Energy Flow and Dissipation in MRI Turbulence

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Introduction

The last 17 years has seen enormous progress in understanding the MRI induced stresses that drive accretion in magnetized disks. The outward angular momentum transport resulting from these stresses has been shown to be very robust.

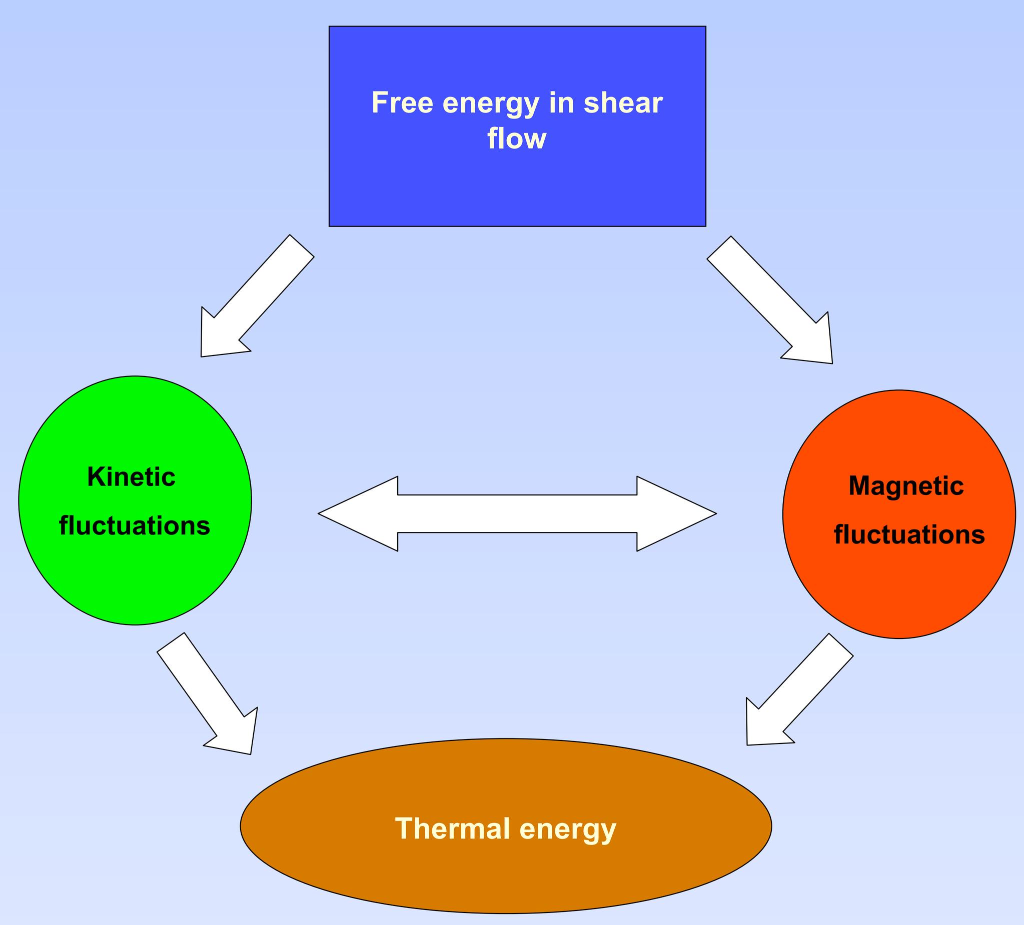
One important next step in accretion disk theory is to understand how accretion energy, which is tapped from differential rotation, ultimately ends up as heat. Such studies could begin to bridge the gap between theory and observation since dissipational heating will influence disk structure and radiative effects.

To begin our study of MRI energetics, we aim to answer these questions:

• The "alpha" model assumes that the accretion energy is dissipated into heat locally and rapidly. To what degree is this assumption valid?

• What path does energy take is it moves from differential rotation to kinetic and magnetic fluctuations to thermal energy?

• What type of dissipation dominates – magnetic versus kinetic?



Method

We solve the equations of ideal magnetohydrodynamics in the shearing box domain using the Athena code. Athena conserves total energy, relying on "numerical dissipation" to thermalize turbulent energy.

Simulations

• 4 different resolutions: N_x x N_y x N_z = 16 x 32 x 16 to 128 x 256 x 128 in factors of 2

• Domain size: L_x = 1, L_y = 2π, L_z = 1

• 2 different initial field geometries: B_z = (2P/_)$^{1/2}$ and B_z = (2P/_)$^{1/2}$ sin(2π/L_x x)

• Initial parameters: _ = 1600, P = 10^{-6}, _ = 0.001, _ = 1, _ = 5/3

Analysis 1

Calculate evolution of volume-averaged energy densities.

$$\frac{\partial \langle E_{\rm tot}\rangle}{\partial t} = \frac{q\Omega}{L_y L_z}\int_X (\rho v_x \delta v_y - B_x B_y)\, dy\, dz$$

