Numerical Simulations of the Princeton MRI Experiment

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Introduction

Numerical simulation has been critical in the development and understanding of the Princeton MRI experiment. Several techniques have been used to investigate the stability properties of the experimental system as a whole, as well as the detailed behavior of the fluid and magnetic field in the experiment. A local WKB analysis, and a more general global stability analysis were used in the initial design of the experiment to explore experimental requirements for observation of the MRI. Both of these approaches are now being revisited in order to gain information on nonaxisymmetric modes of the system. An astrophysical MHD code, ZEUS, was modified to enable investigation of realistic boundary conditions of the experiment, and to characterize the expected growth and saturation of the MRI in the experiment. These simulations can be compared to experimental observations in order to form a more complete understanding of the behavior of the fluid than could be formed by experimental observation alone.

Local Stability Analysis[1]

The equations of incompressible, dissipative MHD are given by

$$\nabla\cdot\vec{V}=0,\qquad \frac{\partial\vec{V}}{\partial t}+(\vec{V}\cdot\nabla)\vec{V}=\frac{(\vec{B}\cdot\nabla)\vec{B}}{\mu_0\rho}-\frac{1}{\rho}\nabla\left(p+\frac{B^2}{2\mu_0}\right)+\nu\nabla^2\vec{V}$$

$$\nabla\cdot\vec{B}=0,\qquad \frac{\partial\vec{B}}{\partial t}=\nabla\times(\vec{V}\times\vec{B})+\eta\nabla^2\vec{B}$$

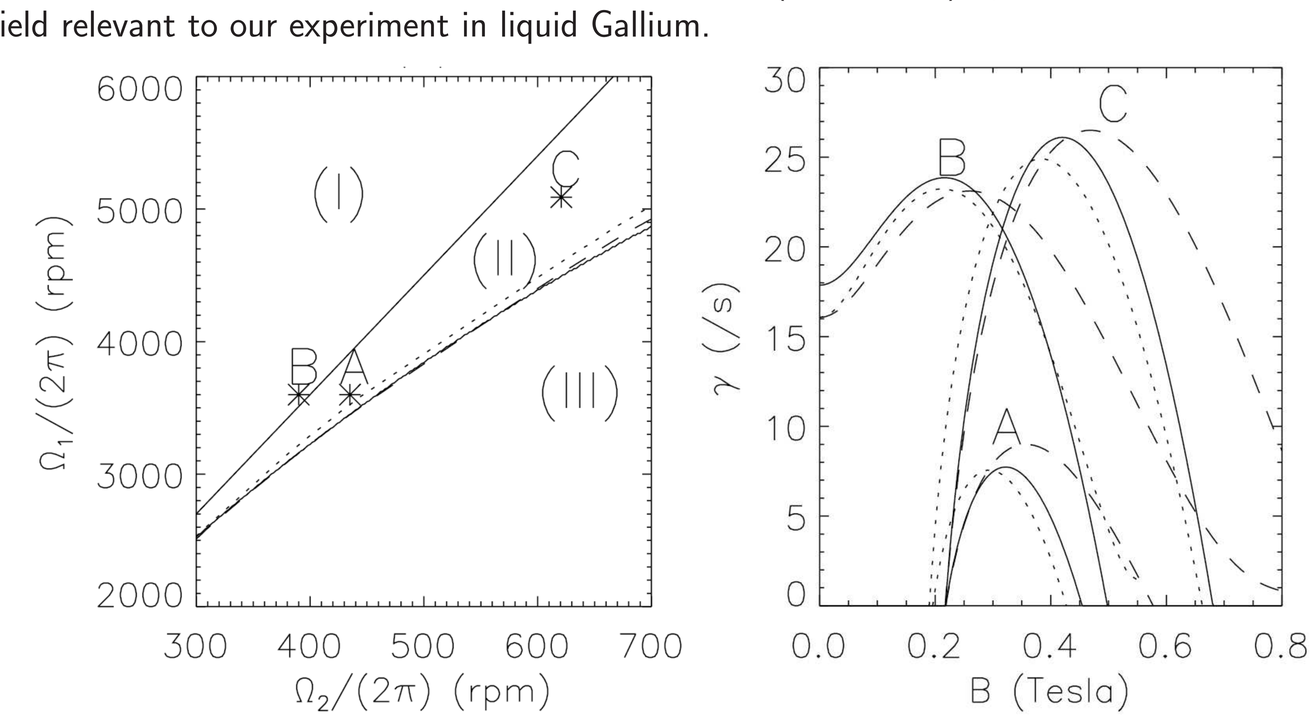
The fluid is assumed to have a background rotation profile given by the Couette solution for flow between two rotating cylinders

$$\Omega(r)=a+\frac{b}{r^2}$$

and there is an initial uniformly applied field B in the $\hat{z}$-direction. Assuming perturbations of the form $\exp(\gamma t-ik_zz-ik_rr)$, and assuming $k\gg 1/r$, a dispersion relation can be derived:

