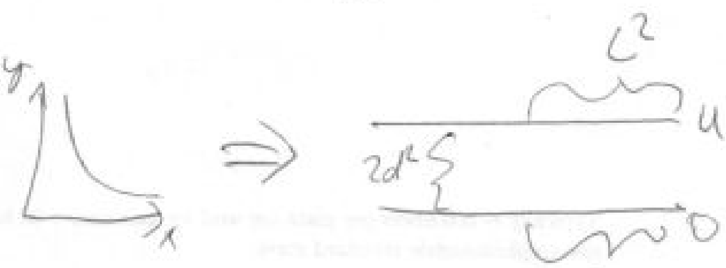
(1.) $z' = z^2$ $\qquad \left(x + i\frac{d^2}{x}\right)^2 = x^2 - \frac{d^4}{x^2} + 2id^2$

(3p) 3



$\tilde{\Phi} = -\frac{i z^2 U}{2d^2}$ $\Longleftarrow$ $\tilde{\Phi} = -i\frac{zU}{2d^2}$

(5p) 2

$\Phi = \mathrm{Re}\,\tilde{\Phi} = 2xy\frac{U}{2d^2}$

$\sigma_{\text{felület}} = E_n \cdot \varepsilon_0 = x=0 \text{ lapon} = \frac{y \cdot U}{d^2}\varepsilon_0$

(7p) 2

$Q = 2\int_{\substack{x=0\\ \text{lap}}} dF\,\sigma = h\cdot\int_0^L dy\,\frac{y\cdot U}{d^2}\varepsilon_0 = \frac{h\cdot U\cdot L^2}{d^2}\cdot\varepsilon_0$

(10p) 3 $\qquad C = \frac{Q}{U} = \frac{h\cdot L^2}{d^2}\cdot\varepsilon_0$

(2.) Határfelt.: $z=0$, h-nál $\Psi=0$

(3p) 3 $\qquad \varphi=0$, $\frac{\pi}{3}$-nál $\Psi=0$

$r=R$-nél $\Psi=0$ $\qquad m = 3\mu$ $\quad \mu = \text{egész}$

(10p) 7 $\qquad \Psi_{mnk} = J_m\left(x_{mn}\frac{r}{R}\right)\cdot\sin\left(\frac{k\pi}{h}\cdot z\right)\cdot\sin(m\cdot\varphi)$

Normálás:

(13p) 3 $\qquad \int \Psi_{mnk}^2\,dV = \int_0^R r\,dr\int_0^h dz\int_0^{\pi/3} d\varphi\,(\Psi_{mnk})^2 = \frac{R^2}{2}\cdot J_{m+1}^2(x_{mn})\cdot\frac{h}{2}\cdot\frac{\pi}{6} = N_{mnk}^2$

$$G = \sum_{\mu=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\frac{\Psi_{3\mu,n,k}(r,\varphi,z)\cdot\Psi_{3\mu,n,k}(r',\varphi',z')}{\lambda_{3\mu,n,k}\cdot N^2_{3\mu,n,k}}$$

(17p) 4 $\qquad \lambda_{3\mu,n,k} = -\frac{x^2_{3\mu,n}}{R^2} - \frac{k^2\pi^2}{h^2}$ $\;\Big|\;$ $\Phi_{\text{pont tölt}}(r,\varphi,z) = \frac{q}{\varepsilon_0}\cdot G\left(r,\varphi,z,\frac{R}{2},\frac{\pi}{6},\frac{h}{2}\right)$

(20p) 3

(3.) $\rho = \frac{q}{64\pi} \cdot r^2 \cdot e^{-r} \cdot \sin^2\vartheta$

$$Q = \int_0^\infty dr \cdot r^2 \int_{-1}^{1} d(\cos\vartheta) \int_0^{2\pi} d\varphi \, \rho = 4! \cdot \frac{4}{3} \cdot 2\pi \cdot \frac{q}{64\pi} = q$$ (2p) 2

$$p_z = \left(\iiint\right) \cdot \rho \cdot r \cdot \cos\vartheta = 0 \quad (\vartheta \text{ int} = 0)$$