Stress at boundaries injects energy

$$E_{\rm tot} = \epsilon + \frac{1}{2}\rho v^2 + \frac{1}{2}B^2 + \rho\Phi$$

Energy eventually ends up as thermal

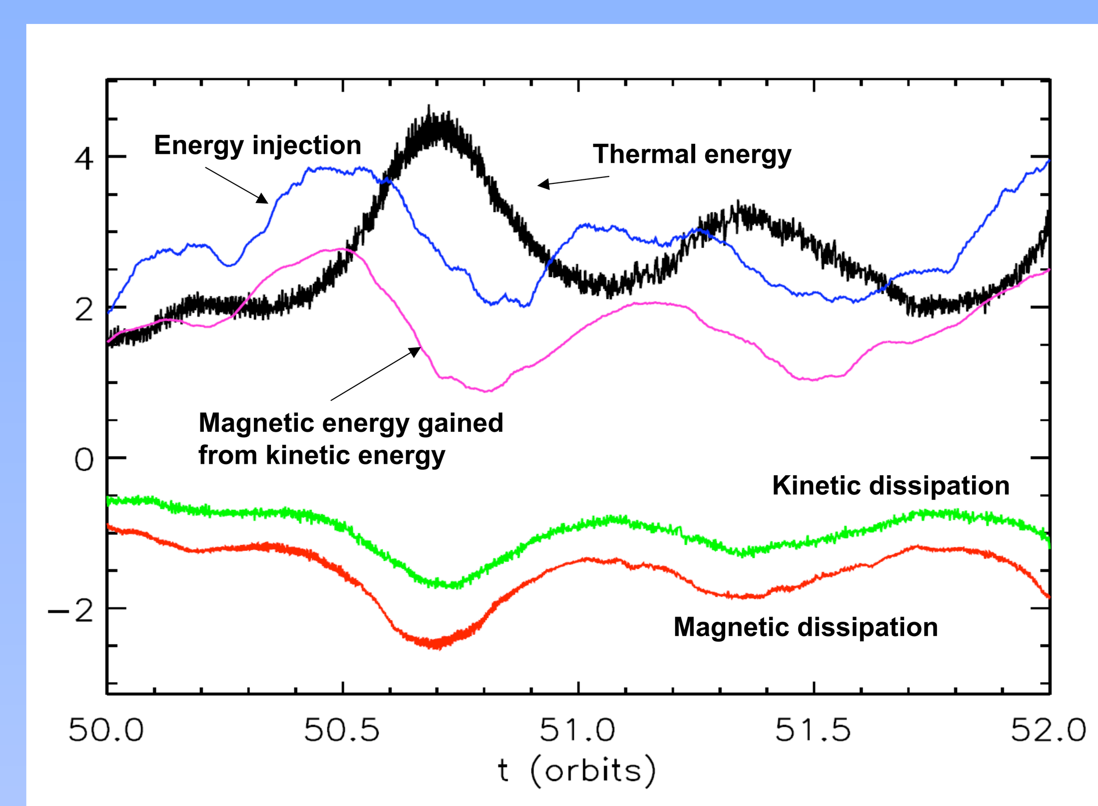
Analysis 2

Calculate Fourier transfer functions, as was done in Fromang & Papaloizou (2007).

$$\eta_{\rm eff}(k) \equiv \frac{D_{\rm mag}(k)}{-k^2|\widetilde{\boldsymbol{B}}(\boldsymbol{k})|^2}$$

$$\nu_{\rm eff}(k) \equiv \frac{D_{\rm kin}(k)}{-k^2|\widetilde{\sqrt{\rho}\boldsymbol{\delta v}}(\boldsymbol{k})|^2}$$