$$\left[(\gamma+\nu k^2)(\gamma+\eta k^2)+(k_zV_A)^2\right]^2\frac{k^2}{k_z^2}+\kappa^2(\gamma+\eta k^2)^2+\frac{\partial\Omega^2}{\partial\ln r}(k_zV_A)^2=0$$

with $V_A=B/\sqrt{\mu_0\rho}$, and the epicyclic frequency given by $\kappa^2=(1/r^3)\partial(r^4\Omega^2)/\partial r$.
The stability properties indicated by this dispersion relation indicate three regions of stability: **(I)** Hydrodynamically unstable, but can be stabilized by a magnetic field; **(II)** Hydrodynamically stable, but can be destabilized by a magnetic field; **(III)** Always stable. These regions are shown below, for values of inner and outer cylinder speeds (Ω_1 and Ω_2), growth rate, and applied field relevant to our experiment in liquid Gallium.



Nonaxisymmetric Local Stability Analysis

The observation of nonaxisymmetric modes in hydrodynamically unstable experiments with an applied magnetic field led us to investigate the effects of nonaxisymmetric perturbations in the experiment. Allowing perturbations of the form $\exp(\gamma t-ik_zz-ik_rr-m\theta)$, the lowest-order modification to the dispersion relation (assuming $m\ll k$) is that the growth rate γ is replaced by $\gamma-im\Omega$. Nonaxisymmetric perturbations are seen to oscillate as they rotate past a fixed observer. For more information of how this relates to experimental observations, see the poster by M. Nornberg.

References

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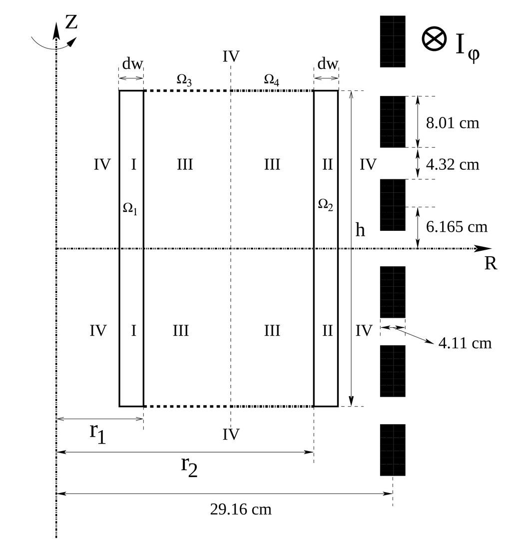
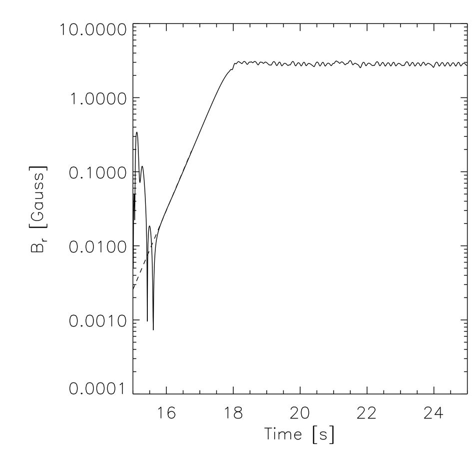
MHD Simulations[3–6]

The Princeton MRI experiment uses the ZEUS-2D and ZEUS-MP 2.0 codes to perform MHD simulations of the finite-height experimental geometry. The codes have been modified by W. Liu to include the affects of resistivity and viscosity, and to allow for partially-conducting, no-slip boundary conditions.
The code has some limitations. ZEUS is a compressible MHD code. But convergence studies show that by enforcing the condition on the Mach number $M\leq 0.25$, the compressibility has little effect on the results of simulations of incompressible liquid gallium.
The rotation speeds of the experiment produce flows with $Re\approx 10^7$. Simulations with such high Reynolds numbers are not currently feasible. We are limited to $Re<10^5$ by time constraints.
The codes have also been used only to perform axisymmetric simulations of the experiment, since the MRI is expected to be most unstable for axisymmetric modes. ZEUS-MP can run with fully 3-dimensional geometries. However, the modifications made to the code to simulate the Princeton MRI experiment, in particular the partially-conducting boundary conditions, have not yet been extended to 3-D. The observation of nonaxisymmetric modes in recent hydrodynamically unstable MHD experiments, as well as the possibility for 3-D instabilities, such as the Kelvin-Helmholtz instability, in shear layers of the experiment, provide an incentive to work on 3-D simulation.



