NEW SHEARING BOX RESULTS

Charles F. Gammie

University of Illinois at Urbana-Champaign

with

Xiaoyue Guan (Illinois) and Bryan Johnson (Livermore)

IAS MRI conference

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Numerical Methods
for
3D Shearing Boxes

Large Boxes

Probe mesoscale ($H \ll \lambda \ll R$) disk structure

- Is turbulent disk stable?

  Do B_{y,k_x}, B_{z,k_x}, Σ_{k_x} grow for $k_x H \ll 1$?

- How does MHD turbulence interact with self-gravity?

  Typical scale of gravitational instability $\gtrsim 2\pi H$

- How does the cold disk interact with corona?

  Corona scales, if $\ll R$, likely $\gg H$.

- What are turbulent transport coefficients?

  Does an imposed B_{z,k_x} grow or decay for $k_x H \ll 1$?

Challenges for large boxes:

Restrictive timestep condition

$$\Delta t = C \Delta x / v \sim 1/L_x$$

Nonuniform truncation error

truncation error varies with v_y

Long runs required

growth rates, if any, $\sim \nu k^2$

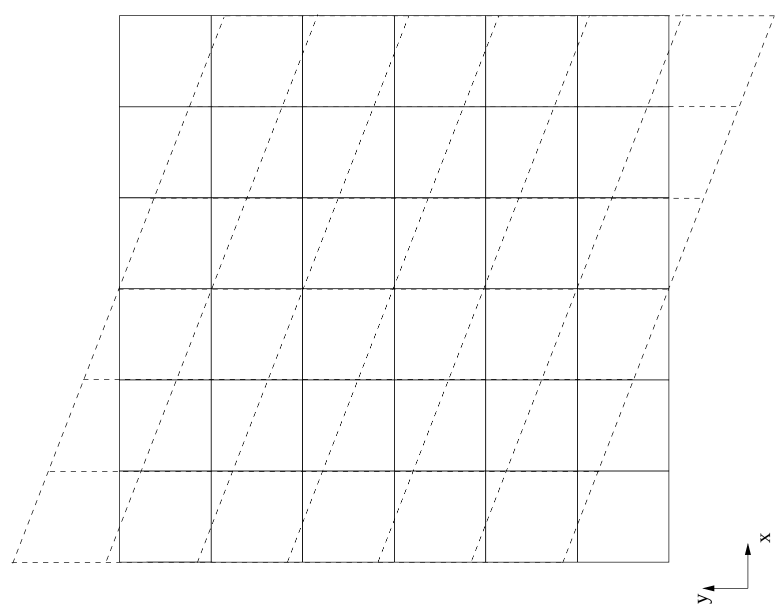
Orbital Advection

Split advection operator:

$$\frac{\mathrm{D}}{\mathrm{D}t} \rightarrow \frac{\partial}{\partial t} + \mathbf{v}_{orb} \cdot \nabla + (\mathbf{v} - \mathbf{v}_{orb}) \cdot \nabla$$

where $\mathbf{v}_{orb} = -q\Omega x$ (Johnson et al. 2008).

Orbital advection handled by interpolation
(no Courant limit!)



Effect of shear on Cartesian grid in r, ϕ plane

Based on ZEUS algorithm

Similar to FARGO (Masset 2000)

But: includes $\mathbf{B}$ interpolation, $\nabla \cdot \mathbf{B} = 0$

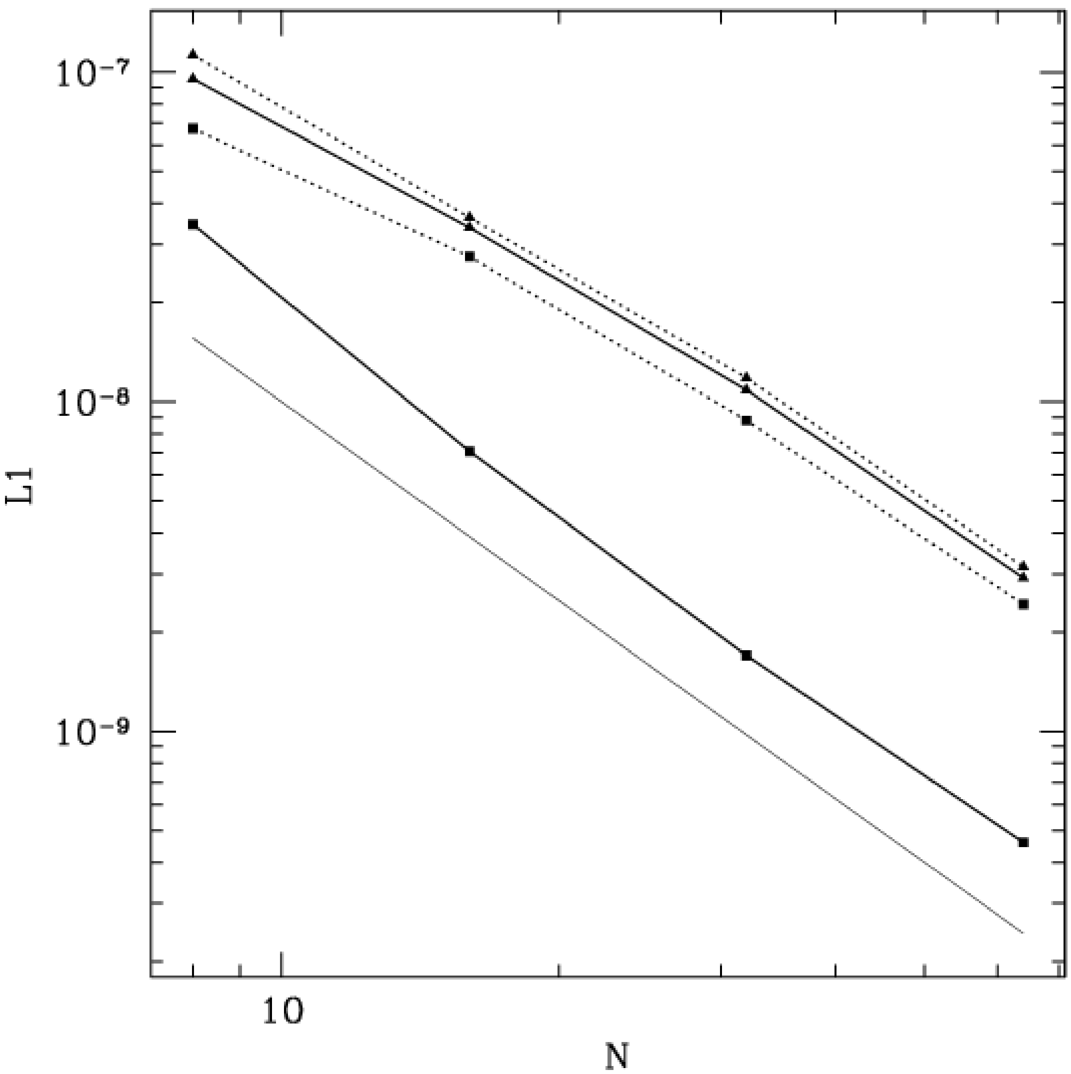
More nearly preserves symmetries of shearing box.

Accuracy and efficiency enhanced

Negligible improvement in small boxes.