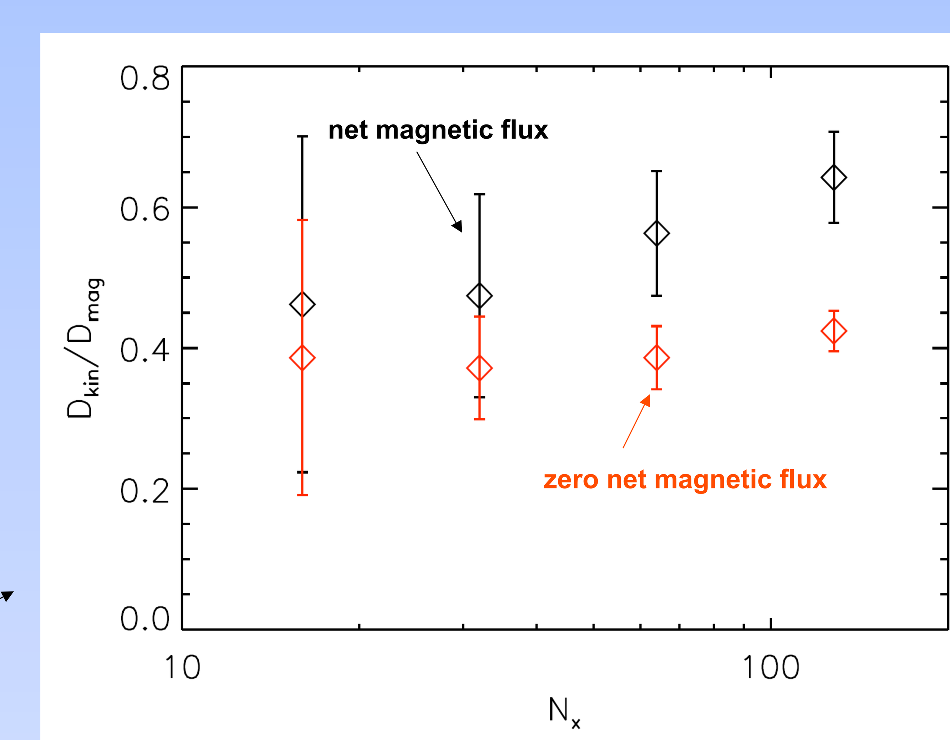
Results



The plot shows several volume-averaged energy rates from Analysis 1. Energy is injected via Reynolds and Maxwell stresses at the radial boundaries. These stresses inject energy in kinetic form, a significant fraction of which is transferred to magnetic. Roughly 0.2 orbits later, this energy is thermalized, primarily through magnetic dissipation.

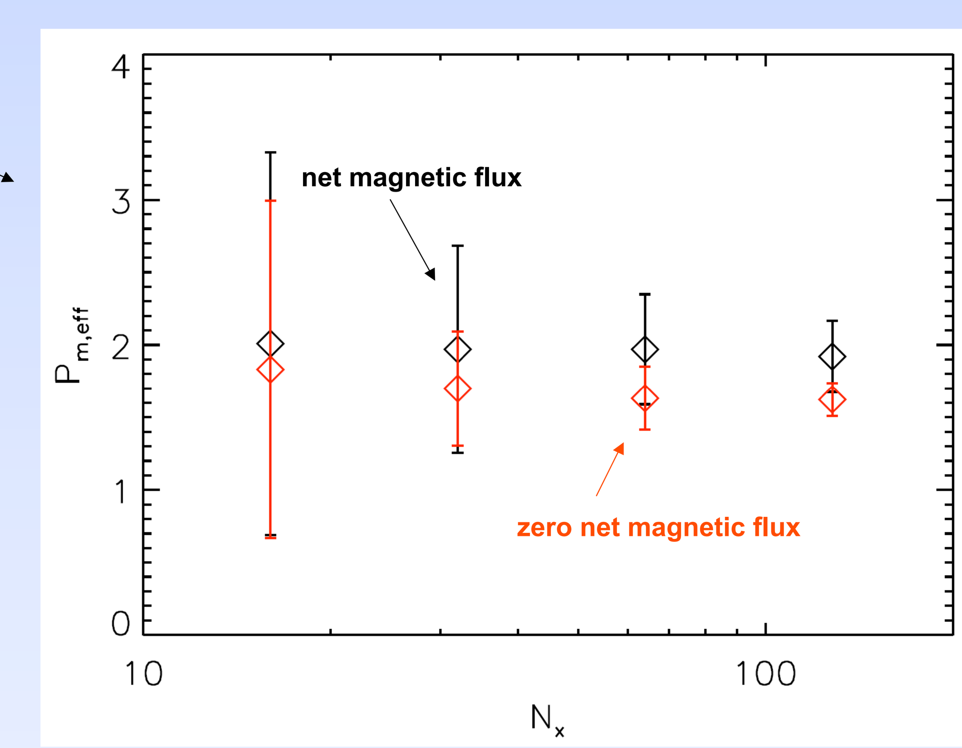
We find the ratio of kinetic to magnetic dissipation to be roughly constant at 0.6.

Analysis 2 calculates the kinetic and magnetic energy evolution in Fourier space as was done in Fromang & Papaloizou (2007). Assuming that in a time-averaged sense, these energies are constant at a given scale, we calculate a kinetic and magnetic dissipation rate.



Ratio of kinetic to magnetic dissipation at small grid scales versus resolution.

Effective magnetic Prandtl number at small grid scales versus resolution.



We obtain an effective numerical viscosity and resistivity, from which we calculate a magnetic Prandtl number. The Prandtl number is independent of resolution.

Conclusions

1. The time lag between features in energy injection and thermalization is on the order of 0.2 orbits => Dissipation happens on a rapid timescale.

2. Two separate analyses have shown that magnetic dissipation dominates over kinetic dissipation.

3. The effective magnetic Prandtl number is constant with resolution and is ~2.

Future Work

Now that we have a baseline of simulations from which to work, we need to add in physical viscosity and resistivity to obtain a more realistic picture of energy flow and dissipation.

Acknowledgments

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REFERENCES

Fromang, S., Papaloizou, J. 2007, A&A, 476, 1113