\Delta\Psi^2}{4}\int\limits_0^L\int\limits_{L'}^\infty F(L'')\,dL''\,dL'\;.$$

- Total magnetic energy in the corona:

$$E_{\rm tot} = \frac{1}{2}\int\limits_0^\infty \bar{F}(L)\,\mathcal{E}(L)\,dL$$
$$= \frac{\Delta\Psi^2}{4}\int\limits_0^\infty dL\int\limits_0^L dL'\;L'\,\bar{F}(L)\,\bar{F}(L')\,.$$

- Total magnetic torque per unit disk area:

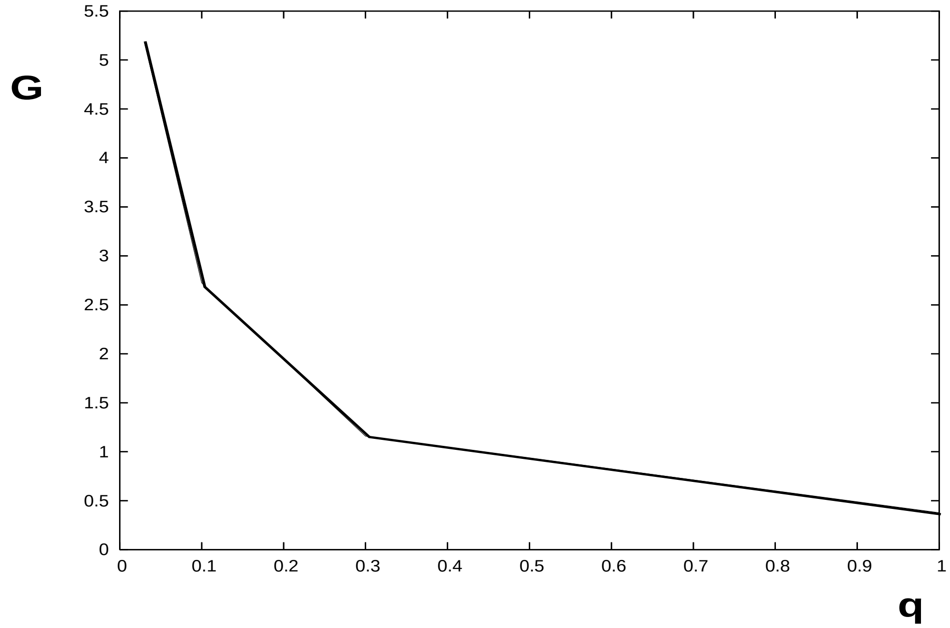
$$G = -\frac{\Delta\Psi B_0}{8\pi}\int\int dL\,d\theta\;F(L,\theta)\,b_{\rm top}(L)\;L\,\sin 2\theta\,.$$

- Energetics of individual reconnection events:

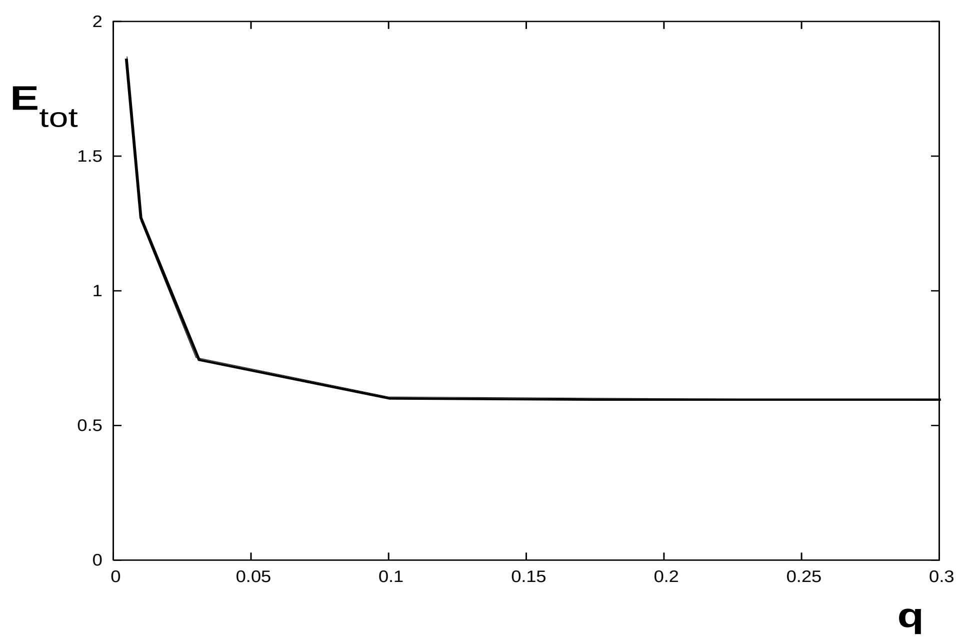
$$E_{\sf flare}^{\mathcal{A}+\mathcal{B}\rightarrow\mathcal{C}+\mathcal{D}} = E(\mathcal{A}) + E(\mathcal{B}) - E(\mathcal{C}) - E(\mathcal{D})$$