Improved Accuracy with Orbital Advection

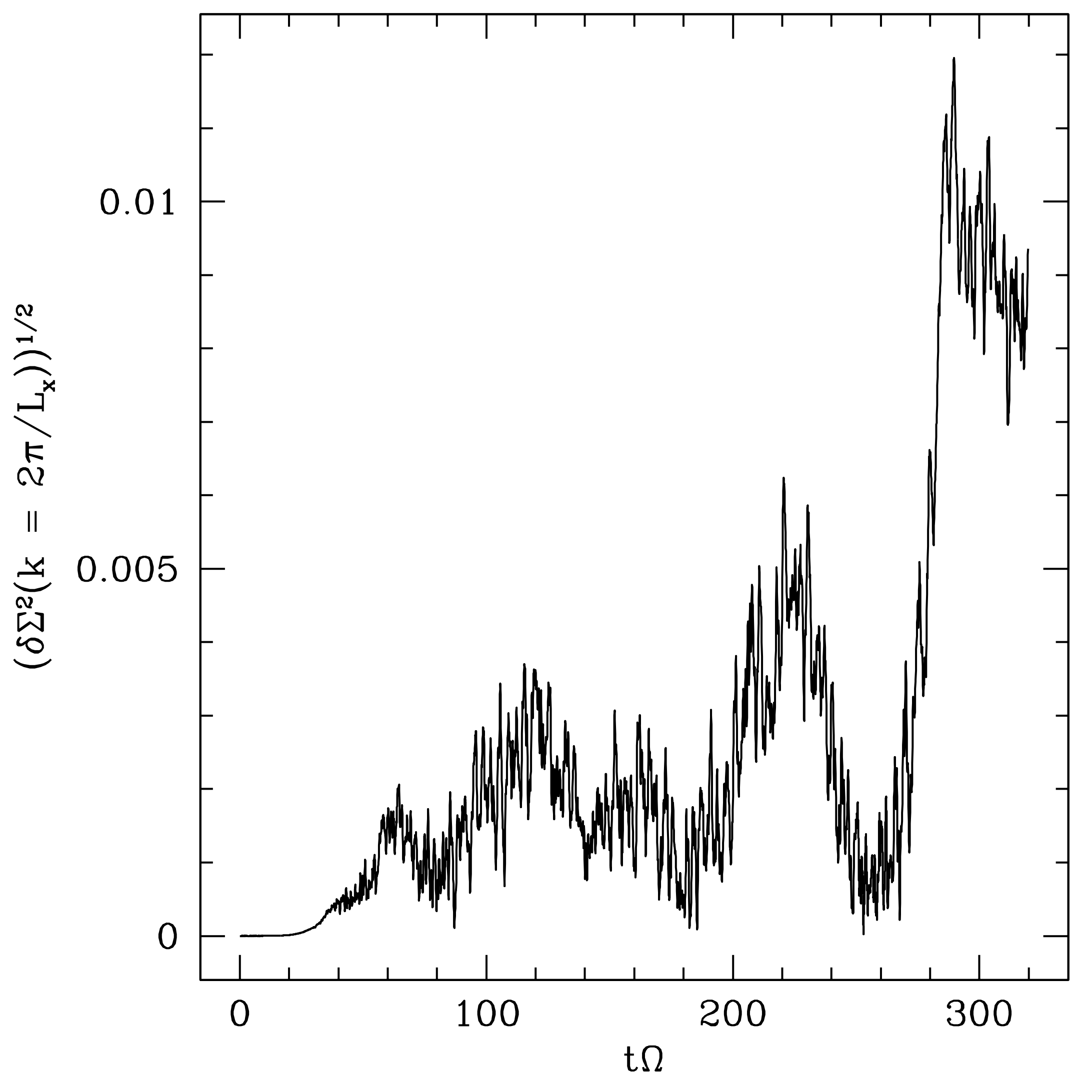
Johnson, Guan & Gammie (2008)



Convergence in magnetized, linear shwave test.
Orbital advection on (solid), off (dashed).
$L_x = H$ *(triangles),* $L_x = 10H$ *(squares).*

Mesoscale Structure?

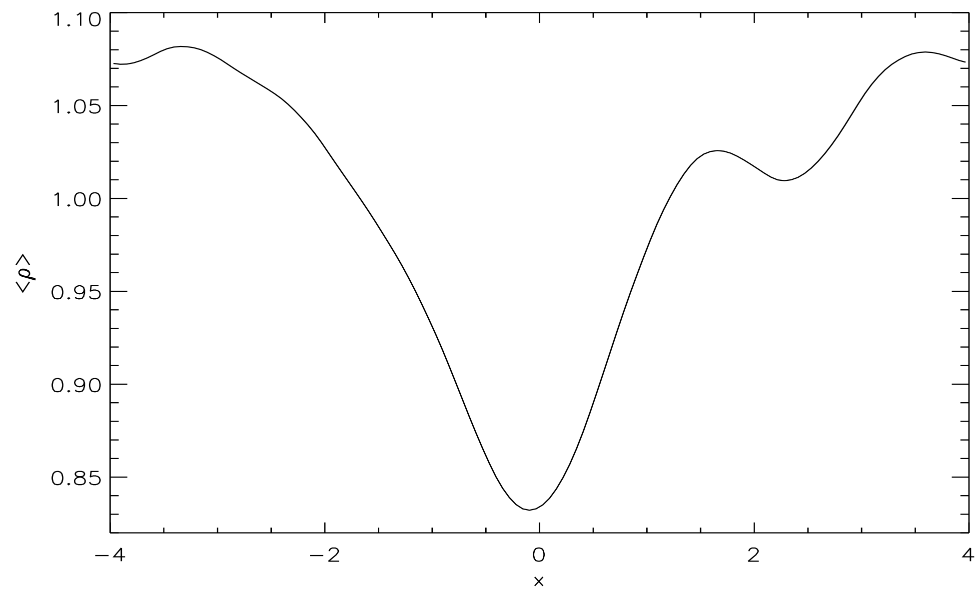
$(L_x, L_y, L_z) = (8, \pi, 1)$, $\langle B_y \rangle$ box with $\beta_o = 400$.

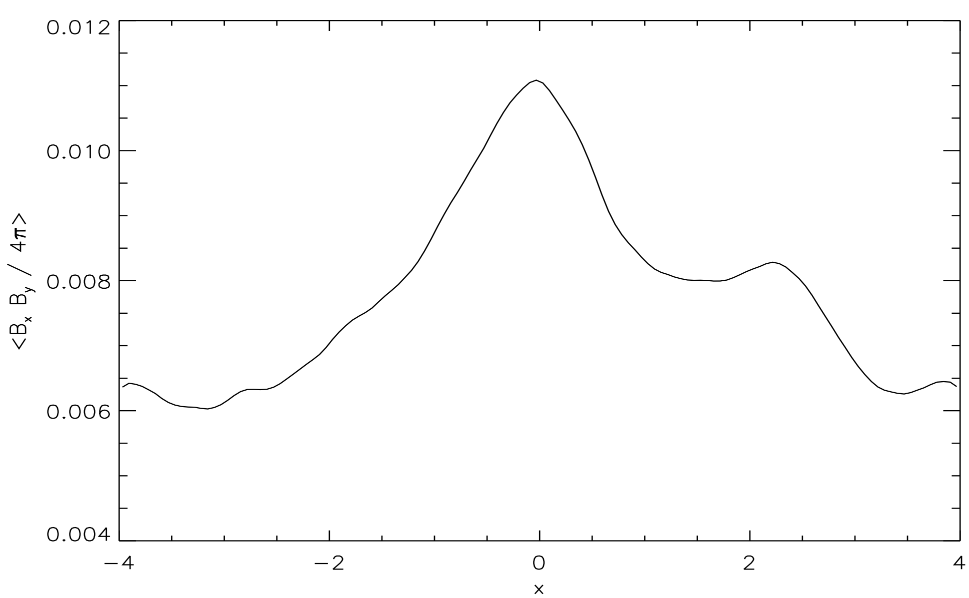


Evolution of $|\delta\Sigma|(k_x = 2\pi/L_x, k_y = 0)$.

