Preliminary MHD Results from the MRI Experiment

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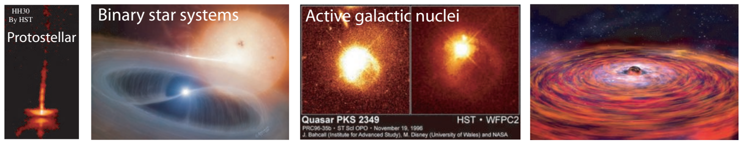
Motivation

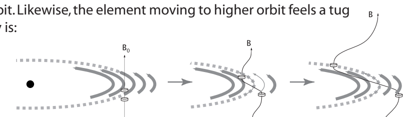
The accretion of gas, dust, and plasma orbiting a strong gravitational source is responsible for the observed luminosity of systems such as binary star systems and active galactic nuclei. Accretion disk dynamics also set the timescale for star and planet formation in protostellar disks. The rate of accretion is governed by how quickly angular momentum can be transported through the disk. Inferred rates of accretion suggest that viscous transport is insufficient, thus turbulence models are invoked to explain observations. The mechanism responsible for turbulent transport in these accretion disks is the Magnetorotational instability (MRI), a linear instability caused by the Maxwell stress introduced by an ambient magnetic field coupled to the Keplerian sheared flow. The MRI is sufficiently generic that it should be observable in any hydrodynamically stable rotating shear flow with a radially-decreasing angular velocity for sufficiently high magnetic Reynolds number with an externally applied axial magnetic field. The Princeton MRI Experiment was designed to study the stability of rotating shear flow in a magnetized conducting fluid. The unique design of this experiment allows the generation of quiescent shear flow at high Reynolds number ($Re > 10^6$) as demonstrated by Reynolds stress measurements using water. The experiment has been filled with a gallium eutectic alloy and operated with an applied axial magnetic field of up to 5 kG. The most recent measurements show the emergence of nonaxisymmetric MHD modes from magnetized turbulent shear flow using an array of radially-aligned induction coils. The modes precess toroidally and display a frequency splitting which scales as the rotation speed, a characteristic of MHD waves in rotating systems.



The MRI Mechanism

Magnetic tension between fluid elements can lead to a runaway instability creating an effective radial flux of angular momentum. As two fluid elements are displaced, the one moving to lower orbit feels a drag from the field line tension (depicted as a spring) and falls to lower orbit. Likewise, the element moving to higher orbit feels a tug from the field and rises farther. The resulting instability is:

- Axisymmetric
- Derives its free energy from the flow shear
- Resistively limited (minimum Rm)
- Stabilized by a sufficiently strong magnetic field (Alfven time shorter than orbit)

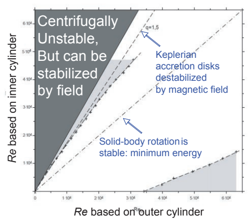


Linear Stability Analysis of magnetized Taylor-Couette flow

The flow between two concentric rotating cylinders (a Taylor-Couette experiment) can be used to explore both hydrodynamic and MHD instabilities related to accretion disk turbulence. Disk flows follow a Keplerian profile $\Omega \sim r^{-3/2}$, but more generally are characterized as being anti-cyclonic and centrifugally stable:

$$\frac{d\Omega}{dr} < 0, \frac{dL}{dr} > 0$$

Such flows are referred to as quasi-Keplerian. Both the nonlinear hydrodynamic instability and MRI are present in incompressible fluids so that we can explore them by generating rotating shear flows in either water or a liquid metal.



For infinitely long cylinders, the rotation profile is the ideal Couette profile

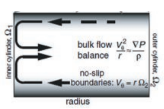
$$\Omega(r) = a + \frac{b}{r^2} \qquad a = \frac{\Omega_2 r_2^2 - \Omega_1 r_1^2}{r_2^2 - r_1^2}; b = \frac{r_1^2 r_2^2(\Omega_1 - \Omega_2)}{r_2^2 - r_1^2}$$

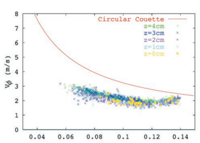
Adjustment of the inner and outer cylinder speeds allow us to explore both hydrodynamically and MHD stable and unstable regimes.