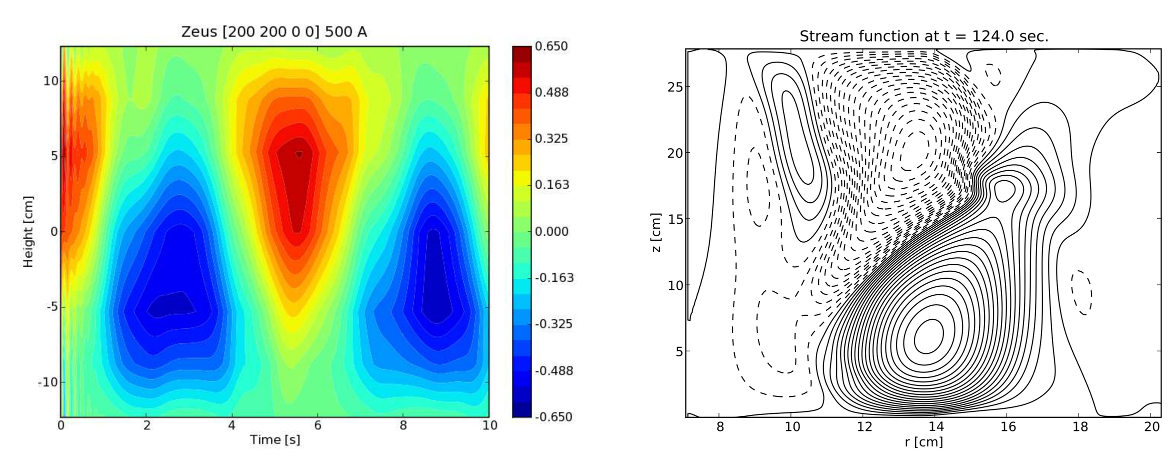
Computational domain of simulations. Region I: Inner partially-conducting stainless steel cylinder. Region II: Outer partially-conducting stainless steel cylinder. Region III: Liquid gallium. Region IV: Vacuum.



B_r at midplane, outside of outer cylinder. Trace shows the MRI's linear growth phase, followed by saturation.

Split-ring Hydrodynamically Unstable Simulation

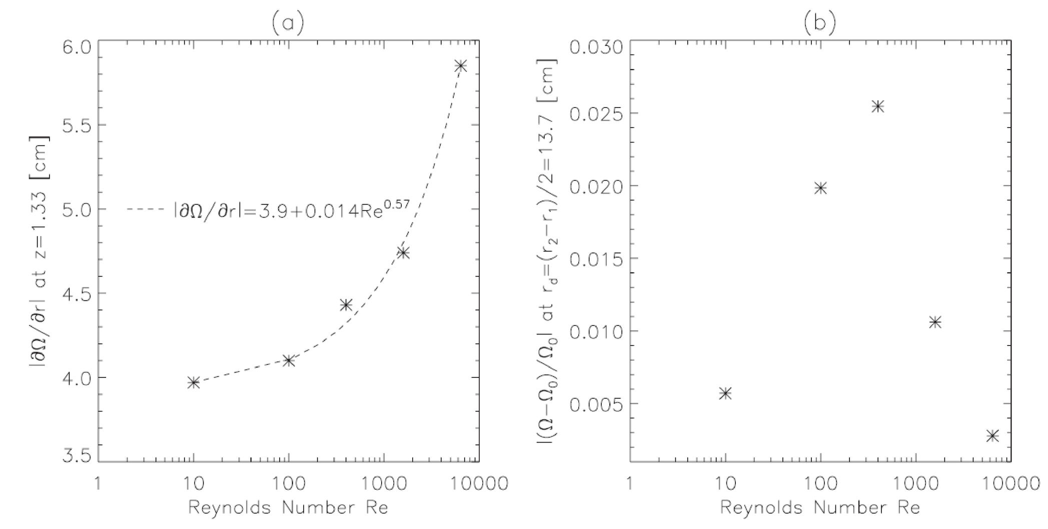
Simulations relevant to the recent investigations of the split-ring, hydrodynamically unstable flows with an applied magnetic field have been performed. These simulations show the existence of a radial jet at the midplane. This jet flaps, producing an oscillating B_r at the location of diagnostic probes located beyond the outer cylinder.



Time series plot of B_r at the location of magnetic probes for a split-ring hydrodynamically unstable case ($\Omega_1=200$RPM, $\Omega_2=0$RPM) with an applied field of 2.5kG). The stream function shows a strong radial jet located near the midplane.

Boundary Layer Effects

The MHD codes are very useful for understanding the effects of boundaries on the system. Recent work by W. Liu on magnetized Ekman and Stewartson layers in a Taylor-Couette flow with split endcaps indicates that Ekman suction may be reduced by an applied magnetic field, and that instabilities in the Stewartson layer may limit its penetration into the bulk flow of the fluid at sufficiently high Re [3].



Effects of increasing Re on Stewartson layer. Plot (a) shows increasing $|\partial\Omega/\partial r|$ at the location of the layer, indicative of a narrowing of the layer at higher Re. Plot (b) shows a decreasing perturbation to the initial rotation profile by the Stewartson layer at the midplane above a critical $Re\sim 600$.