Mesoscale Structure?

Not yet.





late time $\langle\rho(x)\rangle, -\langle B_x B_y\rangle/4\pi$ *from* `ATHENA`
$L_x = 8H$; *courtesy Jake Simon*

Solutions

Possible strategies:

- use spectral code

- use shearing coordinate code

- minimize features in finite difference code

Our strategy: shift and clean regularly

- shift model radially by integral number of zones

- do this only at periodic points

- clean $\nabla \cdot \mathbf{B}$ before shifting

  $\nabla \cdot \mathbf{B} \neq 0$ at boundaries

- eliminate mean field before shifting

Σ_k still grows; resolution dependent.

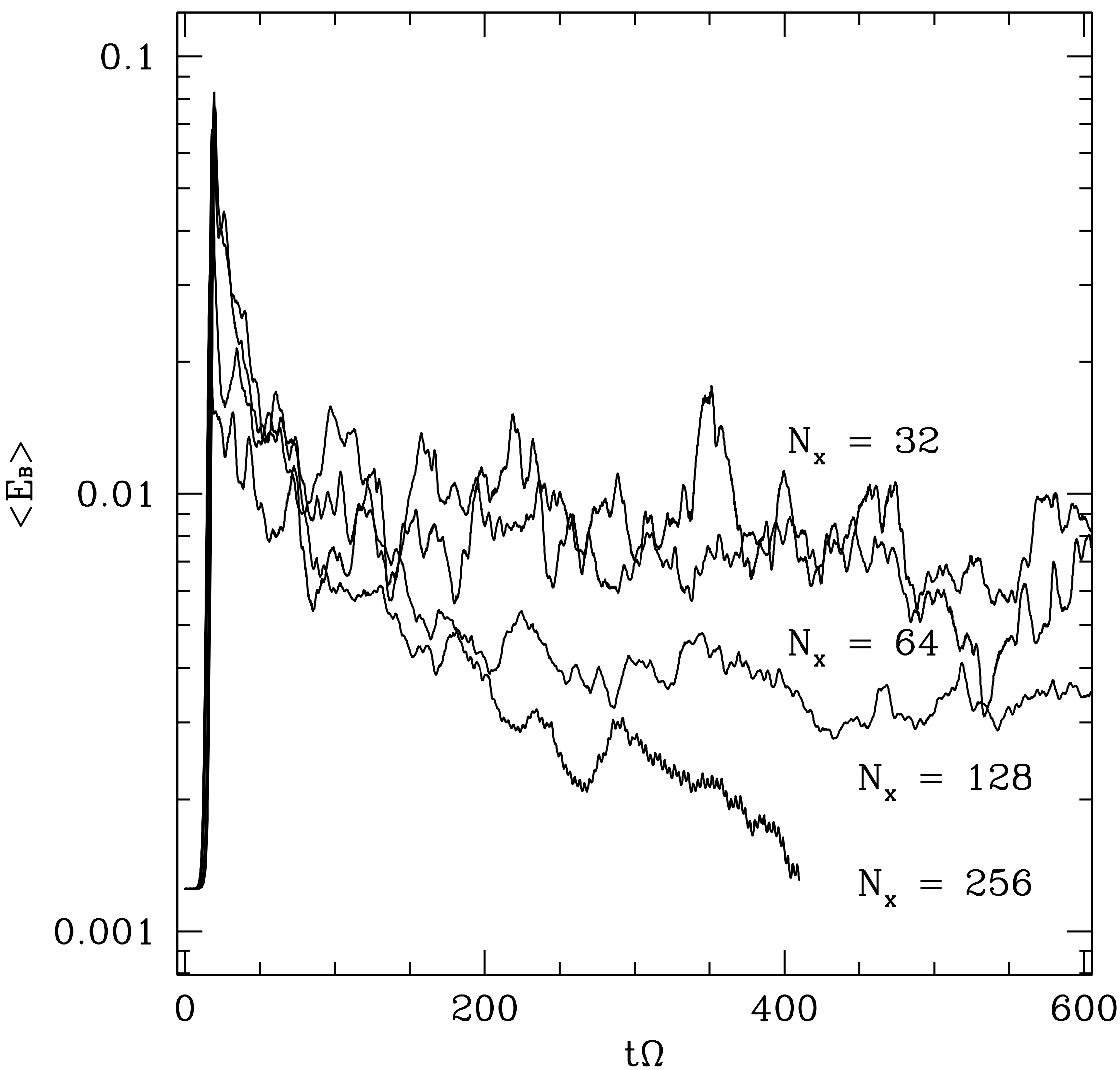
Applications
to
Isothermal, Unstratified
3D Shearing Boxes

Small $\langle \mathrm{B} \rangle = 0$ Boxes

$(L_x, L_y, L_z) = (1, \pi, 1)H$

No convergence

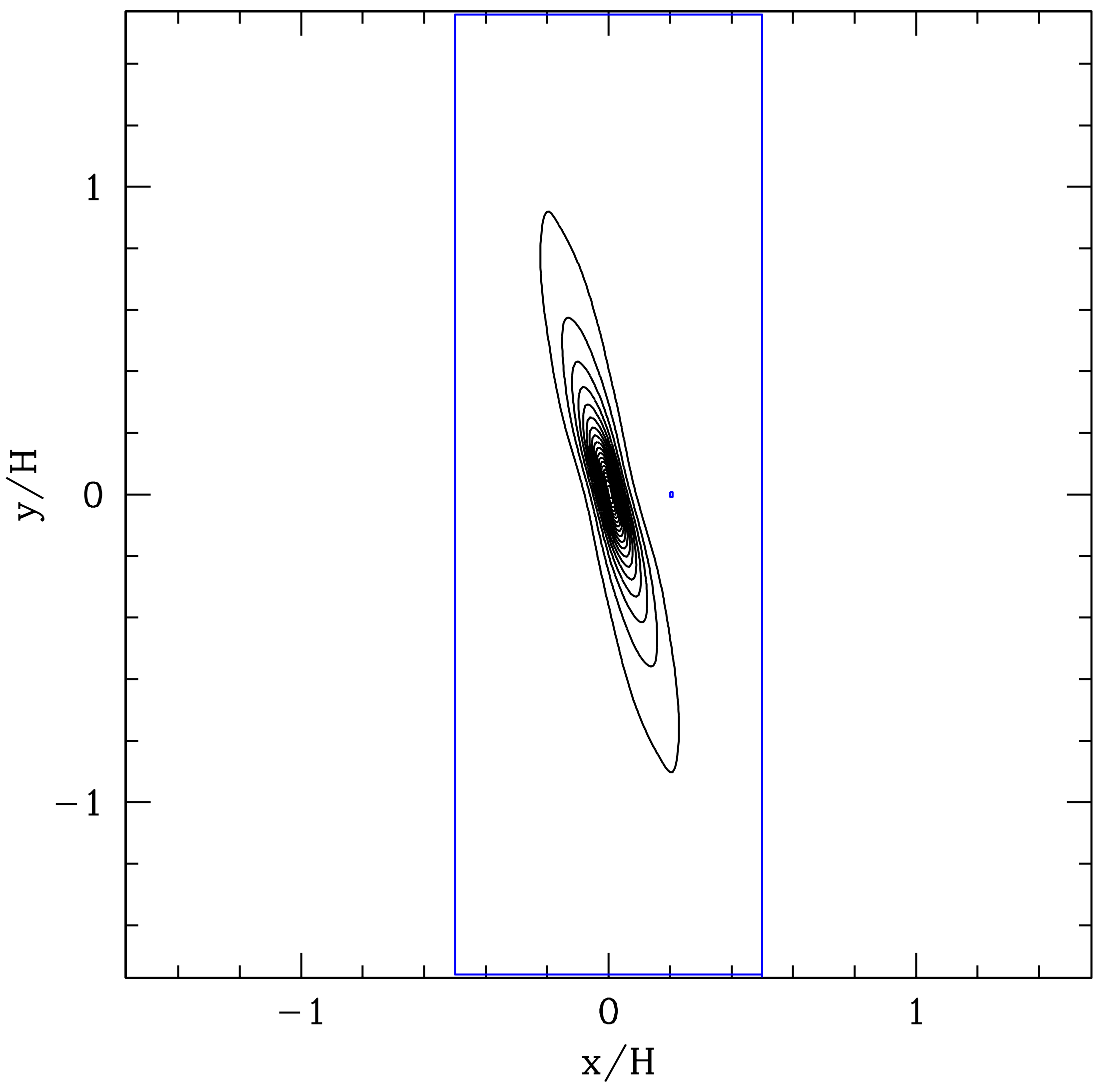
Consistent with Fromang & Papaloizou (2007)