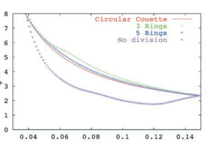
Three regions of stability:
(I) Centrifugally unstable, but can be stabilized by the magnetic field
(II) Centrifugally stable, but can be destabilized by the magnetic field
(III) Always stable

Boundary effects in finite cylinders

Since the endcaps of the apparatus rotate with outer cylinder, a pressure imbalance creates a boundary layer that drives radial flow at the endcaps. The flow circulates into the bulk flow creating large Ekman circulation cells. This secondary circulation is quite effective at transporting angular momentum and causes the rotation profile to deviate from the ideal Couette profile.

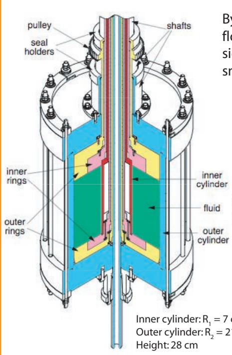




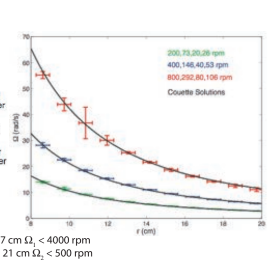


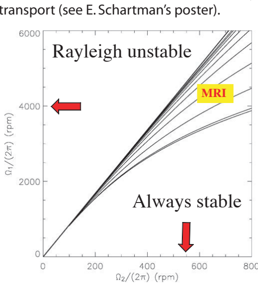
Mitigation of secondary circulation using segmented endcaps

By splitting up the endcaps into independently-driven differentially rotating rings, the flow profiles can be tailored to fit the ideal Couette profile, thereby minimizing the residual secondary circulation. Fluctuations in the bulk flow were found to be extremely small and inefficient for angular momentum transport (see E. Schartman's poster).

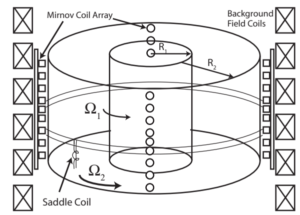


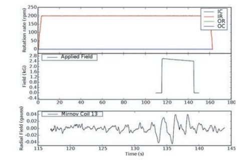
Inner cylinder: R_1 = 7 cm Ω_1 < 4000 rpm
Outer cylinder: R_2 = 21 cm Ω_2 < 500 rpm
Height: 28 cm





Liquid metal rotating shear flow experiments





Example run: the motors are turned on and the flow is allowed to equilibrate for about 2 minutes. Then the magnetic field is turned on and the pickup coils observe the perturbed magnetic field.

Machine Parameters:

Liquid metal: GaInSn eutectic alloy (melts at 10° C)
r = 6.361 gm/cm^3
η = 2430 cm^2/s   Pm = $\nu/\eta = 10^{-6}$
$\nu = 3.402\times10^{-3}$ cm^2/s

Geometry: R_1 = 7.06 cm, R_2 = 20.30 cm, h = 27.9 cm
$\varepsilon = h / (R_2 - R_1) = 2.1$

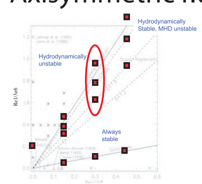
Max Speeds: Ω_1 = 418.9 rad/s (at 4000 rpm or 66.7 Hz),
Ω_2 = 55.8 rad/s (at 533 rpm or 8.88 Hz)
$Re = (R_2 - R_1) R_1 \Omega_1 / \nu < 1\times10^7$
$Rm = (R_2 - R_1) R_1 \Omega_1 / \eta < 16$

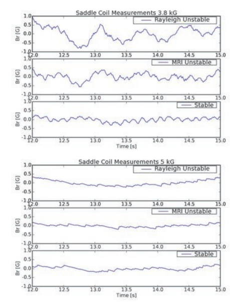
Applied Field: B_0 ranges from 500 to 5000 gauss
$S = B_0 (R_2 - R_1) / \eta\ (4\pi\rho)^{1/2}$ ranges from 0.3 to 3

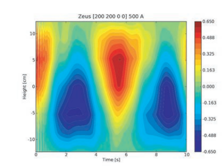
Diagnostics: Array of 39 radially-aligned Mirnov (Pickup coils)
Eight Hall probes (4 vertical, 4 radial)
6 Saddle Coils (to measure axisymmetric fluctuations)

Diagnostics in development:
Internal Hall probe and pressure sensors mounted in a hydrodynamic wing for internal Reynolds and Maxwell stress measurements
Potential probe measurements
Ultrasound Doppler Velocimetry