(3p) 3

$$p_x = \left(\iiint\right) \cdot \rho \cdot r \cdot \sin\vartheta \cdot \cos\varphi = 0 \quad (\varphi \text{ int} = 0)$$

$p_y = 0$ ugyanígy.

$$Q'_{\alpha\beta} = \int r_\alpha r_\beta \rho \, dV \qquad Q'_{11} = \left(\iiint\right) \cdot \rho \cdot r^2 \cdot \sin^2\vartheta \cdot \cos^2\varphi = \frac{q}{64\pi} \cdot 6! \left(\frac{2}{3} + \frac{2}{5}\right) \cdot \pi$$

$$Q'_{22} = Q'_{11}$$

$$Q'_{33} = \left(\iiint\right) \rho \cdot r^2 \cdot \cos^2\vartheta = \frac{q}{64\pi} \cdot 6! \cdot \left(\frac{2}{3} - \frac{2}{5}\right) \cdot 2\pi$$

$$Q_{31} = Q_{32} = 0 \quad (\vartheta \text{ int} = 0)$$

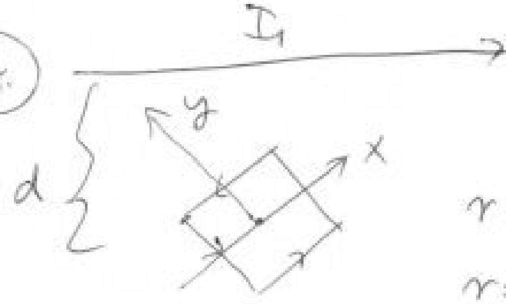
φ int

$$Q'_{12} = \left(\iiint\right) \cdot \rho \cdot r^2 \sin^2\vartheta \cdot \sin\varphi \cos\varphi = \frac{q}{64\pi} 6! \left(\frac{2}{3} + \frac{2}{5}\right) \cdot 0 = 0$$

$$Q_{\alpha\beta} = \int (3 r_\alpha r_\beta - \delta_{\alpha\beta} \cdot r^2) \rho \, dV$$

$$Q = \begin{pmatrix} 2Q'_{11} - Q'_{22} - Q'_{33} & 0 & 0 \\ 0 & 2Q'_{22} - Q'_{11} - Q'_{33} & 0 \\ 0 & 0 & 2Q'_{33} - Q'_{11} - Q'_{22} \end{pmatrix}$$ (10p) 5

4.



$$B = \frac{I_1}{2\pi}\cdot\mu_0\cdot\frac{1}{r}$$

$$r = d - \frac{a}{2}\cos\varepsilon - x\sin\varepsilon = d - \frac{a}{2} - x\cdot\varepsilon \quad \text{felső oldal}$$

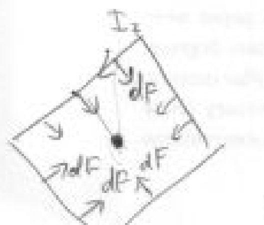
$$r = d + \frac{a}{2} - x\cdot\varepsilon \quad \text{alsó oldal}$$

$$r = d + a\cdot\overset{\sin}{\varepsilon} - y\cos\varepsilon = d + a\cdot\varepsilon - y \quad \text{baloldal}$$

5p ✓

$$r = d - a\varepsilon - y \quad \text{jobboldal}$$

$$d\underline{F} = I_2\cdot d\underline{l}\times\underline{B}$$



$$M = \int_{\text{felső old}} dF\cdot(-x) + \int_{\text{alsó}} dF\cdot x + \int_{\text{jobb}} dF\cdot y + \int_{\text{bal}} dF\cdot(-y).$$