$$\alpha \simeq 0.002 \left(\frac{N_x}{128}\right)^{-1}$$

Small $\langle \mathrm{B} \rangle = 0$ Boxes

Correlation function: $\langle B_i(\mathbf{x}) B_i(\mathbf{x} + \Delta\mathbf{x}) \rangle$



Correlation function in $\Delta z = 0$ plane.

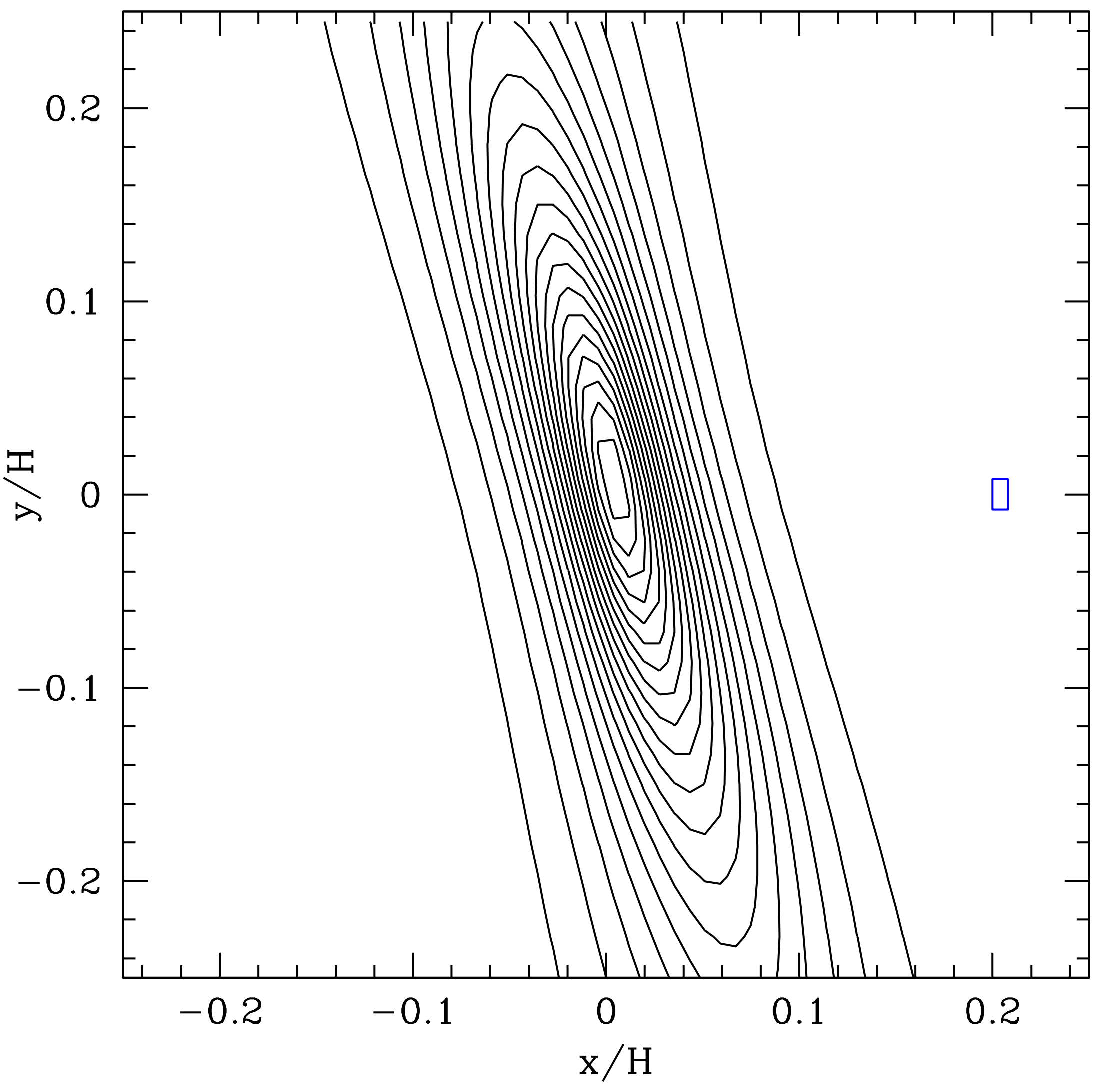
Large box = box size.

Small box = zone size.

Resolution is $128 \times 200 \times 128$.

Small $\langle \mathrm{B} \rangle = 0$ Boxes

Correlation function: $\langle B_i(\mathbf{x}) B_i(\mathbf{x} + \Delta\mathbf{x}) \rangle$



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Small $\langle B \rangle = 0$ Boxes