Results: Magnetic Energy and Torque

Coronal Angular Momentum Transport:



Total Coronal Magnetic Energy:



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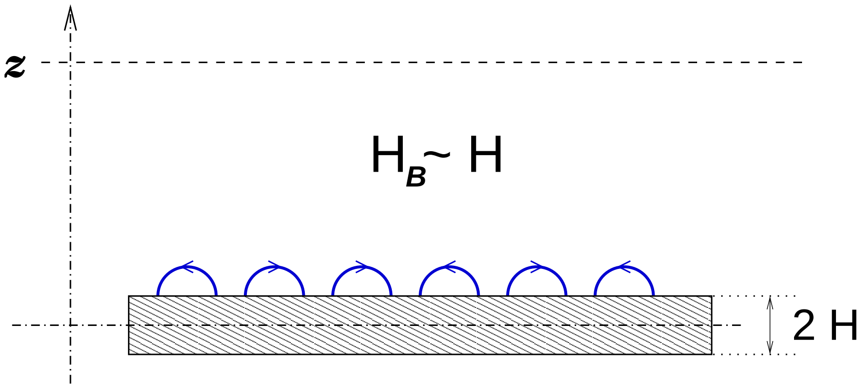
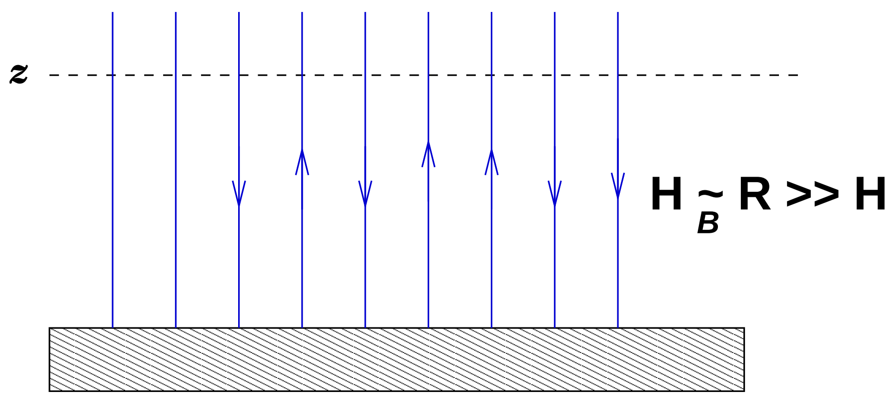
ROLE of RECONNECTION in ADC



<u>Reconnection is too efficient:</u>
magnetic field is close to potential, $E_{\rm magn} \simeq E_{\rm pot}$; free magnetic energy and magnetic dissipation are small.



<u>Reconnection is forbidden:</u>
forrest of open field lines, $E_{\rm magn} \simeq E_{\rm open} \gg E_{\rm pot}$; free magnetic energy is large but dissipation is not allowed.

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SUMMARY

- Magnetic field in Accretion Disk Corona (ADC) contributes to Angular Momentum Transfer **(Coronal MRI)**, especially if $H_B \gg H$.

- **Magnetic flux emergence** driven by magnetic buoyancy/Parker instability in a stratified disk leads to the formation of ADC.

- Rate of flux emergence is set by competition between **Parker instability** and other **parasitic instabilities** feed by the primary nonlinear MRI mode.

- ADC as a **boiling magnetic foam.**

- **Statistical description** of ADC magnetic field: ensemble of loops, Loop Kinetic Equation ...

- **Magnetic reconnection** controls $\bar{B}^2(z)/8\pi$, coronal magnetic energy and torque.