12p ✓

$$\int_{\text{felső}} \frac{I_1 I_2}{2\pi}\mu_0\frac{dx\cdot(-x)}{d-\frac{a}{2}-x\cdot\varepsilon} = -\frac{I_1 I_2}{2\pi}\mu_0\cdot\int_{-\frac{a}{2}}^{\frac{a}{2}} dx\,\frac{x}{d-\frac{a}{2}}\left(1+\frac{x\cdot\varepsilon}{d-\frac{a}{2}}\right) =$$

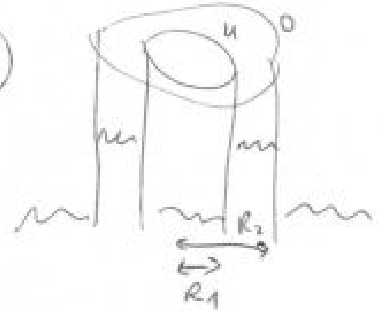
$$M_{\text{alsó}} = \frac{I_1 I_2}{2\pi}\mu_0\frac{\varepsilon\left(\frac{a}{2}\right)^2}{\left(d+\frac{a}{2}\right)^2} = -\frac{I_1 I_2}{2\pi}\mu_0\frac{\varepsilon\cdot\left(\frac{a}{2}\right)^2}{\left(d-\frac{a}{2}\right)^2}$$

15p ✓

$$M_{\text{jobb, bal}} = \int_{-\frac{a}{2}}^{\frac{a}{2}} dy\cdot y\cdot\frac{I_1 I_2}{2\pi}\mu_0\left(\frac{1}{d-a\varepsilon-y} - \frac{1}{d+a\cdot\varepsilon-y}\right)$$

$$\int\frac{y}{c-y}\,dy = -y - c\ln(c-y) \longrightarrow \text{behelyettesítés.}$$

(5.)



Energia sűrűség $= \frac{\varepsilon}{2} E^2$

$$\phi = C \cdot \ln r + D = C \cdot \ln \frac{r}{R_2}$$

(3p) 3

$$C = \frac{U}{\ln \frac{R_1}{R_2}}$$

$$E^2 = C^2 \cdot \frac{1}{r^2}$$

$$\frac{\text{energia}}{dz} = \int_0^r dr \cdot r \int_0^{2\pi} \frac{\varepsilon}{2} \cdot E^2 = \frac{\varepsilon}{2} C^2 \cdot \int \frac{1}{r} \cdot 2\pi = \cancel{\S} = \varepsilon\pi C^2 \cdot \ln \frac{R_2}{R_1} = \varepsilon\pi \cdot \frac{U^2}{\ln \frac{R_2}{R_1}}$$ (5p) 2

$$\frac{\text{töltés}}{dz} = \frac{\int D \cdot dF}{dz} = R_1 2\pi \cdot \varepsilon \cdot \frac{C}{R_1} = 2\pi \cdot \varepsilon \cdot C$$ (3p) 3

belső palástra

telep munkája

h függvényében

$$\Delta E = h \left(R_2^2\pi - R_1^2\pi\right) \cdot \frac{h}{2} \cdot \rho_{víz} g +$$

$\Delta E = ?$

$$+ (\varepsilon - \varepsilon_0) \cdot \pi \cdot \frac{U^2}{\ln \frac{R_2}{R_1}} \cdot h - h \cdot 2\pi (\varepsilon - \varepsilon_0) \cdot C \cdot U$$

$$= \frac{h^2}{2} \pi \cdot \left(R_2^2 - R_1^2\right) \rho_{víz} g + (\varepsilon - \varepsilon_0) \cdot \pi \cdot C \cdot h - 2\pi C h \cdot (\varepsilon - \varepsilon_0) \cdot U$$

minimum $\frac{d\Delta E}{dh} = 0$

$$h = \frac{\pi \cdot (\varepsilon - \varepsilon_0) \cdot C \cdot U}{\left(R_2^2 - R_1^2\right) \rho_{víz} \pi g}$$ (10p) 2