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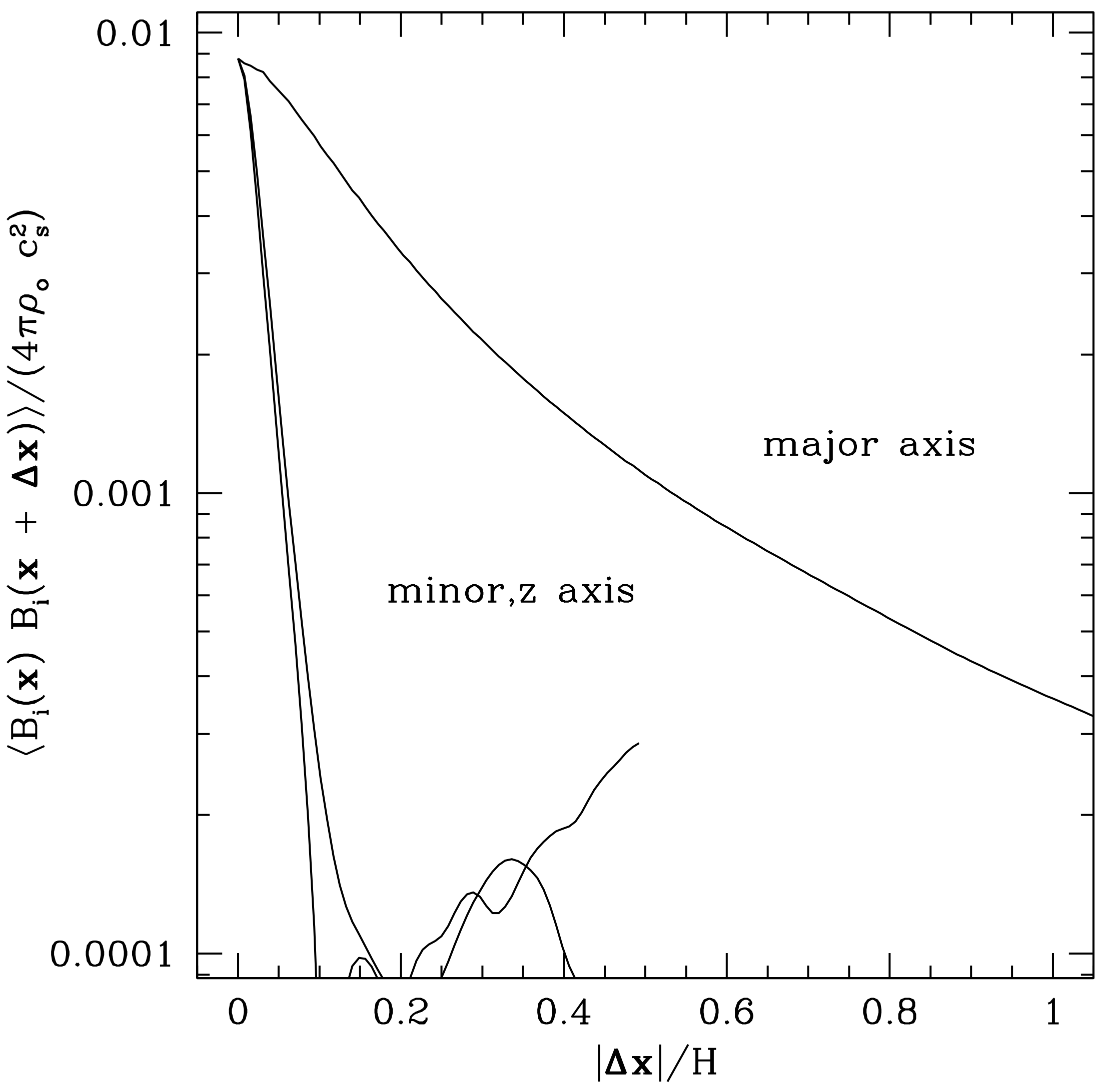
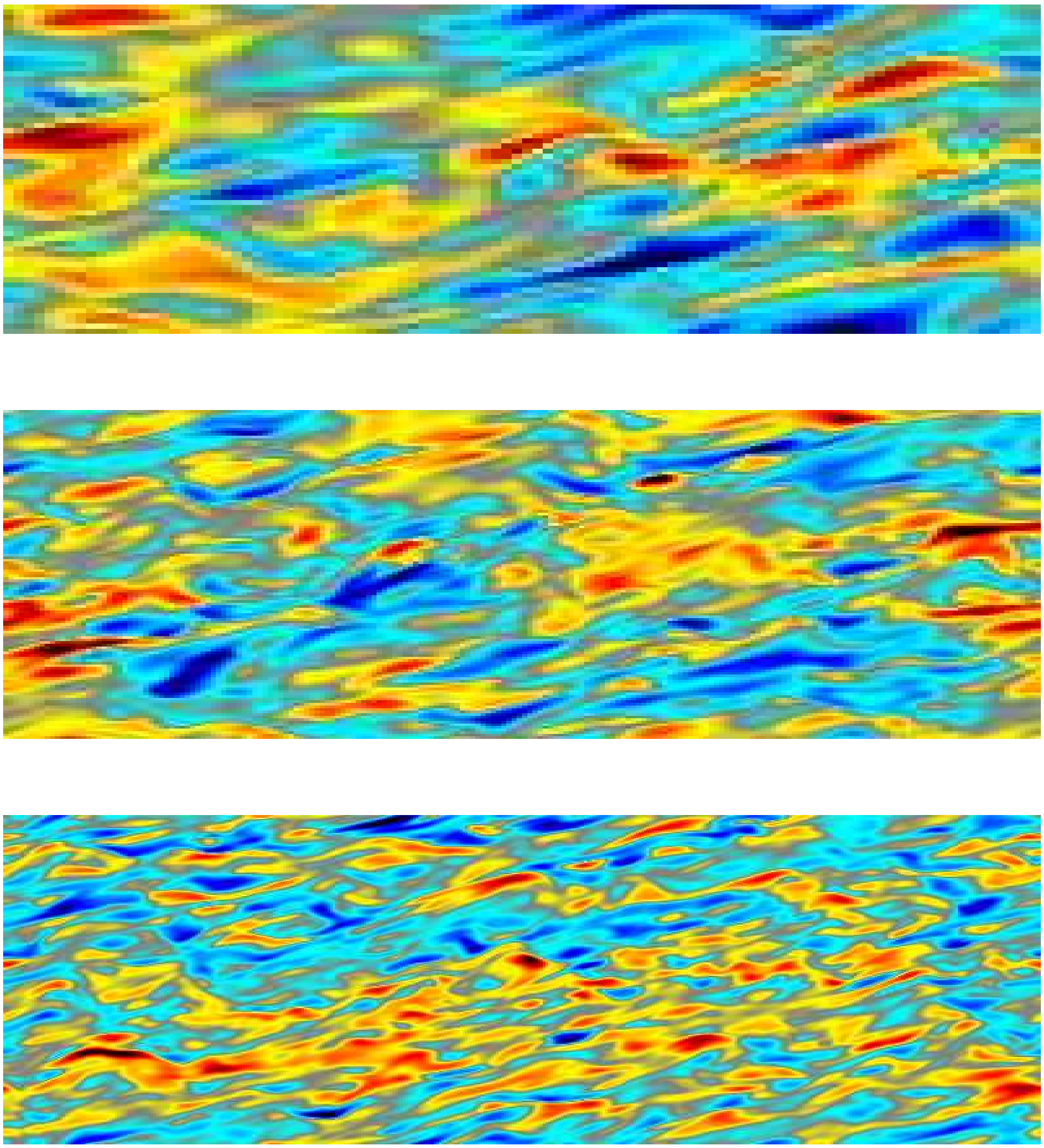


$(\xi_{min},\ \xi_{maj},\ \xi_z) \simeq (0.04,\ 0.22,\ 0.03)H$ scales $\sim \Delta x$