Axisymmetric fluctuations increase with flow shear





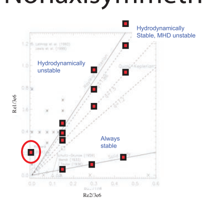


Axisymmetric fluctuations measured with the saddle coil were found to

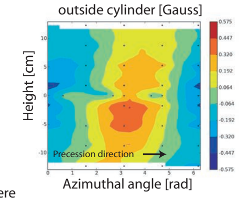
- Increase with flow shear
- Decrease with field

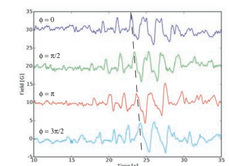
Axisymmetric simulations of the Rayleigh unstable flow using the Zeus MP code reveal the signature of a flapping jet at the midplane (see A. Roach's poster)

Nonaxisymmetric modes observed



Radial field fluctuations outside cylinder [Gauss]

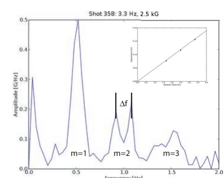


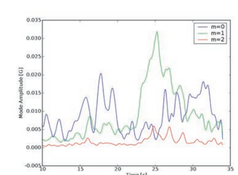


A plot of the measured field at four azimuthal angles is used to determine the phase speed of the mode.

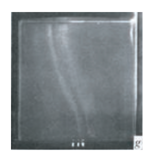
Nonaxisymmetric fluctuations were observed in hydrodynamically unstable flows

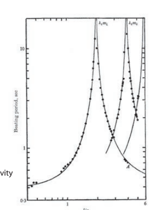
- Dominant m=1 mode
- Higher harmonics
- Mode rotates at 1/6 speed of inner cylinder
- Mode appears to be due to beating waves
- Frequency splitting of m=2 mode proportional to rotation rate





A Fourier decomposition of the measured radial magnetic field on the surface shows excitation of several modes.





A rotating cylinder of aspect ratio ε=2 is a resonance cavity for forced inertial oscillations with mode number m=1 (McEwan, J. Fluid Mech., 1970).

$$\begin{aligned}
(\gamma - im\Omega + \nu k^2)\,\tilde{v}_r \quad -2\Omega\tilde{v}_\phi \quad -ik_r\tilde{p} + \frac{iB_0}{\mu_0\rho}\left(k_z\tilde{b}_r - k_r\tilde{b}_z\right) &= 0\\
(\gamma - im\Omega + \nu k^2)\,\tilde{v}_\phi \quad +\Omega(2-q)\tilde{v}_r \quad +\frac{iB_0}{\mu_0\rho}k_z\tilde{b}_\phi &= 0\\
(\gamma - im\Omega + \nu k^2)\,\tilde{v}_z \quad -ik_z\tilde{p} &= 0\\
(\gamma - im\Omega + \nu k^2)\,\tilde{b}_r \quad +ik_zB_0\tilde{v}_r &= 0\\
(\gamma - im\Omega + \nu k^2)\,\tilde{b}_\phi \quad +q\Omega\tilde{b}_r \quad +ik_zB_0\tilde{v}_\phi &= 0\\
(\gamma - im\Omega + \nu k^2)\,\tilde{b}_z \quad +ik_zB_0\tilde{v}_z &= 0
\end{aligned}$$

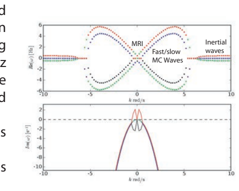
To lowest order, the only nonaxisymmetric modification to the local (WKB) analysis is a frequency shift mΩ due to the co-rotating mode.

Magnetocoreolis (MC) waves

A rapidly rotating conducting fluid subject to a strong magnetic field can support waves where the restoring force is a combination of the Lorentz and Coreolis forces. The Alfvén wave branch is split between a fast and slow MC wave

- Fast MC wave: Lorentz and Coreolis restoring forces are in phase
- Slow MC wave: Lorentz and Coreolis restoring forces are out of phase

These waves are also present in rotating sheared flow.



Dispersion relation for magnetized rotating shear flow

Given that we have operated below the threshold for the MRI, and that we are observing a frequency splitting proportional to the inner cylinder rotation rate, it is likely we are observing a resonance of magnetocoreolis waves:

- Circularly polarized
- Alfvén frequency split by 2Ω
- Damped by both resistivity and viscosity
- Driven by boundary layer perturbations
- Responsible for secular variation of the geomagnetic field