$\theta_{tilt} = 15\,\text{deg.} \ \Rightarrow\ \alpha \simeq (2\beta)^{-1}$

$\xi_{min} \simeq \xi_z \simeq 4\Delta x.$

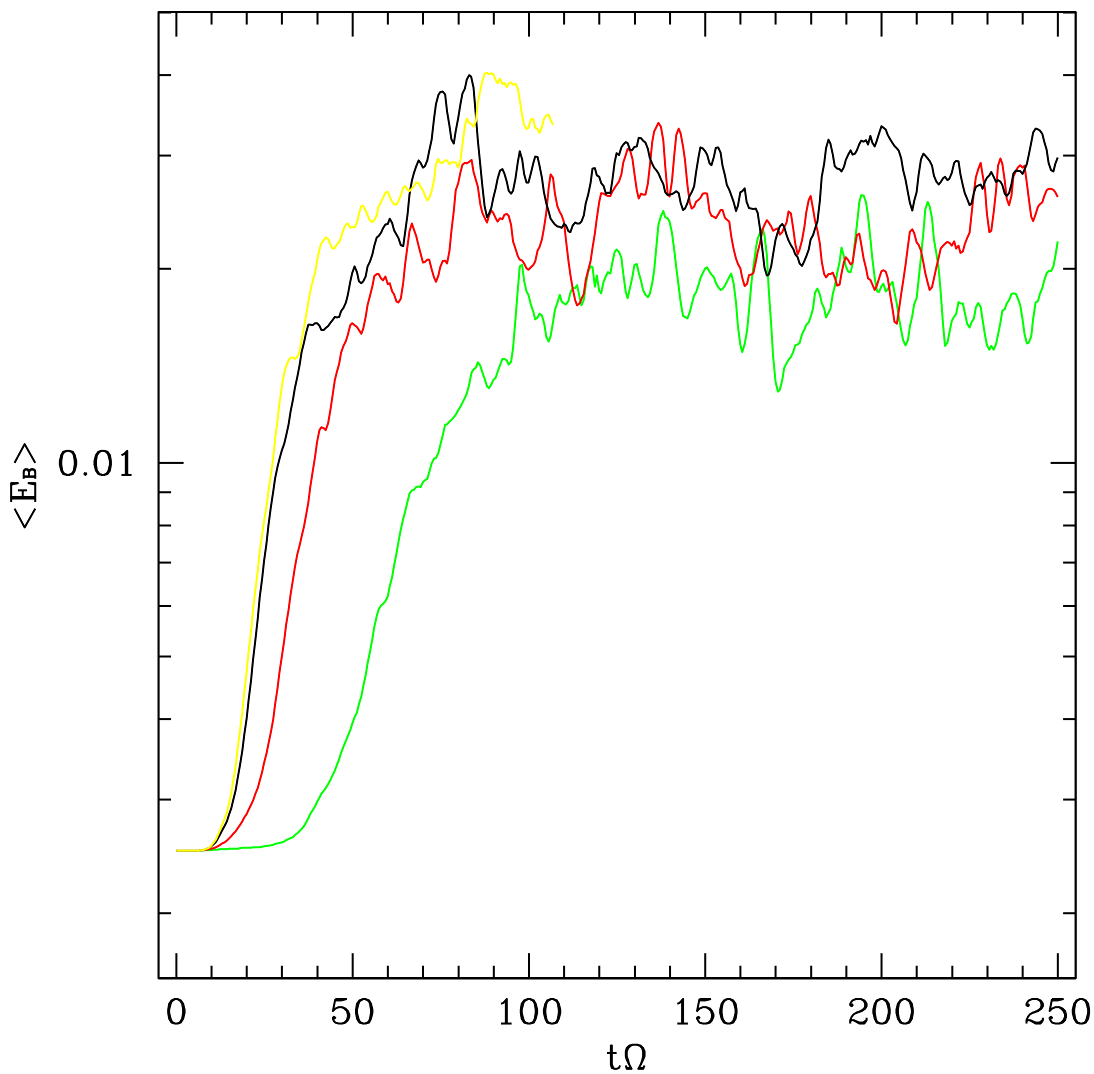
Small $\langle \mathrm{B} \rangle = 0$ Boxes



$\delta v_y(x, y, z = 0)$ at $N_x = 64, 128, 256$

Small $\langle B_y \rangle \neq 0$ Boxes

$\langle B_y \rangle \neq 0$ models possibly most relevant to real disks.



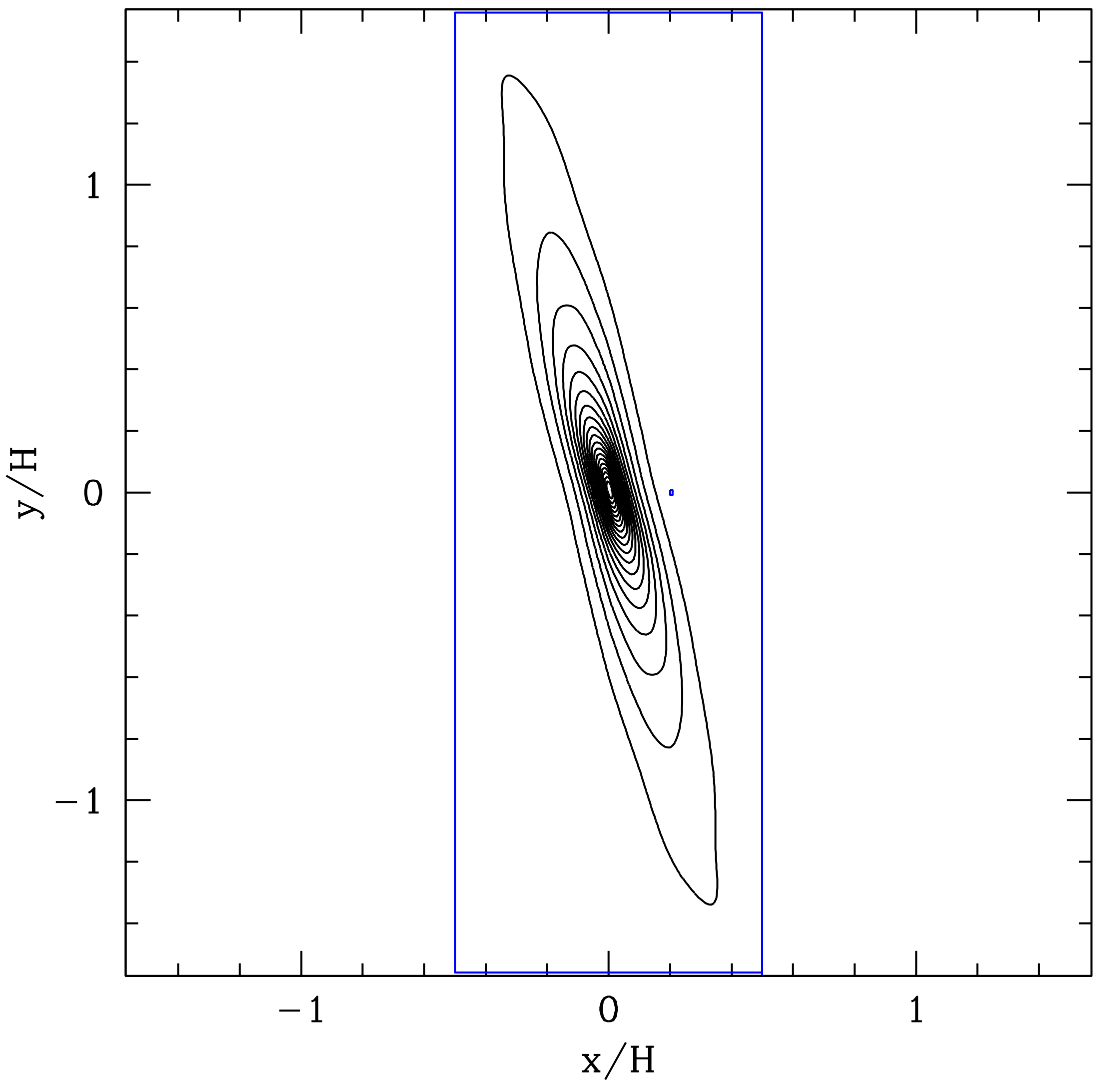
$(L_x, L_y, L_z) = (1, \pi, 1)$ box with $\beta_o = 400$.

$\frac{\langle E_B \rangle}{\rho_o c_s^2}(N_x = 32,\ 64,\ 128,\ 192) = (0.018,\ 0.023,\ 0.028,\ 0.035)$

Converged?

Small $\langle B_y \rangle \neq 0$ Boxes

Correlation function: $\langle B_i(\mathbf{x}) B_i(\mathbf{x} + \Delta\mathbf{x}) \rangle$



Correlation function in $\Delta z = 0$ plane.

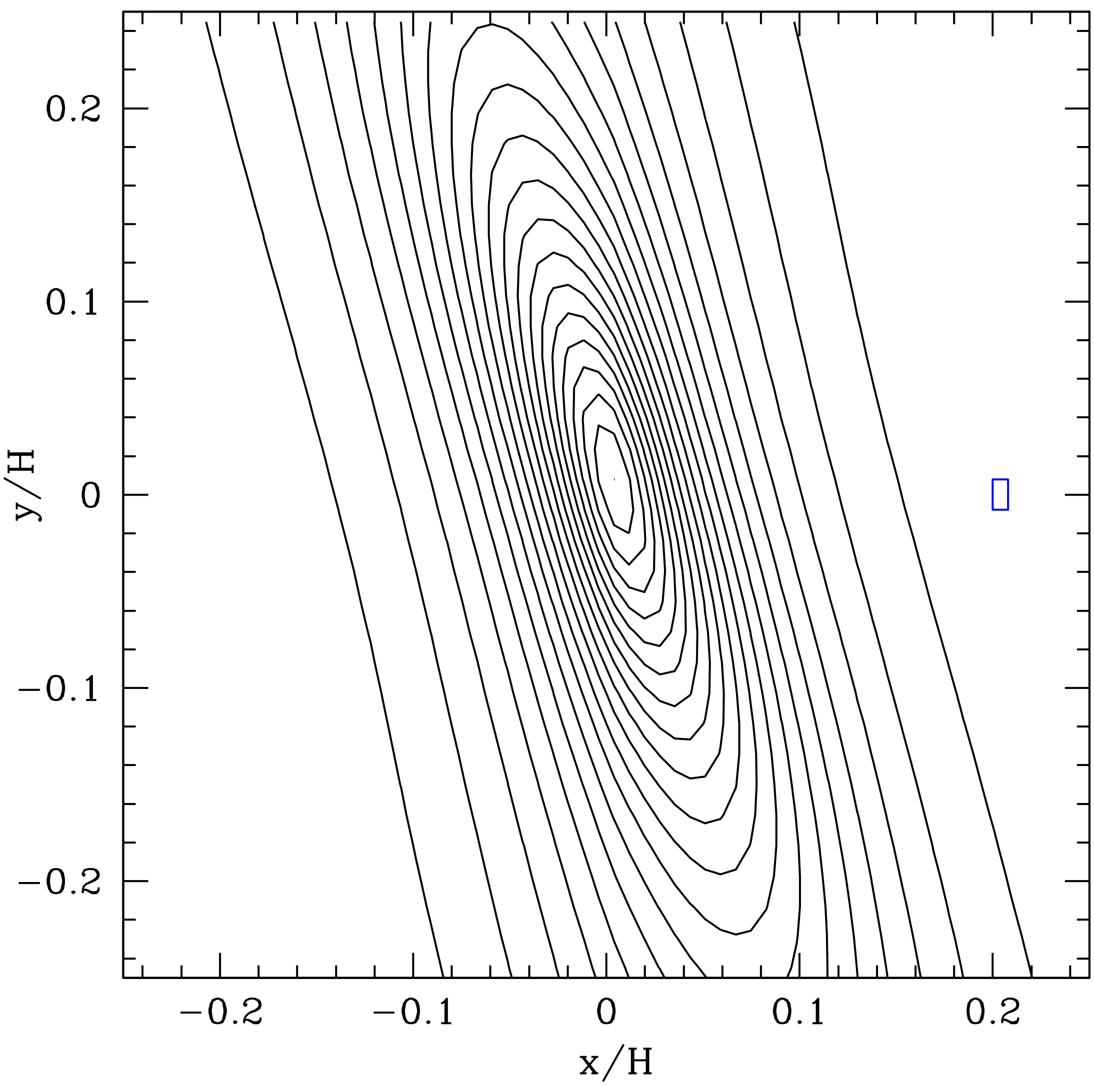
Large box = box size.

Small box = zone size.

$128 \times 200 \times 128$, $\beta_o = 400$

Small $\langle B_y \rangle \neq 0$ Boxes

Correlation function: $\langle B_i(\mathbf{x}) B_i(\mathbf{x} + \Delta\mathbf{x}) \rangle$



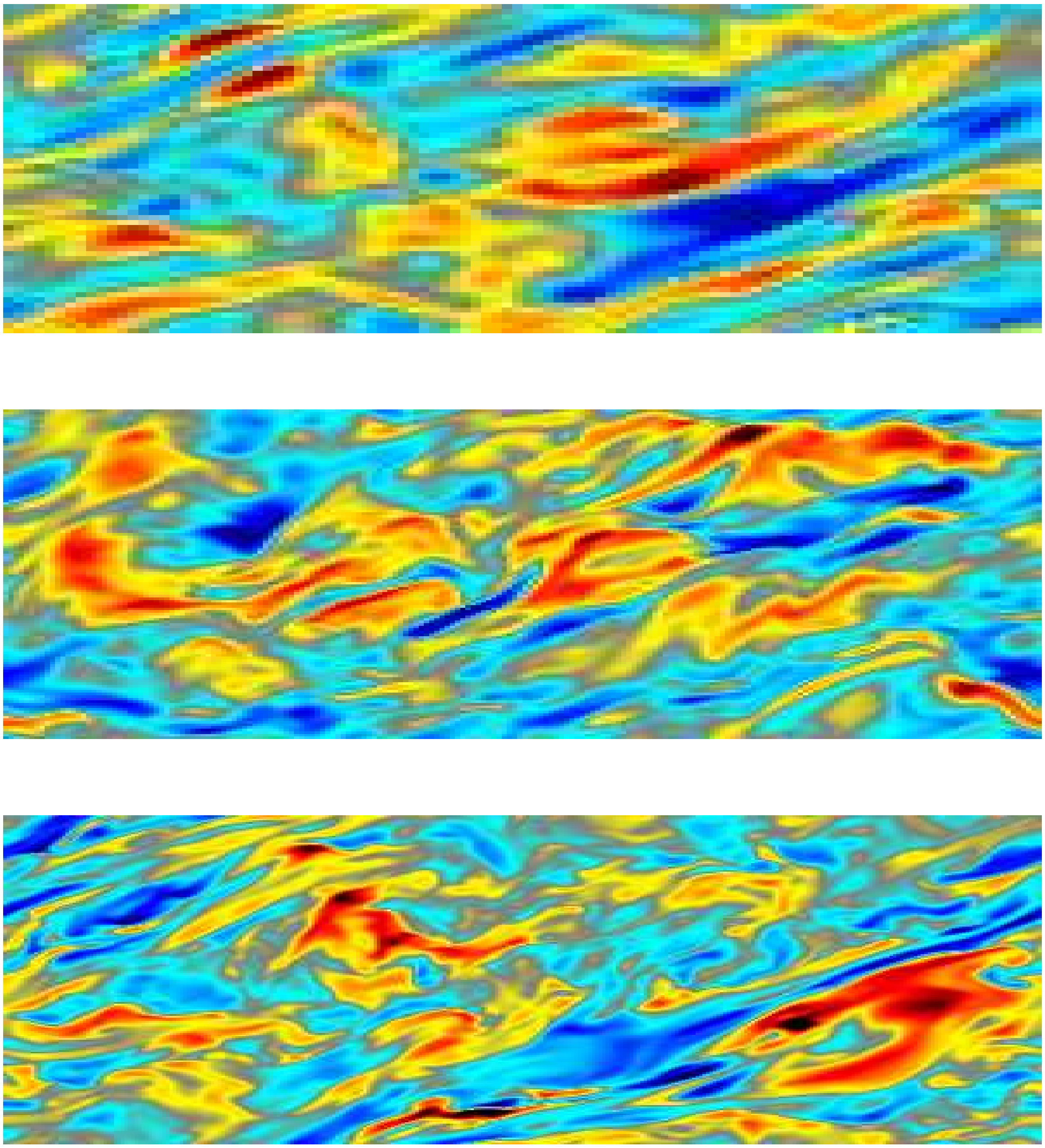
Correlation function in $\Delta z = 0$ plane.

$(\xi_{min},\ \xi_{maj},\ \xi_z) \simeq (0.07,\ 0.35,\ 0.06)H$

$\theta_{tilt} = 15\,\text{deg.}$

$\xi_{min} \simeq 8\Delta x$

Small $\langle B_y \rangle \neq 0$ Boxes



$\delta v_y(x, y, z = 0)$ at $N_x = 64, 128, 192$

Larger $\langle B_y \rangle \neq 0$ Boxes

Outcome weakly dependent on box size:

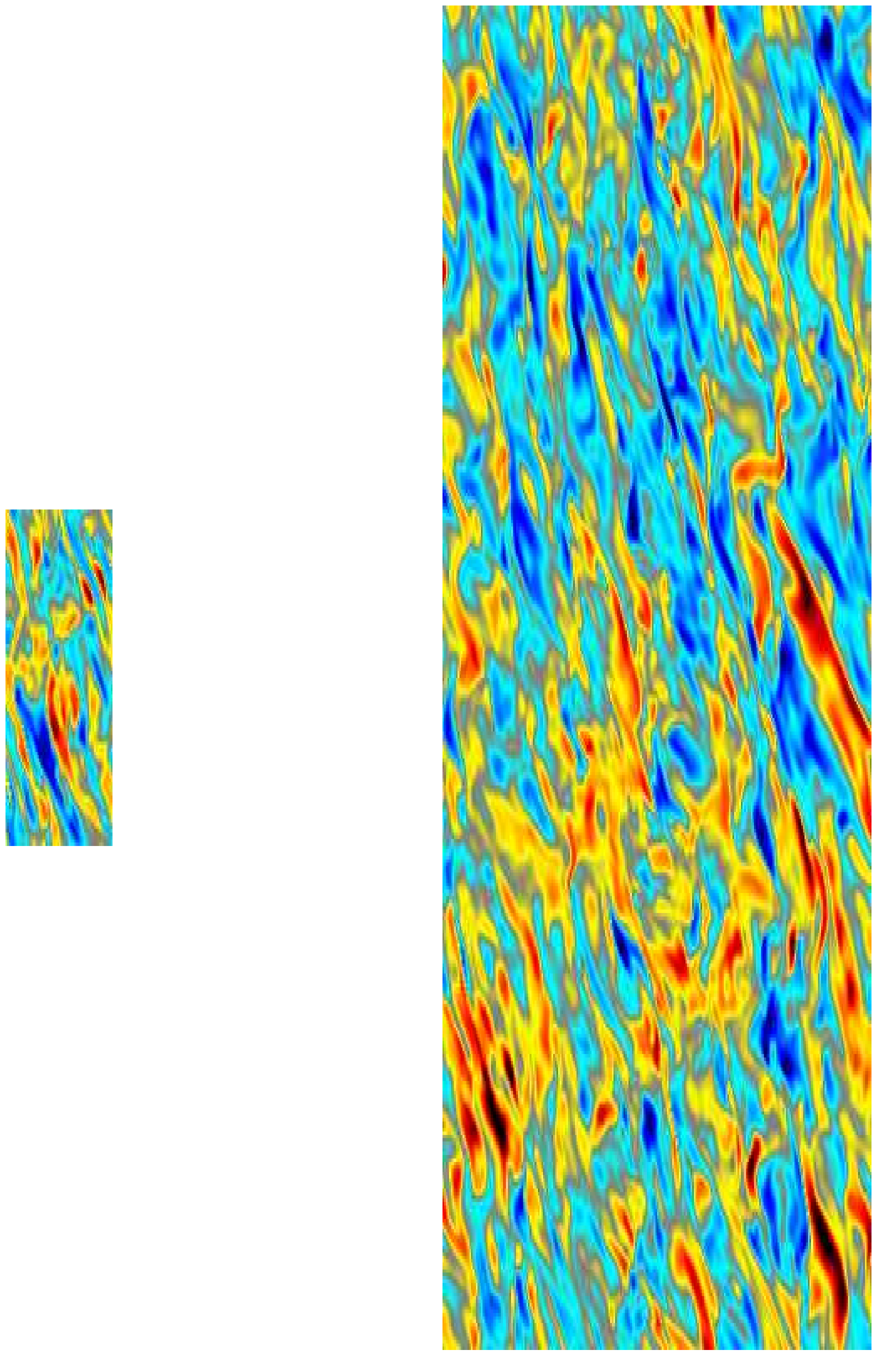
$\langle E_B \rangle (L_x, L_y, L_z)/H$ at 64 zones/H.

$\langle E_B \rangle (1, \pi, 1) = 0.024\ \rho_o c_s^2$
$\langle E_B \rangle (2, \pi, 1) = 0.027\ \rho_o c_s^2$
$\langle E_B \rangle (1, 2\pi, 1) = 0.034\ \rho_o c_s^2$
$\langle E_B \rangle (4, 4\pi, 1) = 0.037\ \rho_o c_s^2$

$\xi_{maj}(L_x, L_y, L_z)/H$ at 64 zones/H.

$\xi_{maj}(1, \pi, 1) = 0.47H$
$\xi_{maj}(2, \pi, 1) = 0.54H$
$\xi_{maj}(1, 2\pi, 1) = 0.60H$
$\xi_{maj}(4, 4\pi, 1) = 0.65H$

Larger $\langle B_y \rangle \neq 0$ Boxes



$\delta v_y(x, y, z = 0)$ at $L_x = 1, 4$

Conclusion

Orbital advection scheme for 3D boxes

- Improved accuracy and performance
- Code described in Johnson et al. (2008)
- Challenges due to nonuniform truncation error

Results for small $\langle \mathbf{B} \rangle = 0$ boxes

- Consistent with Fromang & Papaloizou
- Correlation lengths $\xi_{min}, \xi_z \sim 4\Delta x$
- Correlation angle $\theta_{tilt} \simeq 15$deg, always
  - $\Rightarrow \alpha \sim 1/(2\beta)$

Results for small $\langle B_y \rangle \neq 0$ boxes

- Possibly most relevant to disks
- $\langle E_B \rangle$ increases slightly with N_x
- $\xi_{min}, \xi_z \sim 0.06H$, $\xi_{maj} \sim 0.5H$ (at $\beta_o = 400$)
- $\langle E_B \rangle$ depends on L_x, L_y, but only for small boxes